MAGNETOHYDRODYNAMIC COUPLING USING A LIQUID METAL FLYWHEEL

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A scheme for a coupling with a liquid-metal flywheel is considered. Rotation of the liquid metal is realized by means of crossed electric and magnetic fields. The steady-state problem of the motion of a conducting fluid in an axial magnetic field in a channel with rotating cylindrical electrodes is solved with allowance for the effect of the nonconducting end plates.

Liquid metals are now widely used as working media in various systems. By virtue of their special properties (good thermal and radiation resistance in the high-temperature region, high bulk modulus, high electrical and thermal conductivity) liquid metals can often be substituted for various solid and gaseous elements in systems working under special conditions.

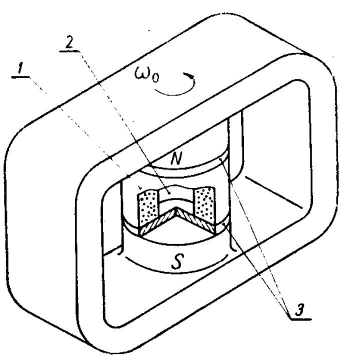

Fig. 1

In particular, in [1, 2] it was shown that liquid metals can be widely used as the working medium in various hydraulic control systems.

We shall consider an angular orientation control system using flywheels. Such a system can be successfully employed to compensate random external perturbations. Usually the flywheel is essentially an electric motor with a rotor capable of creating the necessary torque. The chief disadvantage of mechanical flywheels is the presence of bearings, which complicates the design and reduces the reliability. The proposed system using liquid-metal flywheels [1, 2] is not only free of this shortcoming but also possesses a number of inherent advantages. Thus, for example, in [2] it was shown that an experimental model of a coupling working on liquid mercury pumped by an electromagnetic pump had about 60 times less inertia than ordinary flywheels.

In our research we examined one possible scheme for a coupling with a liquid-metal flywheel (Fig. 1). The magnetic field is created by a permanent magnet or solenoid. The control voltage is applied to cylindrical electrodes 1 and 2. The end plates 3 are made of nonconducting material. Henceforth these elements will be referred to collectively as the stator. When the control voltage is turned on, a radial current flows in the liquid metal filling the space between the electrodes. The liquid metal then starts to rotate. The equation of conservation of total moment of momentum of the system (stator plus liquid metal) when there is no external forces moment is written in the form

$$\mathfrak{J}\omega_0+\mathfrak{L}=0, \qquad (1)$$

where $\mathfrak{J}$ is the moment of inertia of the stator, ω_0 is the angular velocity of rotation of the stator, and $\mathfrak{L}$ is the moment of momentum of the liquid-metal rotor. In this case, therefore, when an electric current flows, the stator and the liquid-metal rotor rotate in opposite directions, the direction of rotation of the stator being determined by the polarity of the control voltage.

In order to establish some of the characteristics of this MHD coupling we shall consider the following problem.

Suppose a conducting fluid occupies the channel formed by two coaxial cylindrical electrodes of radius r_1 and r_2 and height h (the electrodes are perfect conductors) and two nonconducting end plates (Fig. 2). The external uniform magnetic field with induction B_0 is directed parallel to the cylinder axis. A constant potential difference U is applied to the electrodes, while the walls of the channel rotate with angular velocity ω_0, whose magnitude we then determine from expression (1).

We will solve the problem on the following assumptions: 1) the flow is steady, 2) the fluid is incompressible, i.e., its density ρ, kinematic viscosity ν, and electrical conductivity σ are constants, 3) the flow is laminar and two-dimensional. It can be shown that in this case $B_r = 0$, and $B_z = B_0 = \text{const}$.

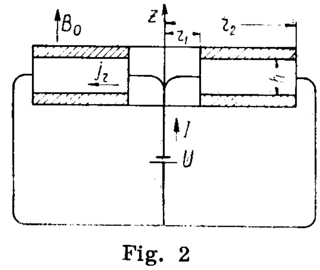

Fig. 2

In our study we shall confine ourselves to the case $h/(r_2 - r_1) \ll 1$. This assumption means that it is possible to neglect the effect of the electrodes on the fluid flow, and take into account only the effect of the nonconducting end plates.

Problems of stationary and nonstationary rotation of a conducting fluid in an axial magnetic field with allowance for the effect of the end plates have been considered in [3–5] and elsewhere. However, in these studies the walls of the channel were assumed to be at rest. The boundary conditions of our problem differ considerably from those adopted in [3–5].

