ON THE DOUBLE LAMINAR BOUNDARY LAYER
AT MHD ROTATION OF THE CONDUCTING LIQUID
IN THE CYLINDER

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By numerical modelling of a three-dimensional axially symmetric flow of a conducting liquid in a cylinder of finite length driven by the action of a rotating magnetic field it is found that under certain conditions a double laminar boundary layer appears in the distribution of the azimuthal velocity along the length of the cylinder near to its end faces in a place of end-wall vortex localization.

Introduction. Ingham [1] investigated the behaviour of a viscous conducting liquid placed in a cylinder of final length. The cylinder rotated with a constant angular velocity in a magnetic field parallel to the axis of the cylinder. The flow looked like “rigid body” rotation, while small perturbations were induced by giving the top and/or bottom surfaces of the cylinder a small additional angular velocity. The author has shown that under certain conditions at the cylinder end faces there appears a double structure of the vertical boundary layer. In Kapusta et al. [2] experiment, local characteristics of a conducting liquid flow were defined in a cylinder exposed to a rotating magnetic field. Azimuthal velocity measurements along the length of the vessel have shown that near the end faces there appears a double laminar boundary layer, and the thickness of the sub-layer next to the end face is very small (∼0.01), while the thickness of the second (internal) layer reaches 0.3 of half-length of the vessel for small Hartmann numbers, decreasing with the increase of this parameter.

The present paper demonstrates the occurrence of a double laminar boundary layer near end walls of the cylinder by modelling of a three-dimensional flow of conducting liquid driven by the rotating magnetic field in the cylinder of a limited length.

1. Formulation of the problem. In [3, 4], a three-dimensional axially symmetric flow of a conducting liquid is studied initiated by a coaxially rotating magnetic field in a cylinder of limited length. The mathematical model of the problem is described by the following system of dimensionless equations

\[
\begin{align*}
\frac{\text{Re}_\omega}{h} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \psi \right) \frac{\partial \varphi}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left( r \varphi \right) \right] &= L \varphi + \frac{H a}{r} \frac{\partial^2 \varphi}{\partial z^2} - 2 \Delta \varphi + \frac{1}{r} \frac{\partial^2 \varphi}{\partial z^2} + 4(p-1) \frac{1}{r} \frac{\partial^2 \varphi}{\partial r^2} \right]
\end{align*}
\]

(1)

with the boundary conditions

\[
\begin{align*}
v_{\varphi} |_{r=0} &= \infty, \quad v_{\varphi} |_{\Gamma} = 0, \quad \frac{\partial v_{\varphi}}{\partial z} |_{z=0} = 0 \quad \text{(the condition of flow area symmetry)}, \\
\psi_{\varphi} |_{r=0} &= \infty, \quad \psi_{\varphi} |_{\Gamma} = 0, \quad \frac{\partial \psi_{\varphi}}{\partial n} |_{\Gamma} = 0.
\end{align*}
\]

(2)
where $\text{Ha}_{\text{ac}} = \frac{Ha}{\sqrt{2}}$ is the Hartmann number based on the active value of induction; $\text{Re}_\omega = \frac{\omega R_0^2}{(pu)}$ is the Reynolds number defined by the relative velocity of the boundary area motion in a magnetic field; $v_0 = \frac{\omega R_0}{p}$ is the characteristic value of velocity; $\psi_i$ is the $\varphi$-component of the velocity vectorial potential ($\nabla = \text{rot} \psi$); $h = Z_0/R_0$; $p$ is the number of pole pairs (an order of rotary symmetry); $\omega$ is the angular velocity of magnetic field rotation; $R_0$ is the characteristic size of the flow field; $\Gamma$ is the internal cylinder surface;

$$\Delta = \frac{\partial^2}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{h^2} \frac{\partial^2}{\partial z^2}; \quad L^2 = \Delta - \frac{1}{r^2}.$$  

The set of equations (1) with the boundary conditions (2) is solved by iterations using the Galerkin method at each step of iteration. At the $r$-step of iteration, from the first equation of set (1) based on the known value $\psi_{i-1}$ a value was found for the azimuthal velocity $v_i$. Then the known values $\psi_{i-1}$ and $v_i$ were substituted into the second equation of set (1), from which the value $\psi_i$ was found. Iteration began with the initial value $\psi_0 = 0$. The computational scheme of a method looks as follows:

$$L v_i - \text{Ha}_{\text{ac}}^2 r^{2p-2} v_i - \frac{\text{Re}_\omega}{h} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \psi_{i-1}) \frac{\partial \psi_i}{\partial z} - \frac{\partial \psi_{i-1}}{\partial z} \frac{1}{r} \frac{\partial}{\partial r} (r \psi_i) \right] = -\text{Ha}_{\text{ac}}^2 \varphi^2 p^{-1},$$

$$L^2 \psi_i - \text{Ha}_{\text{ac}}^2 r^{2p-2} \left[ 2 \Delta \psi_i + \frac{\partial^2 \psi_i}{\partial z^2} + 4 (p-1) \frac{1}{r} \frac{\partial}{\partial r} (r \psi_i) \right] - \frac{\text{Re}_\omega}{h} \left[ \frac{\partial (L \psi_{i-1})}{\partial z} \frac{1}{r} \frac{\partial (r \psi_i)}{\partial z} - r \frac{\partial}{\partial r} \left( \frac{L \psi_i}{r} \right) \frac{\partial \psi_{i-1}}{\partial z} \right] = 2 \frac{\text{Re}_\omega}{h} \frac{v_i}{r} \frac{\partial \psi_i}{\partial z}.$$

