GENERATION OF A MAGNETIC FIELD BY MIRROR-SYMMETRIC TURBULENCE

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A model of two-scale turbulence is proposed; being statistically mirror symmetric, it is capable of generating a turbulent magnetic field. The similarities and differences between the model and the experimentally observed flows with intermittence of active and less active regions are discussed.

Investigations of the sources of the magnetic field of astrophysical objects have posed the methodological problem of whether the field can be excited by the turbulent motion of an incompressible electrically conducting fluid. Hitherto, this has been established only for a particular case [1-7] when the turbulence on the average is not mirror-symmetric and the correlation $\overline{\mathbf{v} \cdot \text{rot}\, \mathbf{v}} \neq 0$ indicates either predominance of right-helical motions over left-helical ones, or vice versa. Moreover, it was also shown that the given correlation between $\mathbf{v}$ and rot $\mathbf{v}$ is formed in a liquid that rotates on the average and contains inhomogeneous turbulence (there are regions with grad $\overline{v^2} \neq 0$) [2, 3]. This Steenbeck–Krause mechanism explains the magnetic field of rotating objects such as the earth and the sun [8, 9].

The methodological question of the generation of a magnetic field by mirror-symmetric turbulence remains open [7, 10]. In [11, 12] it was shown that simplifications such as the neglect of correlations of third and higher orders [11] or Kraichnan's "approximation of direct interaction" do not ensure self-excitation and if it can arise at all, it is solely on the basis of more complicated correlations.

The aim of the present paper is to study the possibility of a choice of the velocity correlation which is such that for statistically mirror-symmetric turbulence, the Steenbeck–Krause mechanism is still capable of generating a turbulent magnetic field. Although it would be natural to treat the problem in the language of correlation functions, we nevertheless restrict ourselves to a qualitative comparison of the model turbulence with the situation found in rotating celestial bodies. This will be more perspicuous, and the quantative relations that follow from a systematic model will in any case have only limited applicability.

In the proposed model, the turbulence in an unbounded fluid of constant electrical conductivity σ is assumed to consist of motions on two scales λ and R (Fig. 1). For simplicity, it is assumed that $\lambda \ll R$. The small-scale ($\sim \lambda$) motion $\mathbf{v}(\mathbf{r}, t)$ is distributed irregularly over the volume and is concentrated within small spatially bounded regions of scale R. According to the adopted terminology, these regions are called activity regions [13], and the concentration phenomenon itself is called turbulence intermittency [14]. In the large-scale ($\geq R$) motion $\mathbf{V}(\mathbf{r}, t)$ the whole liquid both within the activity regions and between them participates. In the large-scale motion the individual regions move relative to one another and are deformed, and, which is especially important, they rotate relative to fixed coordinate axes.

The statistical properties which follow from averaging over the complete statistical ensemble or over the complete (infinite) volume or over a sufficiently long interval of time are, for this turublence model, isotropic, homogeneous, stationary, and mirror symmetric. The mirror symmetry entails the vanishing of the mean values of all variables that change their sign under reflection, for example, $\overline{(\mathbf{V}+\mathbf{v}) \cdot \text{rot}\,(\mathbf{V}+\mathbf{v})} = 0$.

Apart from the complete averaging over infinite volume (and time), the inequality $\lambda \ll R$ enables one to perform a partial averaging (symbol ....). Identical results are obtained both from partial spatial averaging over volumes l^3 for l such that $\lambda \ll l \ll R$, and from partial time averaging over intervals T that

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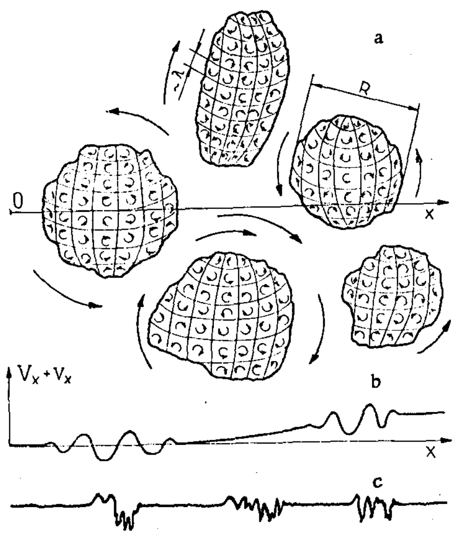

Fig. 1. a) Two-scale turbulence. b) Approximate dependence of $V_x + v_x$ on x on the x axis. c) Time trace of the turbulent velocity in a magnetic field measured in [24].

greatly exceed the time of small-scale correlations τ_v and are much shorter than the time of large-scale correlations τ_V: $\tau_v \ll T \ll \tau_V$. The partial averaging affects only the small-scale motion: $\overline{v}=0$, and $\overline{v^2}$ is obtained as both the spatial and the time dependence with characteristic scale τ_V and R.