With the above assumptions, equations of magnetohydrodynamics [6] for steady flow may be written in the form

$$\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{B_0}{\rho\nu\mu_0}\frac{\partial B_\varphi}{\partial z}=0, \quad (2)$$

$$\frac{\partial^2 B_\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial B_\varphi}{\partial r}-\frac{B_\varphi}{r^2}+\frac{\partial^2 B_\varphi}{\partial z^2}+B_0\sigma\mu_0\frac{\partial v}{\partial z}=0, \quad (3)$$

where v and B_φ are the azimuthal velocity and magnet magnetic field, and the rest of the notation is that usually employed. The current density j and electric field E are expressed in terms of v and $B\varphi$ as follows:

$$\mu_0 j_r=-\frac{\partial B_\varphi}{\partial z}, \quad (4)$$

$$\mu_0 j_z=\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi), \quad (5)$$

$$E_r=-\frac{1}{\mu_0\sigma}\frac{\partial B_\varphi}{\partial z}-vB_0, \quad (6)$$

$$E_z=\frac{1}{\mu_0\sigma}\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi). \quad (7)$$

As a result of the perfect conductivity of the electrodes

$$\frac{\partial\ (rB_\varphi)}{\partial r}=0$$

at $r = r_1$ and $r = r_2$. Since outside the channel the current density is equal to zero, in those regions rB_φ = const. In this case in the upper region $B_\varphi = 0$, while in the lower region, embraced by the external current loop, $B_\varphi = \mu_0 I/2\pi r$, where I is the total current in the external circuit. Since the tangential component of the magnetic field is continuous, the boundary conditions for B_φ may be written in the form

$$B_\varphi=\mu_0 I/2\pi r \quad \text{at} \quad z=0; \quad (8)$$

$$B_\varphi=0 \quad \text{at} \quad z=h. \quad (9)$$

The boundary conditions for the velocity are $v = \omega_0 r$ at all boundaries.

By virtue of the assumption $h/(r_2 - r_1) \ll 1$ it is obvious that the axial current density (which at the nonconducting end plates should be zero) may be neglected as compared with the radial current density. We set $j_z = 0$, then using (3) and (5) we find that

$$B_\varphi=\frac{1}{r}\left[b(z)+\frac{\mu_0 I}{4\pi}\right], \quad (10)$$

$$v=\frac{w(z)}{r}+f(r). \quad (11)$$

Satisfying the boundary condition for the velocity at z = 0 and z = h, we get

$$v=\frac{w(z)}{r}+\omega_0 r \ \text{and} \ w(h)=w(0)=0. \quad (12)$$

We note that in this case the boundary conditions for the velocity at the electrodes ($r = r_1$ and $r = r_2$) are not satisfied. However, keeping in mind that for $h/(r_2 - r_1) \ll 1$ the effect of the electrodes on the fluid flow may be neglected, we assume that the profile (12) describes the channel flow except for the boundary layers at the electrodes.

Substituting (10) and (12) in (2) and (3), we get a system of equations for b(z) and w(z):

$$\frac{d^2 w}{dz^2}+\frac{B_0}{\rho\nu\mu_0}\frac{db}{dz}=0, \quad (13)$$

$$\frac{d^2 b}{dz^2}+B_0\sigma\mu_0\frac{dw}{dz}=0 \quad (14)$$

with boundary conditions

$$w(0)=w(h)=0, \quad b(0)=-b(h)=\mu_0 I/4\pi. \quad (15)$$

To obtain a solution in terms of the potential difference U, we integrate Eq. (6) with respect to r from r_1 to r_2, substituting v and B_φ from (10) and (12). We have

$$\frac{db}{dz}=-B_0\sigma\mu_0 w(z)-\frac{\sigma\mu_0}{\ln a}\left(U+B_0\omega_0 r_1^2\frac{a^2-1}{2}\right), \quad (16)$$

where $a = r_2/r_1$.

Substituting (16) in (13), for w(z) we get

$$\begin{cases} \dfrac{d^2 w}{dx^2}-M^2 w=\dfrac{M^2}{B_0\ln a}\left(U+B_0\omega_0 r_1^2\dfrac{a^2-1}{2}\right), \\ w(0)=0, \\ w(1)=0, \end{cases} \quad (17)$$

where x = z/h is the dimensionless axial coordinate, and $M = B_0 h\sqrt{\sigma/\rho\nu}$ is the Hartmann number. The solution of (17) is

$$w=\frac{\left(U+B_0\omega_0 r_1^2\dfrac{a^2-1}{2}\right)}{B_0\ln a}\left\{1-\frac{\mathrm{ch}\left[M\left(\dfrac{1}{2}-x\right)\right]}{\mathrm{ch}\dfrac{M}{2}}\right\}. \quad (18)$$

Using (15) and (16), we find

$$b(x)=\frac{\mu_0\sigma h}{M\ln a}\left(U+B_0\omega_0 r_1^2\frac{a^2-1}{2}\right)\frac{\operatorname{sh}\left[M\left(\frac{1}{2}-x\right)\right]}{\operatorname{ch}\frac{M}{2}}. \quad (19)$$