Other schemes of iterations are possible as well, however, direct numerical experiment has shown that the scheme of iteration used above exhibits the best convergence for the parameters.

The values $v_\varphi$ and $\psi_\varphi$ were defined by decomposition in series on two full systems of functions satisfying the boundary conditions (2). The extent of the iteration convergence was estimated with reference to relative root-mean-square errors for $v_\varphi$ and $\psi_\varphi$.

2. Numerical results. Numerical experiment made it possible to study the driven spatial flow patterns and to track their evolution at the change of the flow parameters. With small values of Hartmann numbers ($\sim 1$), the determining influence on the flow pattern is rendered by the relative height of a vessel. At a small relative height, one intensive end-wall vortex appears at the half of the vessel’s length. If the relative height increases, this vortex is pushed towards the end-wall, while some weak wall-adjacent meridional vortices with alternating direction of rotation appear in the central zone of the vessel. These vortices arise by detaching from the end-wall vortex, separating from it in pairs. With the increase in relative vessel height, the amount of these small vortices increases; in the middle part of a rather long vessel a pattern of Taylor-type vortices appears (Fig. 1), and the relative sizes of the vortices achieve the corresponding sizes of Taylor vortices, which appear in an infinitely long cylinder [5, 6]. With the increase of Hartmann numbers ($\sim 15–25$), the centre of the end-wall vortex shifts downwards at distances 0.05–0.04 of half-length of the vessel from the end wall, being slightly displaced towards the vessel axis at the increase of the cylinder relative height.

The calculation of the radial profiles of the azimuthal velocity has shown that the character of the velocity distribution practically remains invariable along the entire height of the cylinder; the value of the velocity does not vary, and only...
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Fig. 1. Three-dimensional hydrodynamic pattern: \( p = 1, \ h = 10, \ Re_\omega = 500, \ Ha_{ac} = 14.2 \).

Fig. 2. Radial (a) and axial (b) profiles of the azimuthal velocity. The parameter values are the same as in Fig. 1.

near to the end-face the velocity values decrease. The velocity profiles at small Hartmann numbers have a parabolic shape, resembling the profile of “rigid body” rotation when the Hartmann numbers increase (Fig. 2a). The maximum of the velocity is shifted thus towards the lateral surface of the cylinder.

The distribution of the azimuthal velocity along the vessel height has some specific features. For small Hartmann numbers, the velocity values remain constant along a sufficiently large part of the cylinder’s height, decreasing towards end-faces. The thickness of the boundary layer constitutes 0.35–0.38 of half-length of the vessel. For sufficiently large Hartmann numbers near the cylinder end-faces in the place of end-wall vortex localization there is a double laminar boundary layer (Fig. 2b), and the thickness of a sub-layer adjoining the end-face approximately coincides with the distance from the center of the vortex to the end wall.
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and constitutes 0.05–0.06 of half-length of the vessel. At a small relative height of the cylinder, the thickness of the inside layer reaches \( \sim 0.3 \) of half-length of the vessel, decreasing down to \( \sim 0.2 - 0.15 \) with the increase in relative height. It is necessary to note that the double laminar layer arises at sufficiently large Hartmann numbers in the area of end-wall vortex location, being displaced along with the vortex when the flow parameters vary. During displacement in the radial direction from the vortex center to its periphery, this effect gets weaker, and outside of the stream function, corresponding to a value of \( \sim 0.3 \) of the maximum value at the center of the vortex, at the cylinder end-face, there appears an ordinary laminar boundary layer. The numerical experiment has shown that the double laminar boundary layer at the cylinder end-faces arises due to the presence of sufficiently intense end-wall vortices. If only the first equation of set (1) is solved at \( \psi_0 = 0 \), the double laminar layer at the end-faces does not arise; the calculations have yielded an ordinary boundary layer.

3. Conclusions. Calculations of the azimuthal velocity of the flow have revealed a characteristic regulation that under certain conditions, at the location of the end-wall vortex, a double laminar boundary layer appears. Parameters, at which this phenomenon takes place, have been determined. It is shown that the double laminar boundary layer arises in the zone of end-wall vortex location, while outside this zone the double structure of a boundary layer disappears and it is replaced by a unary boundary layer.

This study has allowed to expand our knowledge of the hydrodynamic phenomena arising at MHD rotation of a conducting liquid in a cylindrical vessel.

REFERENCES


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