CONVECTION IN A VERTICAL LAYER OF STRATIFIED MAGNETIC FLUID

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Infrared camera visualisation of convective flows in differentially heated vertical layers with the height-to-thickness ratios of 30 and 62.5 was used to demonstrate the influence of concentration stratification caused by gravitational sedimentation of magnetic particles and aggregates on the structure of convective motion. It is found that when a density gradient is established in an isothermal layer over a period of several weeks prior to the experiment, cellular structures occur after the heating begins. They persist for a significant time even after the magnetic field is applied and vertical thermomagnetic convection rolls appear eventually mixing the fluid.

1. Introduction. Magnetic fluids are typically considered as homogeneous media due to the fact that in small cavities with a characteristic height \( d \) of about 1 cm the density of magnetic colloids remains uniform over a period of up to several hours or even days [1–3]. However, an oscillatory behaviour of flows and a subcritical character of transition between various convection regimes detected experimentally [4–6] indicate a noticeable non-uniformity of experimental fluids. The appearance of concentration gradients as a result of particle and aggregate sedimentation was investigated in detail experimentally and numerically in [7–10]. The formation of a steady equilibrium barometric distribution of particles requires a sufficiently long time of the order of \( d^2/D \), where \( D \sim 10^{-11} \text{ m}^2/\text{s} \) is the diffusion coefficient. However, since the buoyancy-driven processes require only about 1–2% density difference to be initiated, the sedimentation effects on convection can be felt long before the equilibrium barometric distribution is established [9].

The investigation of specific features of convection in nanofluids of various compositions and in various applications is a challenging heat and mass transfer problem [11–13]. In the case of magneto-polarisable nanofluid, it becomes even more complicated due to the necessity of accounting for the volumetric non-uniformity of the nanoparticle distribution caused by gravitational sedimentation and thermo- and magnetophoreses [14–16]. Theoretical studies of the influence of the resulting concentration gradients on the stability of convective flows in magnetic colloids have been reported in [4, 17–20]. In particular, the features of oscillatory convection regimes observed in spatially non-uniform nanofluids have been considered that are important both for the development of appropriate theoretical models of convection and for the practical use of nanocolloids in various sensors and heat management applications, where oscillations could have an adverse effect on the accuracy of measurements and on overall performance of the equipment. In the present work, we focus on demonstrating experimentally the strong influence of gravitational sedimentation of magnetite nanoparticles on the formation of various cellular flow patterns that might exist in non-uniformly heated magnetic nanocolloids for a considerable time.
2. Experimental setup. The schematic view of an experimental setup is presented in Fig. 1. A uniform magnetic field was created by Helmholtz coils 1. The coils had a diameter of 600 mm, thickness of 60 mm and an axial width of 50 mm. The diameter of the working zone inside the coils was 450 mm. The coils were capable of creating a magnetic field of intensity up to 35 kA/m. The uniformity of the field was controlled using a pair of compensating coils 2. To avoid overheating of the coils, which carried a current of up to 5 A, they were continuously cooled by cold water pumped through a system of copper pipes 3 inside the coils wiring. A DC power source 4 was used to feed the coils. A rectangular enclosure with magnetic fluids was placed symmetrically between the coils. The side of the enclosure facing an infrared camera 8 was maintained at room temperature, while the uniform temperature of the opposite side was controlled by a jet thermostat 6. The infrared images were transmitted to a computer 7 for post-processing and analysis.

The chamber containing a magnetic fluid had the thickness $d = 6.0$ mm, length $l = 375$ mm and the width $w = 180$ mm. Its wall facing the camera was made of 0.80 mm thick textolit with the coefficient of thermal conductivity 0.65 W/(m·K). Two metal rods, each with 4 calibration screws, were attached to this wall to prevent bulging. The opposite wall was made of 10 mm thick aluminium (the coefficient of thermal conductivity 218 W/(m·K)) plate attached to a heat exchanger. A 6 mm thick Plexiglas frame was inserted between the textolit and the aluminium walls to form the perimeter of the experimental chamber.

