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LONGITUDINAL ELECTROMOTIVE FORCES AND HALL EFFECT FOR CHANNEL FLOW OF AN IONIZED GAS

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Estimates are made of the electric field that may arise along a channel containing a flow of ionized gas due to the thermoelectric effect, the electron pressure gradient, large ion slip, and the Hall effect. The author presents certain formulas characterizing energy conversion in a channel with a fixed direction of the electric field.

In experiments on the motion of an ionized gas in various models of MHD generators two types of voltages must be considered: transverse (across the channel) and longitudinal (if the electrodes are sectionalized). It is easy to determine the transverse voltage on opposite electrodes using Faraday's law of induction, and there is no reason for the appearance of other emf's comparable with the Faraday emf. However, in the presence of pressure and temperature gradients, due to the addition of ionizing agents in powder form and also the Hall effect, commensurable emf's may develop along the length of the channel. These can be determined if we write Ohm's law as obtained from the momentum equation for an electron gas in a multicomponent plasma [1]:

$$j_x=\sigma\left[E_x+\frac{1}{en_e}R_{T_x}+\frac{1}{en_e}\frac{dp_e}{dx}-\frac{m_e}{e\tau_e}(v_n-v_i)+(\mathbf{v}_e\times\mathbf{B})_x\right]. \quad (1)$$

Here

$$\sigma=\frac{e^2n_e}{m_e}\tau_e;$$

j_x, E_x – are the current density and electric field along the channel;

m_e, n_e, e – are the mass, concentration, and charge of the electrons;

R_T – is the thermal force [1], and R_T/en_e the thermo-emf;

$p_e = n_ekT_e$ – is the electron gas pressure;

τ_e – is the electron free time;

v_e, v_i, v_n, – are the electron, ion, and neutral gas velocities;

x – is the longitudinal channel coordinate; and

B – is the magnetic induction.

All the terms in the square brackets in Eq. (1), except the first, are emf's (we have in mind the emf per unit length) capable of creating the observed potential differences along the channel. The term $(v_e \times B)_x$ is the Hall emf. Clearly, there are also three other possible emf's.

a) For the longitudinal thermo-emf in the case of a two-component (fully ionized) plasma in the absence of a magnetic field we get the following expression [1]:

$$\frac{1}{en_e}R_{T_x}=-(0.71-1.52)\frac{k}{e}\frac{dT_e}{dx}. \quad (2)$$

The numerical coefficient varies from 0.71 to 1.52 with variation of the ionic charge from one to infinity.

For $T_e < 3000°$ K the greatest potential difference due to the thermo-emf is

$$-\Delta\varphi=\int_1^2 E_x dx=(0{,}71-1.52)\frac{k}{e}(T_{e_2}-T_{e_1})<0.4 \text{ V}. \quad (3)$$

The potential difference on lowering the temperature in the channel is quite small. Its effect on the experiments is insignificant. In the presence of a magnetic field the thermo-emf R_{T_x}/en_e diminishes. If ion-electron collisions can be neglected the thermal force generally vanishes.

b) The electron pressure gradient gives an emf several times greater:

$$\frac{1}{en_e}\frac{dp_e}{dx}=\frac{1}{en_e}\left[kT_e\frac{dn_e}{dx}+n_ek\frac{dT_e}{dx}\right]. \tag{4}$$

Here, the second term in the square brackets is analogous to expression (2), while, to estimate the first term, we assume that in the weakly ionized gas in question (with an ionizing additive) $n_e = cT_e^{\gamma}$.

Then the ratio of the terms

$$\frac{dn_e}{dT_e}\cdot\frac{T_e}{n_e}=\gamma. \tag{5}$$

If the additive is only partially ionized, we usually take γ = 10-15, and hence, in expression (4) the first term is one order greater than the second. Thus, the potential difference due to the electron pressure gradient between points with electron concentrations n_1 and n_2 has the form

$$\Delta\varphi=-\int_1^2 E_x dx=\frac{kT_e}{e}\int_1^2\frac{dn_e}{n_e}=\frac{kT_e}{e}\ln\frac{n_2}{n_1}. \tag{6}$$

When the electron concentration along the channel is reduced due to a fall in temperature and the associated recombination, or due to a reduction of the total pressure in the stream of ionized gas, then the potential difference is negative and may amount to 5 V for a change of six orders in n_e. Note that we must get the same negative potential with respect to the cooled walls across the channel, too. This value is commensurable with the voltages observed in small experimental devices. The resulting currents are evidently very small, since essentially we get ambipolar diffusion.

