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AMPLIFICATION IN A DECAYING PLASMA

L. I. Gudzenko and L. I. Shelepin

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The properties of a nonequilibrium plasma as a medium amplifying electromagnetic radiation at frequencies in its discrete spectrum are discussed. A qualitative analysis is made of the three decay stages of a nonequilibrium plasma corresponding to different degrees of ionization. It is assumed that the degree of plasma ionization is considerably greater than the equilibrium value corresponding to the mean energy of its free electrons.

In considering processes accompanying the recombination of an almost fully ionized nonequilibrium plasma with a fairly small initial mean free electron energy, it is convenient to distinguish three stages which follow each other in time:

1) a strongly ionized plasma with "instantaneously" cooled free electrons, when the lower discrete atomic levels are still practically unpopulated;

2) a substantially ionized plasma in which a "steady drain" from the overpopulated upper levels to the ground state has been established;

3) a weakly ionized plasma.

From the point of view of designing a laser based on a nonequilibrium plasma [1], the behavior of the plasma in the first stage of decay is of greatest interest. The very possibility of such rapid cooling of free electrons that the initial small populations of the lower discrete levels are not notably increased during the cooling time is a result of the fact that the time to attain the free electron distribution for a strongly ionized plasma of mean density ($N_e \sim 10^{11} - 10^{17}$ cm^{-3}) is many times less than the relaxation times for electrons over the lower discrete levels. Papers [2–6] are devoted to the relaxation of a plasma with a high degree of ionization which considerably exceeds the equilibrium value (corresponding to a free electron temperature T_e). As a rule, recombination begins with the capture of a free electron at one of the upper levels in triple collisions; inelastic collisions of the corresponding excited atom with free electrons play the basic part in transitions to somewhat lower levels. Since inelastic collisions sharply reduce the specific role in relaxation of the discrete levels which are metastable for spontaneous radiative transitions, one may expect the well known similarity of relaxation behavior in highly ionized one-component plasmas of different composition; we shall confine ourselves to estimates for the decay of a hydrogen plasma. The system of equations for the population of the energy levels in an optically thin plasma may be represented in the form

$$\frac{dN_n}{dt} = -N_e N_n \sum_{m\neq n} V(n,m) - N_e N_n B_\varepsilon(n) - N_n \sum_{m<n} A(n,m) + \\ + N_e \sum_{m\neq n} N_m V(m,n) + N_e^3 B'_\varepsilon(n) + N_e^2 A_\varepsilon(n) + \sum_{m>n} N_m A(m,n), \\ N = N_e + \sum_n N_n, \tag{1}$$

where N is the total number of electrons (free and bound) in 1 cm^3, N_e is the free electron density, N_n is the number of electrons in 1 cm^3 at the discrete level with principal quantum number n. The quantities $A(n, m)\,dt$ and $A_\varepsilon(n)\,dt$ are equal to the probabilities of the radiative transition $n \to m$ and spontaneous recombination to level n, respectively, in a time dt; $V(n, m)$ corresponds to the nonradiative transition $n \to m$, $B_\varepsilon(n)$ and $B'_\varepsilon(n)$ to collision ionization and recombination.

For densities $N \sim 10^{13} - 10^{16}$ cm^{-3} and temperatures $kT_e \sim 0.1 - 0.5$ eV the general pattern of relaxation is as follows: after the capture of an electron in one of the uppermost bound states "slipping" of the electron occurs through discrete levels to the ground state. Transitions between the upper levels are mainly characterized by collision relaxation, the probability of which falls off rapidly as n decreases; on the other hand, the probability of radiative transitions increases as the electron nears the ground state. Close to $n = n^*$ these probabilities turn out to be equal; the minimum transition probability corresponds to levels with $n \simeq n^*$, thus the speed of electron transition through $n \simeq n^*$ may serve as an estimate of the over-all rate of relaxation of the plasma. The electron distribution at the upper levels ($n > n^*$)

rapidly reaches a quasi-equilibrium state:

$$\tilde{N}_n = n^2 N_e^2 \left(\frac{2\pi\hbar}{mkT_e}\right)^{3/2} \exp\left(\frac{E_n}{kT_e}\right) \tag{2}$$

