ACCURACY OF THE DIPOLE APPROXIMATION OF EXTERNAL MAGNETIC
FIELDS IN MHD MACHINES

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In designing the shielding containers that house k elements of MHD machines and other
electrical equipment (sources), one must resort to mathematical or physical simulation of the
fields of the individual sources. If such models are constructed with the required accuracy,
they simplify the summation of the fields of the individual sources inside the containers and
allow us to calculate the shielding functions by analytical methods. The mathematical models
employed in [1, 2] are dipoles with arbitrary orientation and distribution in space D(P, r);
each of them is determined uniquely by six parameters: the components of the dipole moment
over the axes (instead of the three components of the dipole moment over the axes, its modu-
lus P and the orientation angles χ, η can be determined) and the coordinates r at its
center. In the study of variable electromagnetic fields, the dipole moment P is a function
time and is written in the quasistationary approximation

\[ P = P_0 \hat{r}_s \exp(i\omega t - \alpha), \]

where \( \hat{r}_s \) is the unit vector, \( \omega \) is the angular frequency of the change in the field, \( \alpha \) is a
coefficient, and \( \hat{z} \) is a coordinate.

In determining the dipole parameters used to replace the external field of the source,
a comparison is made between the intensities or scalar potentials of the external field of
the source \( H_s(\psi_s) \) and the dipole \( H_d(\psi_d) \), which are determined on one of the coordinate sur-
faces \( S(x_1, x_2, x_3) \) (the analysis is performed in an orthogonal curvilinear coordinate system \( x_1, x_2, x_3 \) at a certain number of uniformly distributed points \( I (i = 1, 2, \ldots, N) \).

The potentials \( \psi_s \) and \( \psi_d \) in the coordinate system \( x_1, x_2, x_3 \), and \( x_3 \) are determined by the relations

\[ \psi_s = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} a_{nm} F_{nm}(x_1) \cdot Y_{nm}(x_2, x_3); \]

\[ \psi_d = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} D_{nm}(P, r_0) \cdot F_{nm}(x_1) \cdot Y_{nm}(x_2, x_3), \]

where \( a_{nm} \) are coefficients expressing the source field, \( D_{nm}(P, r_0) \) are functions through which
the dipole field is determined, \( Y_{nm}(x_2, x_3) \) are surface harmonics, and \( F_{nm}(x_1) \) are distance
functions of the second kind \( [F_{nm}(0) = \infty, F_{nm}(\infty) = 0] \).

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By the vector $r_0$ we mean a known collection of coordinates $x_{10}$, $x_{20}$, and $x_{30}$.

By equating Eqs. (2) and (3), the relationship between the coefficients of these series is found in the form of algebraic equations

$$a_{nm} = \Phi_{nm}(P, r_0),$$

which make it possible to solve the following problems:

a) from the known coefficients $a_{nm}$ find the parameters of the approximating dipole $P_{x_1}$, $P_{x_2}$, $P_{x_3}$ (or $P_0$, $\chi$, $\eta$), $x_{10}$, $x_{20}$, $x_{30}$.

b) from the known dipole parameters determine the coefficients $a_{nm}$.

The algebraic equations (4) are nonlinear, and their number depends on the number of terms retained in the series. In engineering calculations, when the expansions are limited to a finite number of terms, the equations can be linearized by one of the well-known methods, the Newtonian method [3], for example, and normalized by the least squares method [4].

Investigation of the system of equations (4) indicates that it can be used to estimate the number of terms $a_{nm}$ to be retained when performing numerical calculations of the fields with the required accuracy.

For the numerical calculations one can use the notation of the equation of type (4) in a spherical coordinate system $R$, $\theta$, $\phi$ [1]:

$$a_{nm} = \frac{P_r R_0^{n-1}(-1)^n}{R_i^{n+2}} \left\{ \frac{m P_{x_n} (\cos \theta_0) \cdot \sin \chi (\cos m \varphi_0 + \sin m \varphi_0) + \frac{m}{\sin \theta_0} P_n^m (\cos \theta_0) \times \right.$$

$$\left. \times \cos \chi \cdot \cos \eta (\sin m \varphi_0 \cos m \varphi_0) \right\} + \frac{m}{\sin \theta_0} P_n^m (\cos \theta_0) \left[ \frac{\sin \chi}{\sin \eta (\cos m \varphi_0 + \sin m \varphi_0)} \right]

\times \cos \chi \cdot \cos \eta (\sin m \varphi_0 \cos m \varphi_0)

\left. + \frac{m}{\sin \theta_0} P_n^m (\cos \theta_0) \right\},$$

(5)

where $R_0$, $\theta_0$, $\varphi_0$ are the coordinates of the dipole center, $\chi$, $\eta$ are the orientation angles of the dipole moment, $P_n^m(\cos \theta_0)$ are associated Legendre functions, $R_i$ is the radius of the surface in which the field is investigated, and $\varepsilon_m$ is the Neumann function: $\varepsilon_0 = 1$, $\varepsilon_m = 2$ ($m > 0$).