c) The following term expresses the emf due to ion slip. If the density of the gas is sufficient, the ion slip for ions of atomic dimensions is so small that it may be neglected. However, in actual experiments we sometimes encounter a large velocity difference $v_n - v_i$. This is due to the inertia of the ions entrained by the neutral gas with acceleration, when the ions are solid particles that have emitted electrons, the dimensions of these particles being many orders greater than atomic dimensions ("large ions"). In the absence of a magnetic field, friction between the neutral particles and electrons held close to the large multiply charged ions by the condition of quasi-neutrality gives rise to a considerable emf. However, this process can not produce appreciable currents, since, thanks to the coulomb forces of attraction between the emitted electrons and the multiply charged ions, the number of free electrons is comparatively small, and apparently sufficient only for emf measurements using high-resistance voltmeters. If $n_n = 10^{25}\,m^{-3}$, the electron-neutral collision cross section $Q_{en} = 3 \times 10^{-19}\,m^2$, the electron thermal velocity $\tilde{v}_e = 1.8 \times 10^5$ m/sec, the thermal velocity of the neutrals $\tilde{v}_n = 0.7 \times 10^3$ m/sec, and the radius of a large ion $r = 10^{-4}$ m, then for an ion concentration $n_i = 10^6\,m^{-3}$ we get:

$$\frac{1}{\tau_e}=n_n\tilde{v}_eQ_{en}=5.4\cdot10^{11}\ \sec^{-1}, \qquad E_x=\frac{m_e}{e\tau_e}(v_n-v_i)=3000\ V/m \tag{7}$$

Here it is assumed that $v_n - v_i = 10^3$ m/sec, which is perfectly possible for particles of the given size in a supersonic flow [2]. The potential difference is negative, when the neutral gas moves faster than the ions. At low velocities, this emf is reduced, but remains large enough to be observed.

d) The final term in Eq. (1) — the Hall emf $(\mathbf{v}_e\times\mathbf{B})_x$ has the largest value, since it may exceed the Faraday emf $\mathbf{v}_n\times\mathbf{B}$ and produces substantial currents.

To analyze effect of the Hall emf on energy conversion we shall use Ohm's law in the form:

$$\mathbf{j}=\sigma[\mathbf{E}+\mathbf{v}_e\times\mathbf{B}]=\sigma[\mathbf{E}+\mathbf{v}_n\times\mathbf{B}-(\mathbf{v}_n-\mathbf{v}_e)\times\mathbf{B}]$$

for $v_n = v_i$

$$\mathbf{j} = \sigma\left[\mathbf{E} + \mathbf{v}_n \times \mathbf{B} - \frac{1}{en_e}\mathbf{j} \times \mathbf{B}\right]. \tag{8}$$

For the conditions applying to MHD channels, it is usually possible to assume $B_x = v_y = v_z = 0$, i. e., $\mathbf{B} = (0, B_y, B_z)$ and $\mathbf{v}_n = (v_x, 0, 0)$. Then from (8) we get for the components of the current density:

$$j_x = \frac{\sigma}{1 + \beta_z^2 + \beta_y^2}[E_x - \beta_z(E_y - v_x B_z) + \beta_y(E_z + v_x B_y)], \tag{9}$$

$$j_y = \sigma(E_y - v_x B_z) + \beta_z \frac{\sigma[E_x - \beta_z(E_y - v_x B_z) + \beta_y(E_z + v_x B_y)]}{1 + \beta_z^2 + \beta_y^2}, \tag{10}$$

$$j_z = \sigma(E_z + v_x B_y) - \beta_y \frac{\sigma[E_x - \beta_z(E_y - v_x B_z) + \beta_y(E_z + v_x B_y)]}{1 + \beta_z^2 + \beta_y^2}. \tag{11}$$

Here

$$\beta_z = \frac{e\tau_e}{m_e} B_z, \qquad \beta_y = \frac{e\tau_e}{m_e} B_y.$$

When $B_y = E_z = 0$, the case treated by Montardy [3], we impose the following relation between the components of the electric field (which corresponds to the conditions for an MHD generator with a fixed direction of the electric field):

$$E_x = \gamma E_y. \tag{12}$$

Introducing the notation for the transverse load parameter:

$$k_y = \frac{E_y}{v_x B_z}, \tag{13}$$

from (9) and (10) we get

$$j_x = \frac{\sigma}{1 + \beta^2} v_x B_z [k_y \gamma - \beta(k_y - 1)]; \tag{14}$$

$$j_y = \frac{\sigma}{1 + \beta^2} v_x B_z [\beta\gamma k_y + k_y - 1]. \tag{15}$$

