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MAGNETOHYDRODYNAMIC SCALING
OF GEODYNAMO AND A PLANETARY
PROTOCORE CONCEPT

S.V. Starchenko$^1$, Y.D. Pushkarev$^2$

$^1$IZMIRAN, Troitsk, Moscow region, 142190 Russia
$^2$IPGG RAN, St.-Petersburg, 199034 Russia
e-Mail: sstarchenko@mail.ru

For fast rotating planets/moons, we derive magnetohydrodynamic (MHD hereafter) scaling laws in the limit of negligible molecular diffusivity, viscosity and magnetic diffusivity effects. Our new scaling law equates the typical magnetic/kinetic energy ratio to the square of a corresponding scales’ ratio. In the Earth’s core, magnetic energy dominates over kinetic one and a typical magnetic field is proportional to the third root of the buoyancy flux, which initiates convection, while the field is independent on conductivity and angular rotation rate. The same scaling has been obtained recently via compilation of numerical simulations. Besides, here we present new scaling laws for typical velocity, buoyancy acceleration, sinus of the angle between magnetic and velocity vectors, electromagnetic and hydrodynamic scales. The established scaling laws allow to estimate typical geodynamo values and long-term geomagnetic consequences for different evolution scenarios of a core-mantle system. The currently accepted scenario with the inner solid core crystallizing from the liquid core provides a too small value of the geomagnetic field for more than 3 billion years after the formation of the liquid core. Since this is inconsistent with the available paleomagnetic records, we suggest another scenario with a solid protocore, which occupies almost the whole core of the just formatted Earth. This protocore is being slowly melted under the surface influence of the overheated liquid core, which grows up to its contemporary size, while the solid core is a small relic of the protocore. Such protocore concept resolves the problem of energy source for geodynamo and for plume activity in the mantle. In case of validity of this concept, the mantle should be supplemented with the silicate material from the protocore with a primitive isotope composition of lead, but which cannot be the result of the liquid core crystallization. An additional argument to the concept validity could be the primitive isotope composition of lead in combination with primary helium enriched by isotope $^3$He. Applications of the protocore concept to the terrestrial planets are discussed.

Introduction. In section 1 of this paper, we consider scaling laws relating unknown typical MHD values hidden in an Earth’s type planetary core with known or reasonably estimated primary values, such as the thickness of the liquid core $H$, its density $\rho$, angular rotation rate $\Omega$, conductivity $\sigma$, and the buoyancy flux power $F$ driving convection there. A review by Christensen [1] demonstrates a variety of such theoretical laws, which do not coincide with the magnetic law following from the compilation of the available numerical dynamo simulations. We resolve this discrepancy and present new scaling laws. In the limit of negligible molecular diffusivity and viscosity, we scale the energy and momentum equations. The fast rotating limit ($\Omega \gg V/H$) is also used, where $V$ is the typical convective velocity. The known scaling energy and the convective laws are summarized, while the new result equates the typical ratio of magnetic and kinetic energies to the square of the corresponding scale ratio. The magnetic diffusivity transport is neglected in the scaling electromagnetic equations. This results is the typical magnetic field $B \sim F^{1/3}$, while $B$ is independent on $\sigma$ and $\Omega$. The same scaling was obtained by compiling numerical simulations, e.g., see [1, 2]. Our new scaling laws are for the typical electromagnetic scale $d$, the typical sinus of the angle between magnetic
and velocity vectors $s$, and for the typical electric field $E$. Using all known [1], [3–5] and our new scaling laws, we estimate the corresponding typical geodynamo values.

In section 2, we use the established scaling laws to estimate long-term geomagnetic consequences for different evolution scenarios of a core-mantle system. Already more than for half a century it has been argued that the geomagnetic field is predominately driven by a compositional convection, which takes place during the solidification of the liquid core [3, 6]. However, the same magnetic field can be the result of the compositional convection, which takes place when the liquid core decomposes the iron-nickel protocore, which contains solid inclusions of silicate material. These two essentially different models with identical results in the form of compositional convection and geomagnetic field generated by this convection can differ both in time of the process onset and in number of geochemical consequences and thus determine two essentially different options of the core-mantle system evolution. It is considered that the crystallization of the liquid core might have begun not earlier than 2 billion years ago [6, 7]. At the same time, traces of the magnetic field are found in rocks aging to nearly 3.5 billion years [9, 10] and thus dispose of the model of protocore decomposition, which could begin soon after the end of accretion, i.e. 4.5 billion years ago. Moreover, in the case of the second model validity, the mantle should be supplemented with the silicate material from the protocore with the primitive isotope composition of lead, but which cannot be the result of the liquid core crystallization (see details in section 4). An additional reason to the validity of the second model could be the primitive isotope composition of lead in combination with primary helium enriched by isotope $^{3}$He. This paper is closed by section 4, where the magnetism and the evolution of the terrestrial planets are discussed in the light of our protocore concept.

