PLANE UNSTEADY MOTION OF A CONDUCTING NON-NEWTONIAN FLUID

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We consider a plane unsteady problem of shear flows of a conducting incompressible fluid with a rheological power law in a transverse magnetic field, whose induction depends on the time, with the velocity of a plate lying on the fluid varying with time according to a power law. A generalized solution is constructed in the form of a power series. The limiting transition to a nonconducting fluid is investigated.

Let a conducting non-Newtonian fluid with a rheological power law occupy the half-space $z < 0$. Below, we restrict ourselves to a consideration of dilatant fluids, for which the nature of the shear flows qualitatively differs from the nature of flows of Newtonian fluids, since shear perturbations in such media propagate with a finite velocity [1-4].

The external magnetic field is directed along the z axis, with the induction of this field varying with time according to the law $B(t) = B_0 t^{-1/2}$. There is no electric field in the medium.

A nonconducting plate that initially lies on the fluid ($z = 0$) leads to a motion such that its velocity varies in time according to the power law $U(t) = u_0 t^{\alpha}$ ($\alpha \geq 0$). The corresponding problem describing unsteady shear flow of a fluid in the noninductive approximation is written in the form ($n > 1$)

$$\begin{cases} u_t = \mathfrak{L} u - \gamma(t) u \quad (-\infty < z < 0), \\ u(0, t) = u_0 t^{\alpha}, \quad u(-\infty, t) = 0, \quad u(z, 0) = 0, \end{cases} \tag{1}$$

where

$$\mathfrak{L} = a \frac{\partial}{\partial z}\left[\left|\frac{\partial}{\partial z}\right|^{n-1} \frac{\partial}{\partial z}\right], \quad \gamma(t) = \frac{\gamma_0}{t}, \quad a = \frac{k}{\varrho}, \quad \gamma_0 = \frac{\sigma B_0^2}{\varrho}.$$

By virtue of the monotonicity of the increase in the velocity of the plate it is evident that for the flow being investigated we have $\partial u/\partial z > 0$.

We seek a solution to the nonlinear boundary-value problem (1) in the form

$$u(z, t) = u_0 t^{\alpha} f(\xi), \tag{2}$$

where

$$\xi = 1 + \frac{z}{ct^{\beta}}, \quad c = \text{const}, \quad \beta = \frac{\alpha(n-1)+1}{n+1}.$$

Substituting (2) into (1), for the function $f(\xi)$ we obtain the ordinary differential equation

$$(f')^{n-1} f'' = \frac{c^{n+1}}{a n u_0^{n-1}} \left[(\alpha + \gamma_0) f + m(1-\xi) f'\right] \tag{3}$$

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with boundary conditions

$$f(1)=1;\quad f(-\infty)=0. \tag{4}$$

We note that $f \equiv 0$ satisfies (3), which is a singular solution of this equation.

In the region $\xi \geq 0$ the particular solution (3) can be represented in the form of a power series

$$f(\xi)=\xi^{\frac{n}{n-1}} \sum_{i=0}^{\infty} b_i \xi^i \left(\sum_{i=0}^{\infty} b_i \right)^{-1}, \tag{5}$$

where

$$b_0=\frac{1}{n},\quad b_1=\frac{(n-1)(\alpha+\gamma_0)-\beta n}{2\beta n(2n-1)(n-1)};$$

$$b_2=b_1 \frac{(n-1)(\alpha+\gamma_0)-\beta(2n-1)-0{,}5 b_1 \beta(n-1)(3n-2)(2n-1)^2}{3\beta(n-1)(3n-2)}, \text{ etc.},$$

where the constant c is defined as

$$c=\left\{ \frac{n\alpha}{\beta(n-1)^n} \left[u_0 \left(\sum_0^{\infty} b_i \right)^{-1} \right]^{n-1} \right\}^{\frac{1}{n+1}}. \tag{6}$$

We note that all the coefficients b_i for $i > 1$ are expressed in terms of the coefficient b_1, such that if $b_1 = 0$, then all the $b_i = 0$ $(i > 1)$.