with parameters $N_e(t)$, $T_e(t)$ varying with time. The first stage of relaxation is characterized qualitatively by τ_1, the time to establish a steady drain over the discrete excited levels at $t < \tau$; one may roughly consider $N_n = \begin{cases} \tilde{N}_n, & n > n^* \\ 0, & n < n^* \end{cases}$. A detailed analysis of the first relaxation stage, where the decaying plasma most effectively amplifies optical radiation, requires the solution of the system of differential equations (1) with corresponding initial conditions. This solution turns out to be strongly oscillatory, which renders calculation by the usual stepwise iteration procedure very difficult. We are obliged to seek an approximate solution by breaking up the time interval of interest into consecutive sections, during which the value N_e is assumed to be constant to a first approximation; system (1) is linear in N_n, which allows us to obtain a solution immediately for an arbitrary interval of time.

The second stage begins after a time τ_1 following the rapid cooling of free electrons (τ_1 is estimated from the lesser of two times: the triple recombination time for the second level and the sum of relaxation times between neighboring levels) and finishes when $\sum_{n=2}^{n_{max}} N_n(t) \simeq N_e(t)$; in calculating the populations $\bar{N}_n$ at stage (2), the "steady drain" stage, we must set $\frac{d\bar{N}_n}{dt} = 0$, $n = 2, 3, \ldots, n_{max}$ in (1) (see also [5, 6]).

Depending on the actual choice of a method for cooling the free electrons, their instantaneous temperature, and thus the coefficients of (1), may be considered as given functions of time to a first approximation. $\bar{N}_n$ was calculated on an electronic computer for the simplest case (from the point of view of calculation) when the kinetic temperature of the free electrons is maintained constant after they are "instantaneously" cooled. In calculating the coefficients of system (1) modified Born cross sections for collision processes were employed [8], the value of n_{max} was determined from the Debye radius, it being assumed that $\bar{N}_n = \tilde{N}_n$ for $n > 10$. Transition cross sections from states with $n > 10$ to lower discrete levels were evaluated by the asymptotic continuation of the formula for triple recombination. The table§ sets out the populations $\bar{N}_n$ of levels $n = 2 - 6$, where $\bar{N}_n$ differs significantly from $\tilde{N}_n$.

N_e / κT_e	10^{12}			10^{13}			10^{14}			10^{15}		
	0.1	0.2	0.4	0.1	0.2	0.4	0.1	0.2	0.4	0.1	0.2	0.4
N_2	$5\cdot10^4$	$5\cdot10^3$	10^3	$2\cdot10^7$	10^6	10^5	$2\cdot10^{10}$	10^9	$5\cdot10^7$	$2\cdot10^{12}$	$3\cdot10^{11}$	$2\cdot10^{10}$
N_3	$2\cdot10^5$	$2\cdot10^4$	$4\cdot10^3$	10^8	10^7	10^6	$3\cdot10^{10}$	$6\cdot10^9$	$4\cdot10^8$	$2\cdot10^{13}$	$3.5\cdot10^{12}$	$6\cdot10^{10}$
N_4	10^6	$8\cdot10^4$	10^4	10^9	$3\cdot10^7$	$3\cdot10^6$	$4\cdot10^{11}$	$4\cdot10^9$	$3.5\cdot10^{1}$	$6.5\cdot10^{13}$	$4\cdot10^{11}$	$4\cdot10^{10}$
N_5	$2\cdot10^6$	$2\cdot10^5$	$3\cdot10^4$	10^9	$2\cdot10^7$	$3\cdot10^6$	$5\cdot10^{10}$	$3\cdot10^9$	$4\cdot10^8$	$5\cdot10^{12}$	$3\cdot10^{11}$	$4\cdot10^{10}$
N_6	$2\cdot10^6$	$2\cdot10^5$	$4\cdot10^4$	$3\cdot10^8$	$2.5\cdot10^7$	$4\cdot10^6$	$3\cdot10^{10}$	$2.5\cdot10^9$	$4\cdot10^8$	$3\cdot10^{12}$	$2.5\cdot10^{11}$	$4\cdot10^{10}$

