INTRINSIC MAGNETIC FIELD IN THE ONE-DIMENSIONAL FLOW OF AN ELECTRICALLY CONDUCTING FLUID

E. I. Yantovskii

Magnitnaya Gidrodinamika, Vol. 1, No. 4, pp. 53–56, 1965

UDC 538.4

The author discusses certain properties of the one-dimensional flow of an inviscid incompressible electrically conducting fluid in a long channel of rectangular cross section formed by insulating and conducting walls.

When a conducting fluid flows along a channel (Fig. 1) in a magnetic field B_1, created by an exciting winding (or existing as a result of residual magnetism), an emf is produced which causes a current I to flow in the fluid.

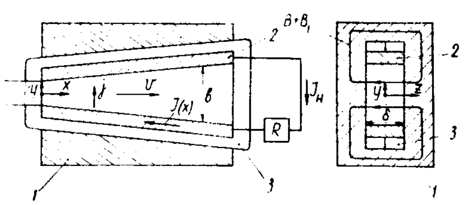

Fig. 1. Flow diagram: 1) magnetic circuit, 2) electrode, 3) exciting winding.

The magnetic field B (called the intrinsic field) of the current fed into and out of the fluid by means of electrodes operates in conjunction with the external field B_1 and may be large for a fluid that is a good conductor.

The equations of magnetohydrodynamics of an inviscid incompressible fluid have the form

$$\operatorname{div} \mathbf{v} = 0; \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B};$$

$$\mathbf{j} = (1/\mu) \operatorname{rot} \mathbf{B}; \quad \mathbf{j} = \sigma(-\nabla\varphi + \mathbf{v} \times \mathbf{B}).$$

In our case of steady-state flow in a long and narrow channel it is assumed that the following approximation is valid:

$$\mathbf{j} = (0, j, 0); \quad \mathbf{B} = (0, 0, B + B_1); \quad \mathbf{v} = (v, 0, 0);$$

$$\nabla p = dp/dx; \quad \nabla\varphi = U/b,$$

where U is the voltage.

The equations assume the following form: equation of continuity of viscosity

$$\rho v b \delta = G; \tag{1}$$

momentum equation

$$\rho v \frac{dv}{dx} = -\frac{dp}{dx} + j(B + B_1); \tag{2}$$

Ohm's law

$$j = \sigma\left[-\frac{U}{b} - v(B + B_1)\right]; \tag{3}$$

Maxwell's equation for the one-dimensional problem

$$j = -\frac{dH}{dx} = -\frac{1}{\mu}\frac{dB}{dx}; \tag{4}$$

Ohm's law for an unsaturated magnetic circuit of negligibly small magnetic resistance

$$B = \mu I/\delta. \tag{5}$$

In the generator mode shown in Fig. 1 the vector components have the following signs: j positive, v positive, B negative and the electric field U/b negative (but U positive).

In what follows we shall treat two particular cases only (these are fairly characteristic and will enable us to obtain intuitive results easily), namely: flow at constant velocity and flow at constant pressure.

a) The case v = const. Here we also have b = const and δ = const. Substituting (5) and (4) in (2), we get

$$\frac{dp}{dx} = -(B + B_1)\frac{1}{\mu}\frac{dB}{dx}. \tag{6}$$

On integrating (allowing for the condition that $p = p_1$ for B = 0, we obtain

$$p - p_1 = -\frac{(B + B_1)^2}{2\mu} + \frac{B_1^2}{2\mu}. \tag{7}$$

Setting (4), (1), and (5) in (3), we obtain the induction equation

$$-\frac{1}{\mu}\frac{dB}{dx} = \sigma\left[-\frac{U\rho\delta}{G}v - (B + B_1)v\right]. \tag{8}$$

Introducing the notation

$$B_0 = -\frac{U\rho\delta}{G}; \quad \bar{x} = x\mu\sigma v, \tag{9}$$

we obtain

$$\frac{dB}{B + B_1 - B_0} = d\bar{x}, \tag{10}$$

whence

$$B = (B_1 - B_0)(e^{\bar{x}} - 1). \tag{11}$$

This case has been treated previously (for example, [1]) and is given for the sake of comparison. The introduction of a virtual magnetic field B_0, which is a function of the voltage, simplifies the analysis of the results somewhat.

The argument of $\bar{x}$ is the analog of the magnetic Reynolds number, where the characteristic length is the length of the channel up to the cross section under consideration.

b) The case p = const. It is assumed here that $\delta = \text{const}$, and that the channel width varies in such a way that a constant pressure results, i.e., $b = G/\rho\delta v$.