A kerosene-based ferrocolloid with magnetite particles of an average size of 10 nm was used as a working fluid. The fluid had a saturation magnetisation of 43 kA/m, magnetic susceptibility of 2.88, and a dynamic viscosity of $7.66 \times 10^{-3}$ Pa·s. The volume fraction of the magnetite was $C_0 = 0.1$ so that the density of the well-mixed fluid was

$$\rho_0 = \rho_k (1 - C_0) + \rho_m C_0 \approx 1300 \text{ kg/m}^3,$$

where $\rho_k \approx 830 \text{ kg/m}^3$ and $\rho_m \approx 5500 \text{ kg/m}^3$ are the densities of kerosene and magnetite, respectively.
The ratio of the coefficients of the thermal conductivity of the magnetic fluid and aluminium was \((0.55 - 0.87) \times 10^{-3}\), and of the magnetic fluid and textolit 0.18–0.29.

The flow was visualized using an infrared camera FLIR-CEDIP Titanium with the matrix resolution 640×512 pixels and the detectable temperature ranging from −20 to 3000°C. The experimental temperature range was 5 – 60°C. Temperatures were determined with an accuracy of ±0.1°C. The infrared images were post-processed using the software package Altair.

3. Results and discussion.

3.1. Homogeneous fluid. The problem of primary convection flow stability in a vertical layer of a homogeneous magnetic fluid side-heated and exposed to a transverse uniform magnetic field was investigated experimentally in [21–23] and theoretically in [24, 25]. It was shown that in this case an additional thermo-magnetic mechanism of heat transfer occurred: a cooler and hence stronger magnetised fluid tends to flow toward regions with a stronger magnetic field. This leads to the formation of stationary vertical thermo-magnetic rolls and obliquely propagating thermo-magnetic waves. Fig. 2 presents a schematic diagram and an infrared photograph of primary thermogravitational flow of a homogeneous fluid.

The fluid raises along the hot (aluminium) wall and descends along the opposite cold (textolit) wall. Light (red) and dark (blue) areas at the top and bottom parts of the cavity show the presence of respective hot and cold zones in the enclosure caused by the temperature stratification existing in the enclosure due to its finite vertical size [26–28].

When a transverse magnetic field is applied, thermomagnetic convection rolls arise in a homogeneous magnetic fluid and aline with the primary shear flow, as shown schematically in Fig. 3. Each roll is seen as a pair of light and dark stripes stretching vertically over the most of the layer in the presented infrared image: the warm fluid from the back aluminium wall impinges the textolit wall along the light stripe, while the cool fluid flows away from the textolit wall along the dark stripe.

![Fig. 2. Primary thermo-gravitational shear flow in the absence of a magnetic field: schematic view (left) and an infrared image of a textolit wall (right). The temperature difference between the textolit and aluminium walls is 40°C.](image-url)
3.2. Fully stratified fluid with no magnetic field. If the magnetic fluid remains isothermal and at rest for a substantial period of time (from several days to few weeks) prior to the start of an experiment, the gravitational sedimentation of magnetite particles leads to its vertical density stratification. The flow patterns in such a vertically stratified layer of ferrocolloid differ significantly from their counterparts observed in homogeneous fluids. Namely, the flow domain breaks into several cells with sharp horizontal boundaries as seen in Fig. 4. The flows within each cell is qualitatively similar to those existing in a cavity filled with a uniform fluid.