The coefficient of negative absorption of radiation at wavelength $\lambda_{n,m}$ (corresponding to the transition $n \to m$) equals

$$\varkappa_{m,n} \simeq \frac{\lambda_{n,m}^2}{4\,\Gamma_{m,n}} A(n,m) \qquad (N_n - N_m) \tag{3}$$

for a 1 cm photon path length. For example, we will evaluate $\varkappa_{5,2}$ for the first and second relaxation stages. Here the line width is determined by collisions with ions [8]: $\Gamma_{m,n} = 12.5\,(m^2 - n^2)\,N_e^{2/3}$. Assuming $N_2 = 0$, $N_5 \simeq \tilde{N}_5$ with $kT_e = 0.1$ eV we find $\varkappa^{(1)}_{5,2} = 2\cdot10^{-28}\cdot N_e^{4/3}$; for the first stage; consequently, for $N_e = 10^{15}$ cm^{-3} we have $\varkappa^{(1)}_{5,2} \simeq 4\cdot10^{-2}$ cm^{-1}. For the second stage, in accordance with the table, we find $\varkappa^{(2)}_{5,2} = 2\cdot10^{-3}$ cm^{-1} for $kT_e = 0.1$ eV, and $N_e \simeq 10^{15}$cm^{-3}. Thus in the course of the entire steady drain period ($\tau_2 \sim 10^{-6}$ sec) a sufficient inverse population is maintained to create a quantum oscillator. In the course of the time $\tau_1 \sim 10^{-8}$ sec, corresponding to stage 1), amplification in a hydrogen plasma allows radiation to be generated at a wavelength of several centimeters.

§So far, the collision cross sections for upper levels, even in the case of hydrogen, have been determined correct only to a factor of ~ 1–3. Thus it is of interest to compare the values of $\bar{N}_n$ computed by us with those calculated in recent papers [6], where quasi-classical cross sections were employed; corresponding values of $\bar{N}_n$ agreed to within an order of magnitude.

In an optically thin plasma the quantities N_n do not depend on $\bar{N}_1$; but in real systems for $N_e > 10^{14}$ cm^{-3} radiation reabsorption must be taken into account, which leads to an increase of $\bar{N}_2$ and $\bar{N}_3$, and their dependence on N_1, as well as to a decrease of τ_2, $\varkappa^{(2)}{}_{n,2}$, $\varkappa^{(3)}{}_{n,3}$ §. It must be noted that hydrogen is not the best medium for a plasma laser, since as a result of the linear Stark effect its values of $\Gamma_{m,n}$ are large; in addition to this, the absence of metastable states in a hydrogen plasma reduces the times τ_1 and τ_2.

Apart from radiation capture, the influence of nondecaying molecules and neutral atoms and also the effect of electron adhesion are not represented in equations (1). In stage 1), and to a certain extent in stage 2), taking all these phenomena into account would only serve to make the calculations more precise, which is meaningless until a concrete method of achieving cooling has been devised. However, in stage 3) such effects may exert a significant influence. The position is further complicated by the fact that in a weakly ionized plasma the significance of nonradiative transitions decreases, which enhances the part played by metastable states. As distinct from stages 1) and 2), the state of population inversion may easily be kept stationary in a weakly ionized plasma, this is certainly so in a gas laser and similar conditions may also occur in MHD converters based on nonequilibrium plasmas.

In a gas discharge heating and cooling of free electrons occur together, the heater being the electric field, the cooler the large thermal capacity of both the heavy particles (ions, atoms, and molecules) and the walls. At low degrees of ionization the electron-electron collision frequency decreases, and the N(E) distribution of free electrons differs from the Maxwell distribution; this is well known for high energies $E > 2kT_e$. It is important to note the departure of N(E) from the Maxwell distribution for low energies. At $E \ll kT_e$ electron collisions with cold heavy particles become increasingly more important as the energy decreases. The loss of cold electrons as a result of recombination is compensated for by the influx from inelastic collisions. As a result of a certain metastability of the upper levels their population exceeds the quasi-equilibrium values, and this may be the chief cause of the population inversion in a gas discharge plasma. This effect should be strengthened if the field is pulse-modulated, and experiment confirms this qualitatively. It should be stressed that from the point of view of laser design pulsed deep cooling methods with times significantly less than the times for establishment of the steady drain stage are the most effective.

It should be noted once again that in analyzing amplification in a nonequilibrium recombining plasma of various degrees of ionization the kinetics of the whole multilevel system must be taken into account; calculations of electron distributions made to date for two or three levels only (see, for example [9]), are not well-founded here.

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§ Apart from technical applications, inverted media are of interest in the study of certain natural phenomena, in particular, stellar atmospheres. Here the great distances may lead to a noticable amplification effect without feedback for comparatively small plasma densities when radiation reabsorption turns out to be insignificant.