1. Planetary MHD scaling and geodynamo typical values. With reference to [1–4], we neglect all the effects related to the transport coefficients. Then the heat and composition energy MHD dynamo equations could be approximated via the integral relations derived in [3] for almost any astrophysical object as

$$AV = F,$$  
(1)

The same scaling follows from numerical simulations [2] and analytical considerations [4]. So, the simple scaling law from Eq. (1) could have a universal nature.

The radial component of curl of the momentum equation without viscosity terms is

$$\Omega \frac{\partial V_r}{\partial z} = \hat{r} \cdot \nabla \times \left( \frac{\rho D V}{D t} + \frac{B \times \nabla \times B}{\mu} \right) \frac{1}{\rho}.$$  
(2)

Here, the velocity $V$ and the magnetic $B$ field vectors are used, but there is no buoyancy force vector, which is parallel to the unit radial vector $\hat{r}$. The Coriolis force term ($\sim \Omega$) is omitted when integrating Eq. (2) along the rotation axis (the $z$-coordinate) from one boundary to another. It is because $V_r = 0$ at those boundaries due to non-penetration conditions. So, we are only left with the magnetic and inertia bracketed terms from Eq. (2) providing us with

$$\frac{B^2}{\mu \rho d^2} = \frac{V^2}{h^2}.$$  
(3)

This equates the typical magnetic and kinetic energy ratio $B^2/(\mu \rho V^2)$ to $(d/h)^2$. Using Eq. (3), the condition of fast rotation and other components of the same curl as in [1, 3, 5] yield

$$\frac{\Omega V}{H} = \frac{V^2}{h^2} = \frac{A}{h}.$$  
(4)
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Solving Eqs. (1), (4) for unknown $h$, $A$, $V$, we obtain the known after Rhines [5] scaling laws for buoyancy driven convection in the fast rotating planets and moons:

\[ h = \left( \frac{F}{H^2 \Omega^3} \right)^{1/5} H, \]  
\[ V = \left( \frac{F}{H^2 \Omega^3} \right)^{2/5} H \Omega, \]  
\[ A = \left( \frac{F}{H^2 \Omega^3} \right)^{3/5} H \Omega^2. \]  

To scale the electromagnetic equation, we use the Ohm’s law with the electric field vector $E$

\[ \frac{1}{\mu \sigma} \nabla \times B = E + V \times B. \]  

The magnetic diffusivity term $\sim 1/(\mu \sigma)$ could be neglected here, if $\mu \sigma V d V d \gg 1$, leading to

\[ E = s V B. \]  

Here, the typical sinus $s$ of the angle between the velocity and the magnetic field must be very small due to almost parallel $V$ and $B$ for such frozen flux approximation. Thus, the Faraday’s law $\partial B/\partial t = -\nabla \times E$ could be scaled with the largest possible typical time $H/V$ as

\[ V B / H = E / d. \]  

Using $d \gg h$ (see confirmation below), the induction equation without the magnetic diffusivity term $\partial B/\partial t \approx \nabla \times (\nabla \times B)$ yields a typical time $h/s V$. This time and the magnetic energy domination are used to balance the buoyancy work power with the magnetic power as

\[ s V B^2 / (\mu h) = \rho A V, \]  

while the kinetic power $\rho V^3 / h$ is always of the order of the buoyancy work power $\rho V A$ due to Eq. (5). Thus, relations (2), (6)–(8), (10)–(12) yield the magnetic scaling law known previously only from the compilation of many numerical simulations [1]:

\[ B = (\mu \rho)^{1/2} (F H)^{1/3} = (\mu \rho)^{1/2} \left( \frac{F}{H^2 \Omega^3} \right)^{1/3} H \Omega. \]  