After partial averaging within an individual activity region we have a situation completely analogous to that in rotating celestial bodies. Each region rotates on the average with angular velocity $\Omega = \frac{1}{2}$ rot **V**, and within it small-scale turbulence is excited with intensity gradient grad $\overline{v^2}$ directed from the boundary to the center of the region. Coriolis forces form the correlation

$$\overline{\mathbf{v}\cdot\mathrm{rot}\,\mathbf{v}} = -\frac{24}{25}\tau_v\Omega\cdot\mathrm{grad}\,\overline{v^2} \qquad (1)$$

which is negative (left-helical motions predominate over right-helical ones) in the half of the activity region in which the angle between Ω and grad $\overline{v^2}$ is less than a right angle and positive (right-helical motions predominate) in the remaining part of the region. The integral of (1) over the whole volume of the activity region is zero.

In each activity region the same partly averaged equations of electrodynamics hold as in celestial bodies:

$$\mathrm{rot}\,\overline{\mathbf{H}} = \overline{\mathbf{j}}, \quad \overline{\mathbf{j}} = \sigma_{ef}(\overline{\mathbf{E}} + [\mathbf{V}\times\overline{\mathbf{B}}]) + \alpha\overline{\mathbf{H}}, \qquad (2)$$

where

$$\alpha = \frac{\lambda^2\mu^2\sigma^2}{18}\,(\overline{\mathbf{v}\cdot\mathrm{rot}\,\mathbf{v}}). \qquad (3)$$

Formally, the derivation of (1)-(3) in [2, 3] is restricted to the inequalities $|\Omega|\cdot\tau_v \ll 1$ and $\mu^2\sigma^2\lambda^2 v^2 \ll 1$, though it was shown in [6, 7] that Eqs. (2) still hold when $\mu^2\sigma^2\lambda^2 v^2 \gg 1$, and only α takes on a factor depending on $\mu^2\sigma^2\lambda^2 v^2$. Therefore, to within a factor ~ 1, the expressions (1)-(3) can also be used when $|\Omega|\cdot\tau_v\mu^2\sigma^2\lambda^2\overline{v^2} \sim 1$. From (1)-(3) there follows the condition

$$|\Omega|\cdot\tau_v\mu^2\sigma^2\lambda^2\,\overline{v^2}_{\max} \geqslant \mathrm{const} \qquad (4)$$

of generation of a magnetic field in the given region. The word "generation" means that a zero magnetic field is an unstable solution of the system (2) and small random fluctuations of the field grow with time exponentially to finite amplitudes. If (3) is satisfied for an appreciable number of activity regions (this is not even required for absolutely all), then in the turbulence there arises not only a velocity component but also a magnetic component.

It should be noted that between the magnetic fields considered here and the fields of [11, 12] there are important differences. In both cases the field is the result of superposition of two parts. But in [11, 12] the main field varies in magnitude and direction only over distances comparable with the dimensions of the whole fluid conductor (global field). The rapidly varying turbulent part is much smaller in magnitude and varies over distances of the order of the turbulence scale. In the present paper both parts of the field are turbulent, though they have very different scales: R for the main part, λ for the weaker part.