More specifically, the series of images presented in Fig. 4 was taken after the enclosure filled with a magnetic fluid was left laying horizontally on its aluminium wall for a month so that the vertical concentration gradients had established due to the particle sedimentation. This process is modelled by the following non-dimensional advection-diffusion equation

$$\frac{\partial C}{\partial t} = V \frac{\partial C}{\partial z} + \frac{\partial^2 C}{\partial z^2},$$

(1)

where the vertical coordinate $z$ and the time $t$ are scaled using the layer thickness $d$ and diffusion time $d^2/D$, respectively. The value of the non-dimensional separation parameter $V = vd/D \approx 0.048$ appearing in Eq. (1) was computed using the independently estimated diffusion coefficient $D \approx 1.9 \times 10^{-11} \text{ m}^2/\text{s}$ and the Stokes rate of sedimentation $v \approx 1.52 \times 10^{-10} \text{ m/s}$ [9]. Since initially the fluid was homogeneous and because the magnetic particles could not penetrate through the enclosure walls, Eq. (1) was solved subject to the following non-dimensional initial and boundary conditions

$$C(z,0) = 1, \ 0 \leq z \leq 1, \ CV + \frac{\partial C}{\partial z} = 0 \text{ at } z = 0, 1.$$  

(2)
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Fig. 4. Transition from multi- to single-cell thermo-gravitational motion in the absence of magnetic field. The temperature difference between the textolit and aluminium walls is 28°C. The snapshots are taken 10, 17, 26, 44, 379 and 400 min after the start of heating. The experimental chamber has a vertical aspect ratio of 62.5. The vertical black lines on the images are the shadows of externally attached ribs used to prevent the observation wall from bulging out.

A standard method of separation of variables then has resulted in

$$C = \frac{Ve^{-Vz}}{1 - e^{-V}} + 16\pi^2V^2 \sum_{n=1}^{\infty} \frac{n(1 - (-1)^n e^{\frac{Vz}{V^2}})}{(V^2 + 4\pi^2n^2)^2} \exp\left\{ -\frac{V^2 + 4\pi^2n^2}{4} t - \frac{Vz}{2} \right\}$$

$$\times \left( \sin(n\pi z) - \frac{2\pi n}{V} \cos(n\pi z) \right).$$  \hspace{1cm} (3)

The first term in expression (3) is the well-known steady equilibrium barometric distribution of solid particles, while the first term in the sum represents the slowest decaying mode that gives the estimate of the characteristic time required to reach this equilibrium

$$t^* = \frac{4}{V^2 + 4\pi^2d^2/D} \approx \frac{d^2}{\pi^2D} \approx 54 \text{ hours.}$$ \hspace{1cm} (4)
Therefore, prior to flow observations, the equilibrium distribution of particles shown by the solid line in the left panel of Fig. 5 was achieved and a maximum separation of 4.8% in concentration between the fluid layers located near the top and bottom plates was established. The corresponding fluid density stratification is shown in the same figure by the dashed line. At the start of the experiment, the cavity was turned to its upright position so that the layer of the fluid with the larger density started sliding down along the aluminium wall displacing upward the less concentrated fluid near the textolite wall, thus creating a vertical density stratification. At the same time, the heating of the aluminium wall was started so that the local thermal expansion of the fluid led to the appearance of the upward buoyancy force. In turn, the competition between the thermal buoyancy force and the gravitational pool has resulted in instability of the expected parallel shear flow, which exhibits itself in the form of a large number of horizontal rolls with a wavelength of the order of the layer thickness (6–12 mm). This is seen in the top left snapshot in Fig. 4 taken 10 minutes after the cavity was rotated from its initial horizontal position. As time progressed, the number of the observed horizontal rolls started to reduce indicating the flow regularization due to fluid mixing. This is seen in the top middle snapshot: in 17 minutes after the start of the experiment only six horizontal rolls were still observed even though their boundaries remained irregular. In 26 min, the formation of larger cell rather than roll-like structures with sharp horizontal boundaries was observed, with the flow within each cell being qualitatively similar to the primary unicellular flow illustrated in Fig. 2. The process of dissolution of small rolls continued towards the bottom of the layer for about 5.8 hours, eventually resulting in a 3-cell structure with very well defined horizontal boundaries shown in the bottom left image in Fig. 4. Such cellular structures are very similar to those observed earlier in the vertically stratified layers of saline solutions heated from the side [29, 30]. The fluid mixing, however, continued to erode individual cells (see the bottom middle image) until a unicellular flow was eventually established in about 6.7 hours after the start of the experiment so that the bottom right image is qualitatively identical to that shown in Fig. 2 for a homogeneous fluid.