Finally, for v, B_φ and I we have

$$v=\omega_0 r-\frac{\left(U+B_0\omega_0 r_1^2\frac{a^2-1}{2}\right)}{B_0\ln a}\frac{1}{r}\left\{1-\frac{\operatorname{ch}\left[M\left(\frac{1}{2}-x\right)\right]}{\operatorname{ch}\frac{M}{2}}\right\}, \quad (20)$$

$$B_\varphi=\frac{\mu_0 h\sigma\left(U+B_0\omega_0 r_1^2\frac{a^2-1}{2}\right)}{M\ln a}\times\frac{1}{r}\left\{\frac{\operatorname{sh}\left[M\left(\frac{1}{2}-x\right)\right]}{\operatorname{ch}\frac{M}{2}}+\operatorname{th}\frac{M}{2}\right\}, \quad (21)$$

$$I=\frac{4\pi h\sigma}{M\ln a}\left(U+B_0\omega_0 r_1^2\frac{a^2-1}{2}\right)\operatorname{th}\frac{M}{2}. \quad (22)$$

Naturally, at $\omega_0 = 0$ Eqs. (20)–(22) go over into the corresponding equations of [3, 4].

We now find the moment of momentum of the liquid:

$$\mathfrak{L}=2\pi\rho h\int_{r_1}^{r_2}\int_0^1 v(r,x)r^2 drdx=-\frac{\pi U\rho h r_1^2(a^2-1)}{B_0\ln a}\left[1-\frac{\operatorname{th}\frac{M}{2}}{\frac{M}{2}}\right]+2\pi\rho h\omega_0 r_1^4\frac{a^4-1}{4}\left\{1-\left[1-\frac{\operatorname{th}\frac{M}{2}}{\frac{M}{2}}\right]\frac{a^2-1}{(a^2+1)\ln a}\right\}. \quad (23)$$

Using expression (1), we can obtain the angular velocity of the stator as a function of the parameters:

$$\omega_0=\frac{\pi U\rho h r_1^2(a^2-1)\left[1-\frac{\operatorname{th}\frac{M}{2}}{\frac{M}{2}}\right]}{B_0\ln a\left\{\mathfrak{J}+\frac{\pi\rho h r_1^4(a^4-1)}{2}\left[1-\left(1-\frac{\operatorname{th}\frac{M}{2}}{\frac{M}{2}}\right)\frac{a^2-1}{(a^2+1)\ln a}\right]\right\}} \quad (24)$$

To determine the power characteristics of such a MHD coupling we substitute (24) in (22). We then get

$$U=IR, \quad (25)$$

where R is the effective dc resistance of the device, which depends on the external magnetic field, the moment of inertia of the stator, the channel geometry, and the properties of the fluid.

In the steady-state regime the electric power $P = |IU|$ will be converted into heat as a result of viscous dissipation and joule heating of the fluid.

To estimate the inertia properties of such a MHD coupling, we use the maximum velocity relaxation time for pulsed operation of the control voltage taken from the solution of the nonstationary problem [3, 4]:

$$\tau_{max}=h^2/\nu(M^2+\pi^2). \quad (26)$$

Consequently, for pulsed operation of the voltage, the time required for the angular velocity of the stator to become established will be of the order of τ_{max}. Therefore with pulsed voltage control, to use up the angular velocity of the stator corresponding to the voltage of a given pulse the duration of the latter should be greater than τ_{max}.

Following [3], we shall call the external magnetic field "strong" if the inequality $B_0^2 \gg 100\pi^2\rho\nu/(r_2 - r_1)\sigma$ is satisfied, or "weak" if the opposite inequality holds. As estimates show [3], in practice all the above equations can be used for "strong" fields at $h/(r_2 - r_1) < 0.4$ and for "weak" fields at $h/(r_2 - r_1) < 0.08$.

Using the equations obtained above, we shall calculate a MHD device of this type for the following data. As working medium we shall take mercury at 20° C ($\rho = 13.55 \cdot 10^3$ kg/m^3, $\nu = 1.17 \cdot 10^{-7}$ m^2/sec, $\sigma = 10^6$ 1/ohm·m) The channel dimensions are: $r_1 = 1$ cm, $r_2 = 10$ cm, $h = 1$ cm. The external magnetic field $B_0 = 0.1$ Wb/m^2. The moment of inertia of the stator $J = 100$ kg · m^2. Then

$$\omega_0\approx 0.2U;\quad I\approx 2\cdot 10^3 U;\quad \tau_{max}\approx 1.3 \text{ sec}$$

It should be noted that the question of the limit of stability of a laminar flow of the type considered above has not, to the best of the author's knowledge, previously been dealt with in the literature. Existing experimental data [7] of a study of the steady-state rotation of a plasma formed as a result of an electrical discharge in air ($r_1 = 3.4$ cm, $r_2 = 9.8$ cm, $h = 3.6$ cm, $H_0 = 2.10^5$ A/m, $I = 0.8$ A, $T = 700°$ K) indicate that in the range of Reynolds numbers $10 \le Re \le 300$ the flow was distinctly laminar.

Reference [8] is devoted to a theoretical study of the turbulent flow of a conducting fluid in a cylindrical homopolar device under the action of Ampere forces.

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