Assuming uniform distribution of all the parameters in a channel of width b and depth δ, we get the total current

$$I = b\delta\left(j_x + \frac{1}{\gamma} j_y\right) = b\delta \frac{\sigma v_x B_z}{1 + \beta^2}\left[k_y\left(\gamma + \frac{1}{\gamma}\right) - \frac{1}{\gamma} + \beta\right]. \tag{16}$$

The voltage over a length l is given by the formula

$$-U = E_x l = k_y \gamma v_x B_z l. \tag{17}$$

The specific power of the generator

$$P_{\text{sp}} = j_x E_x + j_y E_y = \sigma \frac{v_x^2 B_z^2}{1 + \beta^2}[k_y(\gamma^2 + 1) + \beta\gamma - 1]k_y. \tag{18}$$

The efficiency of such a generator

$$\eta = \frac{P_{\text{sp}}}{j_y v_x B_z} = \frac{k_y[k_y(\gamma^2 + 1) + \beta\gamma - 1]}{k_y - 1 + \beta\gamma k_y}. \tag{19}$$

If we introduce the "longitudinal" load parameter

$$k_x=\frac{E_x\left(1+\frac{1}{\gamma^2}\right)}{\left(\frac{1}{\gamma}-\beta\right)v_xB_z}, \tag{20}$$

then from (9) and (10) we get a formula for the specific power

$$P_{\text{sp}}=E_x\left(j_x+\frac{1}{\gamma}j_y\right)=k_x(k_x-1)\frac{\sigma v_x^2B_z^2\left(\frac{1}{\gamma}-\beta\right)^2}{(1+\beta^2)\left(1+\frac{1}{\gamma^2}\right)}, \tag{21}$$

that agrees exactly with the results of Montardy [3], who showed that when $\gamma=\infty$ we get the formulas for a Hall current generator, and when $\gamma=0$ the relations for a generator with continuous electrodes, while $\gamma=-\beta$ gives the maximum specific power.

When $|\gamma|\to\infty$ we must put $k_y\to 0$ in (18) so that $\gamma k_y=\Gamma=\text{const}$. Then the specific power and efficiency of the generator assume the form:

$$P_{\text{sp}}=\frac{\sigma}{1+\beta^2}v_x^2B_z^2(\Gamma^2+\Gamma\beta);\qquad \eta=\frac{\Gamma^2+\Gamma\beta}{\Gamma\beta-1}. \tag{22}$$

For a Hall current generator we obtain the familiar formula

$$P_{\text{sp}}=-\frac{\sigma\beta^2}{1+\beta^2}v_x^2B_z^2k_x(1-k_x),$$

$$k_x=\frac{-E_x}{\beta v_xB_z}. \tag{23}$$

If the component B_y acts, in addition to B_z, the resulting magnetic field will be intensified, and there will be a corresponding increase in specific power. In the special case $B_y=B_z$ and $\beta_y=\beta$, from (9), (10), and (11) we get

$$P_{\text{sp}}=j_xE_x=-\frac{\sigma}{1+2\beta^2}(2\beta v_xB_z)^2k_x(1-k_x);$$

$$k_x=\frac{-E_x}{2\beta v_xB_z}. \tag{24}$$

For $\beta\gg 1$ (which is characteristic of a Hall current generator) and the same k_x, a comparison of (23) and (24) shows that there is a two-fold increase in specific power.

Note two other properties of plasmas consisting of large ions, electrons, and neutral particles (without ordinary ions):

1. The Hall effect and large ion slip give emf's that act in opposite directions and may compensate each other. If there is no longitudinal current, then the longitudinal electric field

$$E_x=\frac{m_e}{e\tau_e}(v_n-v_i)-(\mathbf{v}_e\times\mathbf{B})_x=\frac{B}{\beta}(v_n-v_i)+(k_y-1)\frac{\sigma B^2}{en}v_n. \tag{25}$$

if

$$k_y=1-\frac{v_n-v_i}{v_n\beta^2},\ \text{then}\ E_x=0. \tag{26}$$

2. In principle, ion slip permits energy conversion even in the absence of a magnetic field. The specific power of this conversion

$$P_{sp}=j_xE_x=k_x(k_x-1)\,n_e m_e n_n v_e Q_{en}(v_n-v_i)^2. \tag{27}$$

Here

$$k_x=\frac{-E_x}{\dfrac{m_e}{e\tau_e}(v_i-v_n)}. \tag{28}$$

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