3.3. Partially stratified fluid in a magnetic field. In a separate experiment, the same enclosure was used, but it was was rotated by 90° in its own vertical plane so that the effective width of the layer increased at the expense of its height. Instead of laying horizontally, it was kept vertical for four weeks prior to the experiment. With this configuration, the separation parameter appearing in Eq. (1) became \( V = \frac{v w}{d} \approx 1.44 \) and the characteristic equilibrium time in-
Fig. 6. Cellular structures with thermo-magnetic convection rolls arising in the magnetic field $H=35$ kA/m applied perpendicularly to the plane of the photos (top two images) and the primary thermo-gravitational flow restored 55 min after the magnetic field was switched off. The temperature difference between the textolit and aluminium walls is $23^\circ C$. The snapshots are taken 64, 69 and 135 min after the start of heating. The experimental chamber has a vertical aspect ratio of 30.

creased to $t^* \approx 4.6 \times 10^4$ hours. Therefore, as follows from Eq. (3), in this case, the vertical distribution of magnetic particles was far from equilibrium and only was non-uniform in two regions near the top and bottom edges of the enclosure, each occupying about 12% of the total height (see the right panel in Fig. 5).

At the start of the experiment, the aluminium wall of the enclosure was cooled relative to the textolit one and a uniform magnetic field was applied normally to the plane of the fluid layer. As a result, a cellular flow pattern depicted in the top photograph in Fig. 6 was driven. The locations of the horizontal convection cell boundaries agree well with those of the edges of the density-stratified regions pre-
dicted computationally. The regular vertical thermo-magnetic roll pattern similar
to that shown in Fig. 3 is clearly visible in the middle part of the layer, where the
fluid remains homogeneous, but it is disrupted in the top and bottom stratified
regions.

Since the fluid layer remained vertical, no initial gravity flow caused by a
sudden rotation of the density-stratified fluid was generated. As a result, the
cell instabilities described in Section 3.2 are not visible in Fig. 6: a well-defined
two-cell flow structure was established from the start and existed for 1.5 hours,
see the top two photographs in Fig. 6. The smaller top cell, where the primary
up-down thermogravitational flow remains relatively weak due to its small vertical
extent, contains inclined, wavy and spiral thermo-magnetic rolls typically observed
in the Rayleigh-Bénard configuration. In contrast, in the larger bottom cell with
a strong up-down flow component, predominantly vertical and slightly inclined
thermo-magnetic rolls and waves [23–25] were observed similar to those in Fig. 3.
As time progressed, the structures in the smaller top cell remained relatively stable,
while the rolls in the larger bottom cell underwent a series of secondary instabilities
eventually leading to the formation of irregular unsteady patterns illustrated in
the middle image in Fig. 6 and evidencing a higher effective supercriticality of the
conditions in the larger cell.

The experiments have also demonstrated that the fluid mixing enhancement
caused by thermomagnetic convection significantly accelerates the process of fluid
homogenisation so that the cellular structures get washed out within a much
shorter time if compared with the situation discussed in Section 3.2 when no
magnetic field was applied. In the experiments described in the current section,
the magnetic field was applied for 80 min from the start of observations of the
 unicellular thermogravitational flow shown in the bottom image in Fig. 6, which
evidences that fluid uniformity takes about two hours to establish compared to
almost 7 hours in the absence of magnetic convection.

4. Conclusions. For the first time, cellular flow structures in a vertically
stratified magnetic colloid were observed experimentally. Even though they are
transient in their nature, they remain stable for a substantial time (up to several
hours) until eventually destroyed by convective mixing. The obtained infrared
images of such cellular structures demonstrate that they can lead to the formation
of local hot spots, and hence the likelihood of their existence must be kept in mind
when comparing theoretical and experimental results as well as when designing
practical heat applications.

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