Finally, the scaling laws for geodynamo and similar planetary dynamos are Eqs. (6)–(8), (13), and

\[ d = \frac{B h}{\sqrt{\mu \rho V}} = \left( \frac{F}{H^2 \Omega^3} \right)^{2/15} H, \]  
\[ s = \frac{d}{H}, \]  
\[ E = s V B. \]  

These equations utilize the new scaling laws, while the laws from Eqs. (6)–(8), (13) were already known. Using in those laws roughly known ($F \approx 2 \cdot 10^{-13} \text{m}^2/\text{s}^3$ from [2], while it could be ten times larger [3, 4]) and well-known ($\Omega = 7.3 \cdot 10^{-5} \text{1/s}$,
\[ \rho = 11 \text{ Mg/m}^3, \ H = 2.3 \text{ Mm and } \mu \sigma = 2 \text{ s}^2/\text{m} \text{ from [7]) input values, we derive typical geodynamo values:} \]
\[ B = 2 \text{ mT, } h = 6 \text{ km} \ll d = 90 \text{ km}, \ V = 1 \text{ mm/s}, \]
\[ s = 0.04, \quad R_m \equiv \mu \sigma V d = 180. \tag{17} \]

The last relation in Eq. (17) defines the electromagnetic Reynolds number \( R_m \), which is large for the Earth. The found scaling laws (Eqs. (6)–(8), (13)–(16) are also applicable to Jupiter, Saturn, and ancient dynamo-active Mars and to fast rotating planets/moons, where \( R_m \) is also large.

2. The protocore concept. The obtained magnetic scaling laws (Eq. (13)) link the typical geomagnetic field intensity \( B \) directly to the buoyancy flux power \( F \) value that is the ratio of the power available to drive convection to the liquid core mass. This convection power could be supported by a thermal process if the heat flux from the core is sufficiently greater than the adiabatic heat flux that was earlier estimated at about 5 TW [3, 6]. The recent work [7] raises this estimation up to 15 TW making thermal convection impossible for any realistic value of the heat flux from the core in the modern epoch. For more ancient time, thermal support to convection could exist but at a very low level. Thus, geodynamo is created by a convection, which is primarily supported by compositional effects [3, 6, 7]. These effects could be due to the inner solid core crystallizing from the liquid core, as it is widely accepted nowadays. Such crystallization could have started not earlier than two billion years ago (2 Ga, shortly), while the most probable start is 1 Ga [4, 6, 7]. Therefore, for more than the first 3 billion years of the Earth history the typical geomagnetic field value should be much smaller than the modern one. However, there are evidences of the ancient geomagnetic field existence [8, 9, and references therein] up to the early Archean [10] (~3.5 Ga), whose intensity is identical to the modern one. This information contradicts to the generally accepted ideas, according to which geodynamo is produced by compositional convection caused by the liquid core crystallization. Thus, if the compositional convection is needed to generate the Archean geomagnetic field, the intensity of which is close to the contemporary one, this convection should have some other nature. At the same time, magmatic derivatives of the mantle material sometimes contain primary lead and noble gases, which contain, in particular, isotope \(^{129}\text{Xe}\). It evidences that somewhere in the Earth there is a material, which had become a geochemically closed system before \(^{129}\text{I}\) completely decayed, i.e. <150 million years after the beginning of accretion.

A concept, which can explain all mentioned above, is proposed. It suggests that the solid core of the Earth has not crystallized from the liquid one, but represents a small relict of the protocore, on which heterogenic accretion had begun. The protocore consists of a mixture of heavy metal iron-nickel alloy and a light chondrite silicate component, which contains primary noble gases and which is marked by chondritic lead, having a primitive isotope composition unreached by \(^{204}\text{Pb}\). Soon after the end of accretion or near to its end, the geosphere of the liquid core had been formed in the external part of the planet. It started to plunge, expanding due to the melting of new portions of the iron-nickel alloy. This expansion was rather fast during the period of initial formation of the liquid core geosphere due to its intensive overheating. Then the expansion rate decelerates. The first reason of this is the decrease of the temperature difference between the liquid geosphere and the solid protocore. It leads to a slower conductive transport of the heat necessary to melt the protocore. Namely, the rate of this conductive heat transport determines the time needed for the protocore dissolution in the
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Fig. 1. A “core-mantle” system and its evolution based on the protocore interaction with the liquid core.

liquid core geosphere. If we use the thermal conductivity [7] of the contemporary liquid core to estimate this time, we will obtain a few billion years, as required for our concept. During the protocore dissolution, the silicate chondritic component of the protocore is set free (see Fig. 1). It floats upwards through the metallic melt of the liquid core and drives compositional convection, which mainly supports geodynamo.