Thus, the Steenbeck–Krause mechanism ensures generation of a magnetic field for the given model of two-scale turbulence. However, the question arises of the extent to which the very model corresponds to the properties of actually observed turbulence, i.e., how turbulence can be collected in fact into isolated activity regions. In an electron–ion plasma such a phenomenon arises with increasing wavelength and energy density in the high-frequency electrostatic turbulence branch [15]. On the transition through the critical threshold, a spatially homogeneous turbulence structure becomes unstable and the turbulence is concentrated in bounded regions with reduced density of the matter. In an incompressible liquid the formation of activity regions was observed long ago [14]. This is revealed, in particular, by the time traces of the velocity, in which turbulent bursts alternate with quiescent sections (Fig. 1c). The observed structure is very similar to Fig. 1b, although in it there are the following three important differences from the model.

a) Each activity region contains a whole spectrum of motions without any definite scale λ. Because of the presence in (3) of the factor $\lambda^2\tau_V$, the magnetic field effectively generates a large-scale wing of the small-scale spectrum, i.e., essentially the most pronounced of the pulsations that form the activity region. This circumstance does not alter the generation principle but only strengthens the following difference.

b) In every region there is not a very large number of the largest pulsations and there is no guarantee that the inequality $\lambda \ll R$ is appropriately satisfied. This inequality enables one to use the averaged equations (2) and is a necessary condition for fulfillment of equation (4) and all the preceding exposition. For the generation process itself it is, together with (4), only a sufficient condition. Unfortunately, a necessary condition has not yet been investigated. Even for rotating celestial bodies it is not known whether the generation ceases with increasing λ/R and, if it does cease, at what value of this ratio. Therefore, we do not yet have the possibility of determining whether the form of the observed turbulent bursts is suitable for the generation of a field or not. One cannot even exclude the possibility of generation in the case of coalescence of all the active regions into a continuous turbulence, when the third characteristic difference considered below is unimportant.

c) In contrast to an electron–ion plasma, in which the formation of active regions characterizes developed turbulence, in an incompressible liquid the majority of observations are for the transition regime of intermittence of laminar and turbulent regions with characteristic numbers $Re_V + RV/\nu \sim 10^3$ and $Re_v = \lambda v/\nu \sim 10$ for the large- and small-scale motions, respectively. For the generation of a field, such values of Re_v are inadequate. Since Re_v^2 on the right-hand side of (4) differs by the factor $(\mu\sigma\nu)^{-2}$, which for molten metals is equal to 10^{12}, generation also arises for Re_v not less than 10^6. In recent years the observation of turbulent bursts was accomplished for a developed turbulent regime with $Re_V \sim 10^6$ [16-19], though Re_v remained small as before. At the present time there is no common opinion as to the reason why turbulence should be concentrated in active regions. If it is a general property [20, 21] of developed turbulence, then with a further increase in Re_V one can expect an increase in Re_v as well. But if it is due to effects in the laminar sublayer, then there are no reasons to expect Re_v to increase to values 10^6. One should add that if the medium is assumed incompressible, then it is even theoretically impossible to imagine the existence in it of arbitrarily large values of Re_V. In an incompressible liquid the height differences are restricted by the insignificant hydrostatic compression, and the velocity by the velocity of sound u, so that Re_V in any case satisfies

$$Re_V \ll \frac{u^3}{\nu g}.$$

Under the conditions of terrestrial gravitation an estimate for liquid metals gives

$$Re_V \ll 10^{16}.$$

The same estimate is obtained under conditions of selfgravitation of a volume of order R^3; under stellar conditions the right-hand side is much lower. Thus, the value of 10^{16} is an upper limit for an incompressible liquid.

Well-defined activity regions are observed on the exchange of laminar and turbulent regimes in an external magnetic field [22-24]. With increasing field, the turbulence boundary is shifted to larger Re_V and in principle the values of Re_v are also in no way restricted. However, one cannot here speak of field generation from zero but only of a certain interconnected regime in which active regions are formed in a strong magnetic field and these in their turn sustain the field. For the transition to this regime there must exist an initial field of finite strength in at least a small region, whence with time it can gradually extend over the whole liquid.

Thus, the proposed mirror-symmetric model can in principle generate a turbulent magnetic field. This model is qualitatively similar to some forms of observed turbulence, and only the quantitative properties of the hitherto investigated flows may be inadequate for generation. One requires further investigation to establish the minimal value of R/λ needed for generation and the conditions under which a sufficiently intense turbulence can arise with such values of R/λ. The long-scale wing of the turbulence spectrum, which according to a) gives rise to generation is sensitive to the origin of the turbulence and therefore one cannot in principle exclude the possibility of realization of the model under certain conditions.

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