The parameters in Eq. (5) vary within the following limits:

$$R_0 \in [0 \to R_i], \theta_0 \in [0 \to 180^\circ], \varphi_0 \in [0 \to 360^\circ],$$

$$\eta \in [0 \to 180^\circ], \chi \in [0 \to 360^\circ].$$

The algebraic equations (5) were calculated on a computer for $n$, $m = 1, 2, 3, 4, 5$, over the entire range of the parameters $R_0$, $\theta_0$, $\varphi_0$, $\eta$, $\chi$.

The damping of the following harmonics was investigated in the calculations: dipole ($n = 1; m = 0, 1$), quadrupole ($n = 2; m = 0, 1, 2$), octupole ($n = 3; m = 0, 1, 2, 3$), sixteenth ($n = 4; m = 0, 1, 2, 3, 4$), and thirty-second ($n = 5; m = 0, 1, 2, 3, 4, 5$), which henceforth will be called first, second, etc.

The following were established in the calculations.

1. If the dipoles are placed in a sphere of radius $R_0 \leq 0.05R_i$, only the first harmonics can be used in the calculation. Inside this sphere the dipoles can be shifted to any point and be located at the origin of the coordinate system. In this case the summation of the dipole moments is algebraic.

2. If the dipoles are in the region $0.05R_i < R_0 < 0.1R_i$, the first and second harmonics must be included in the calculations. Summation of the corresponding harmonics of the dipole fields is performed without including the orientation angles of the dipole moments.

3. When the dipoles are in the region $0.1R_i < R_0 < 0.2R_i$ the first three harmonics must be included in the calculation. Summation of the dipole fields should include all the dipole parameters (the orientation angles $\chi$, $\eta$ and the angular coordinates of the center $\theta_0$, $\varphi_0$).

4. If the dipoles are in the region $R_0 > 0.2R_i$ all five harmonics must be included in the calculation. Special methods should be used for summation of the dipole fields.
The present investigation leads to the following conclusions.

1. In investigating the source field bounded by a convex surface with a diameter $d$, where $d$ is the maximum distance between two of its points, the model of the source field at distances $R \gg 10d$ can be represented by one central dipole.

2. In investigating a source field at distances $10d > R > 5d$, the model of the source field can be represented by several central dipoles of different orientations.

3. In investigating the source field at distances $R < 5d$, the model of the field should be composed of dipoles arbitrarily oriented and distributed in space.

LITERATURE CITED


ERRATA

1. In the article by N. G. Taktarov, "Disintegration of streams of magnetic fluid," Magnetohydrodynamics (Magnitnaya Gidrodinamika), No. 2, 156-158 (1975), in the first term in the right-hand side of Eq. (12) instead of $(\mu - 1)I_m(kr_0)$ it should read $(\mu - 1)^2 I_m(kr_0)$. In the numerator of the first term in the right-hand side of Eq. (13) instead of $I_0(kr_0)$ it should read $I_0(kr_0)$. The results of the calculations based on these equations should be corrected as follows: for $H = 10$ Oe, $(k_c r_0)^{-1} = 1.11$ and $\lambda_m = 10.1$ cm; for $H = 15$ Oe we have 1.25 and 11.3 cm, and for $H = 20$ Oe we have 1.43 and 13 cm. The computer calculations were carried out by S. I. Martynov at the request of the author.

2. In the article by O. A. Kukainis, Magnetohydrodynamics (Magnitnaya Gidrodinamika), No. 4, 500-502 (1977), in Eq. (4) and the two unnumbered expressions following it instead of 147 read 7 $\Pi^6$.

In the same article, in the sixth line from the bottom of p. 501, the expression for $j$ should read $j \propto 0.75 \cdot 10^{-8}$ H/m$^2$; on the same page, in the fourth line from the bottom instead of 0.17 read 0.37; the same reading applies to the top line of p. 502.

With all of the corrections we then obtain $\lambda \propto 157$ sec$^{-1}$ and $t_d \propto 6 \cdot 10^{-8}$ sec.