From (2), (4), and (5) we have

$$\rho v \frac{dv}{dx} = -\frac{B+B_1}{\mu}\frac{dB}{dx}, \tag{12}$$

whence

$$\rho\frac{v^2}{2} - \rho\frac{v_1^2}{2} = -\frac{(B+B_1)^2}{2\mu} + \frac{B_1^2}{2\mu}$$

or

$$v = v_1\sqrt{1 - \frac{B^2}{\rho\mu v_1^2} - 2\frac{BB_1}{\rho\mu v_1^2}}, \tag{13}$$

where the condition $B = 0$ for $v = v_1$ has been employed. Introducing the notation

$$a^2 = 1/\mu\rho v_1^2, \tag{14}$$

we obtain the induction equation

$$d\bar{x} = \frac{dB}{(B - B_0 + B_1)\sqrt{1 - 2a^2B_1B - a^2B^2}}, \tag{15}$$

whence, in view of the condition that $B = 0$ for $\bar{x} = 0$, we obtain

$$\bar{x} = \frac{1}{\sqrt{1 + B_1^2a^2 - B_0^2a^2}} \times \ln\frac{(B + B_1 - B_0)[1 + a^2(B_1^2 - B_0B_1) + \sqrt{1 + B_1^2a^2 - B_0^2a^2}]}{(B_1 - B_0)[1 + a^2(B_1^2 - B_0B_1) - a^2BB_0 + \sqrt{1 - a^2B^2 - 2BB_1a^2}\sqrt{1 + B_1^2a^2 - B_0^2a^2}]}. \tag{16}$$

In both the cases considered a conversion of fluid energy to electrical energy, liberated in the resistance R, may take place, in case a) potential energy is converted, and in case b) kinetic energy. The conversion efficiency is determined by the relation

$$\eta = \frac{IU}{G\left(\frac{p_1}{\rho} + \frac{v_1^2}{2} - \frac{p}{\rho} - \frac{v^2}{2}\right)}. \tag{17}$$

Expressing all quantities in (17) in terms of the magnetic field in accordance with (5), (9), (7), or (13), we obtain for both cases

$$\eta = \frac{B_0}{B_1 + B/2} = \frac{k_{y_1}}{1 + B/2B_1}, \tag{18}$$

where $k_{y_1} = U/b_1B_1v_1$ (in case a) $k_{y_1} = U/B_1bv$) is the "load parameter" in general use.

It is clear from (18) that when the intrinsic magnetic field B is used to strengthen the external field B_1 a diminution in the efficiency of energy conversion results.

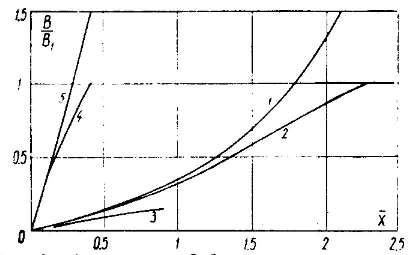

Fig. 2. Increase of the intrinsic magnetic field along the length of the channel: 1) v = const, $B_0/B_1 = 0.8$; 2) p = const, $B_1^2a^2 = 0.3$, $B_0/B_1 = 0.8$; 3) p = const, $B_1^2a^2 = 3$, $B_0/B_1 = 0.8$; 4) p = const, $B_1^2a^2 = 0.3$, $B_0/B_1 = -2$; 5) v = const, $B_0/B_1 = -2$.

Apparently, the same result is also valid for the flow of a compressible fluid. Thus it is pointless to use the intrinsic magnetic field in MHD generators with circuits like that given in Fig. 1.

Figure 2 shows the increase of magnetic field at constant velocity and constant pressure (weak and strong magnetic fields B_1).

In the case where v = const the magnetic field along the channel increases exponentially (and the pressure differential increases correspondingly); moreover, in unsaturated magnetic systems this increase in unlimited.

In the case where p = const the solution becomes meaningless at finite values of the channel length (the smaller the values the larger the external magnetic field), where the velocity falls to zero.

Curves 2 and 3 in Fig. 2 terminate at that B/B_1 for which $v \to 0$, but, in fact, the solution becomes meaningless even earlier, since as a result of the large increase of channel width the one-dimensional approximation becomes meaningless at low velocity.

Figure 2 also shows the increase of magnetic field when a current from an external source flows through the moving fluid (the sign of B_0 is changed, so that $B_0/B_1 = -2$). The magnetic field grows sharply, but the decrease of velocity in the channel results in the field increasing more slowly than in the case v = const.

In Eq. (12) for the case p = const the derivative dB/dx is negative, since B increases along the channel only in modulus, but the induction decreases algebraically.

If the signs of $B + B_1$ and dB/dx are different, then the device becomes an accelerator in which the kinetic energy of the fluid increases and the magnetic field decreases.

From (13) we find that the maximum quantity $v_{max} = v_1\sqrt{1+a^2B_1^2}$, where $B = -B_1$, i.e., the total effective magnetic field falls to zero.

The author wishes to thank E. I. Khanzhina for drawing the graphs.

REFERENCE

1. L. R. Blake, Proc. Inst. Electr. Eng., p. A., 13, February 1957.

19 March 1965.