3. Verification of the protocore concept. There are some ways to verify the hypothesis on protocore disintegration. If this hypothesis is valid, the noble gases and lead, having the isotope composition similar to the chondritic one, together with the silicate chondrite component should be superinduced from the core into the mantle. The main isotope feature of this component is the primitive ratio of lead isotopes $^{206}\text{Pb}/^{204}\text{Pb}$, $^{207}\text{Pb}/^{204}\text{Pb}$ and $^{208}\text{Pb}/^{204}\text{Pb}$ because, unlike the terrestrial material in chondrites, the $^{238}\text{U}/^{204}\text{Pb}$ ratio is ten times smaller.

Thus, there is a possibility for protocore hypothesis verification. On the basis of such studies, investigations of the mantle magmatic derivatives for the lead with the isotope composition shifted in the direction to the chondrite component should be performed. Preliminary results of such studies (Fig. 2) testify that in the mantle magmatic derivatives aging to the culmination stage of the endogen activity of a megacycle of 1.7 billion years, an impurity of 3% of such lead is considerable. According to the megacycle age reduction, the concentration of such Pb consistently decreases almost until its total disappearance in the modern mantle magmatic rocks.

Collection of the paleomagnetic data, testifying to the long periods with invariable geomagnetic polarity in the early Precambrian, can become a basis of one more option to verify the hypothesis of the eroding protocore. If the assumption that such phenomenon is caused by the stabilizing role of the solid core for the geomagnetic polarity [11] proves to be true, this will be a unequivocal recognition of the solid core existence in the Archean. In a combination with the results of assessment of the liquid core crystallization onset [6, 7], it once again will confirm the invalidity of the existing ideas for the geodynamo nature explanation.
Fig. 2. Pb-Pb isotope systematic of lead ore minerals and feldspars in the Earth magmatic rocks (compilation of Kramers and Tolstikhin [12]), between which there are 130 samples of the mantle magmatic derivatives of basic composition. Dashed lines correspond to two-component mixes of the “mantle-protocore” system for the culmination stages of megacycles of endogenic activity of 2.65, 1.65 and 0.3 billion years. Dot ellipses along these dashed lines reflect an up to 3% additive of the protocore component (for a megacycle of 1.65 billion years).

Depending on the distribution of the iron-nickel and silicate components in the protocone under the power regime of evolution of the “liquid core-protocore” system, the culmination stage in the Archean is possible. This is the basis for identification of the Archean culmination in geomagnetic field intensity and, hence, for realization of one more option to verify the model, which has predicted the protocone.

4. The protocone concept for the terrestrial planets. There are several factors determining the evolution of the “mantle–protocone” system. Their combination defines one or the other scenario of the planet evolution. Using the concept of eroded protocone, it is easy to characterize the features of its evolution. In the case of Venus, which does not have its own magnetic field but has the liquid core, it is natural to assume that there is either no convection at all or it is very insignificant. Such conditions can occur after the protocone already has fully reacted with the liquid core and the crystallization has not yet begun or has begun quite recently. Neither Mars nor the Moon has their own modern magnetic field, but residual magnetization is found and liquid cores are supposed. Such situation can correspond to the early fading of the compositional convection either because of the small protocone or due to the deficit of released energy needed to maintain the temperature assuring this protocone erosion. A similar situation can also be expected on other quite large moons of giant planets or even on asteroids. The
specific features of the magnetic field formation and evolution on Jupiter satellites most likely are defined by the existence of a liquid core arising under the influence of gravitational disturbance from the central body. Being almost identical in size, Mercury and Ganymede (the satellite of Jupiter) have the hydromagnetic dynamo, which generates a magnetic field, which is considerably smaller than the Earth's one and very different in terms of the structure. It is possible to assume that both the tidal forces of the Sun and Jupiter, correspondingly, and the weak compositional or/and thermal convection support the magnetic field generation in the thin layer of the liquid core of these objects.

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