Equation (5) satisfies the conditions $f(1) = 1$, $f(0) = 0$, and $f'(0) = 0$. This allows us to construct a generalized solution of (3), satisfying the boundary conditions (4), matching the singular solution $f(\xi) = 0$ for $\xi \leq 0$ with the particular solution (5) for $\xi > 0$ at the point $\xi = 0$. In this case at any point, the physical conditions of continuity of the velocity and tangential stresses will be satisfied. The constructed solution belongs to class C^m, since at the point $\xi = 0$ a discontinuity is undergone by derivatives beginning with the derivative of order $(m + 1)$, where m is the maximum integer such that $m < n/(n - 1)$.

Such a generalized solution corresponds to a shear wave whose front propagates from the plate along the unperturbed fluid with a finite velocity, where the position of the shear-wave front (point $\xi = 0$) at any time $t \geq 0$ is determined by the equation

$$z_0(t)=-ct^{\beta}. \tag{7}$$

In particular, for $\alpha = n/(n - 1)$ the shear-wave front moves in the medium with a constant velocity.

As we see from (7), in the problem being considered, the effect of the shear-wave front coming to a halt [1] does not take place. This agrees with the results of [3], where it was indicated that the shear-wave front can move off to infinity if the induction of the external magnetic field decreases with time, and the plate velocity increases.

We separately consider the case when

$$\gamma_0=\frac{n}{n^2-1}-\frac{\alpha}{n+1} \quad \left(\alpha \leqslant \frac{n}{n-1}\right). \tag{8}$$

In this case for $i \geq 1$ all the $b_i = 0$, and Eq. (3) has the generalized solution

$$f(\xi)=\begin{cases} \xi^{\frac{n}{n-1}} & \text{for } \xi>0, \\ 0 & \text{for } \xi \leqslant 0, \end{cases} \tag{9}$$

satisfying the boundary conditions (4).

The solution obtained for problem (1) admits two limiting transitions, $\gamma_0 \to 0$ and $\alpha \to 0$. The first of these corresponds to motion of a fluid without a magnetic field. The case $\alpha = 0$ corresponds to the

Rayleigh problem for a dilatant conducting fluid, when the plate is initially set in motion with a constant velocity. This problem was considered in [4, 5].

In [5] in the construction of a solution to the Rayleigh problem, variational methods were used. However, by virtue of the fact that the minimization of the corresponding functional was carried out in a rather narrow class of functions, the corresponding solution should be considered as approximate. This, in particular, is why Vujanovic et al. [5] were unable to detect the finite propagation velocity of the shear perturbations for dilatant fluids.

In [4] a solution of the Rayleigh problem was constructed in the form of an expansion in the parameter γ_0, which assumes generally that this parameter is small. Furthermore, in the solution by such a method of the Rayleigh problem for a dilatant fluid, considerable difficulties arise in the construction of the solution near the shear-wave front.

The solution of problem (1) constructed above does not have the indicated deficiencies and allows us to obtain a solution of the Rayleigh problem for dilatant fluids in the form of a power series in the self-similar variable ξ, if we set $\alpha = 0$ in all the preceding equations. In particular, for $\gamma_0 = n/(n^2 - 1)$ the exact solution of the Rayleigh problem has the simple form

$$u(z, t) = \begin{cases} u_0 \left(1 + \dfrac{z}{ct^{\frac{1}{n+1}}}\right)^{\frac{n}{n-1}} & \text{for } |z| < ct^{\frac{1}{n+1}}, \\ 0 & \text{for } |z| \geqslant ct^{\frac{n}{n+1}}, \end{cases} \tag{10}$$

where

$$c = \left\{ a(n+1) u_0^{n-1} \left(\frac{n}{n-1}\right)^n \right\}^{\frac{1}{n+1}}.$$

In conclusion, we note that the problem can be solved similarly, when there is blowing or suction of a conducting fluid past the plate.

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