# MAGNETOHYDRODYNAMICS OF NON-NEWTONIAN FLUIDS

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Soviet work on the magnetohydrodynamics of fluids with intrinsic mechanical properties is reviewed. The effect of a magnetic field on the character of steady and unsteady flows of conducting nonlinear-viscosity media is discussed.

The magnetohydrodynamics (MHD) of non-Newtonian fluids is one of the problems of the mechanics of continuous media the importance of which was noted even in the very first issue of the present journal [1] and which has been subsequently widely discussed within its pages. The present review makes no pretense to completeness of bibliographic citations, and the list can be augmented by material from the fundamental monograph of Vatazhin, Lyubimov, and Regirer [2]. In the present paper we attempt to review the work on the MHD of non-Newtonian fluids published in "Magnitnaya Gidrodinamika" in the first 10 years of its fruitful existence.

It is well known that the mechanical properties of the media used in MHD installations sometimes differ considerably from the properties of an incompressible viscous (Newtonian) fluid. For example, the formal apparatus of the MHD of Newtonian fluids proves to be incapable of satisfactorily describing the motion of multiphase dispersed conducting media and the flow of liquid metals at temperatures slightly in excess of the melting point [3]. It is also known that many media change or even acquire non-Newtonian properties in strong electric and magnetic fields [4-7]. In these cases it is imperative in a more refined analysis to allow for the intrinsic mechanical properties of the moving medium. Naturally, as more facts of this sort have been discovered, a need has arisen for studying the magnetohydrodynamics of non-Newtonia fluids (magnetorheology) [8].

## 1. The Equations of Magnetorheology

1.1. In phenomenological rheology non-Newtonian media are classified in accordance with the character of the relationship between the deviator of the stress tensor $\mathfrak{s}$ and the rate of deformation tensor $\mathfrak{f}$ [9]. One of the classes of non-Newtonian media most often encountered in practice is that of nonlinear-viscosity media, for which this relationship (the rheological law) has a comparatively simple form. Nonlinear-viscosity media include: viscoplastic media (Shvedov–Bingham plastics), for which the rheological law has the form

$$\begin{cases} s_{ij}=2\left[\eta+\dfrac{\tau_0}{\sqrt{2f_{\alpha\beta}f_{\alpha\beta}}}\right]f_{ij} & \text{when } \sqrt{2s_{\alpha\beta}s_{\alpha\beta}}\geqslant\tau_0, \\ f_{ij}=0 & \text{when } \sqrt{2s_{\alpha\beta}s_{\alpha\beta}}<\tau_0, \end{cases} \tag{1}$$

where $\tau_0$ is the limiting shear stress and $\eta$ is the coefficient of dynamic viscosity; and fluids with a power rheological law (Ostwald–de Waele media), for which

$$s_{ij}=2k\,(2f_{\alpha\beta}f_{\alpha\beta})^{\frac{n-1}{2}}f_{ij},\quad k,n-\text{const}>0. \tag{2}$$

In accordance with the accepted terminology, fluids with a power rheological law are said to be pseudoplastics if $n < 1$ and dilatant if $n > 1$. Making in (1) and (2) the limiting transitions $\tau_0 \to 0$ and $n \to 1 \pm 0$ gives the case of a Newtonian fluid.

1.2. If an isotropically conducting incompressible medium is characterized by an arbitrary rheological equation

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$$R(\mathfrak{s}, \mathfrak{f}) = 0, \tag{3}$$

then, provided the displacement currents, the convective currents, and the Hall currents are negligible, the set of equations describing the motion of such a medium in electromagnetic fields

$$\varrho\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u}\right] = \nabla(-p+\mathfrak{s}) + \mathbf{j}\times\mathbf{B} + \mathbf{F}; \tag{4}$$

$$\nabla\mathbf{u} = \nabla\mathbf{B} = 0; \quad \nabla\mathbf{E} = \gamma/\varepsilon\varepsilon_0; \quad \mathbf{j} = \sigma(\mathbf{E} + \mathbf{u}\times\mathbf{B});$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{j}; \quad \nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t,$$

and closed by rheological equation (3) can be called the equations of magnetorheology. The notation here is as follows: $\rho$ is the density of the medium; **u** is the velocity vector; p is pressure; **j** is the vector current density; **B** is the vector magnetic induction; **E** is the vector electric field intensity; **F** is the vector force (per unit volume) of nonelectromagnetic origin; $\gamma$ is the electric charge density; $\varepsilon$ is the dielectric constant of the medium; $\varepsilon_0$ and $\mu_0$ are the electric and magnetic constants. As in ordinary MHD we are concerned with the motion of nonferromagnetic media, and accordingly $\mu$ has been set equal to unity.

1.3. Attempts to solve the equations of magnetorheology for the general case of an arbitrary rheological equation (3) run up against serious difficulties. In the first instance this comes about because of the equation of motion, which is essentially nonlinear even for one-dimensional flows. This difficulty can be overcome in the case of nonlinear-viscosity media, and it is precisely for this reason that, out of all non-Newtonian media, it is the MHD of nonlinear-viscosity media that has been developed most fully.

The rheological equation of nonlinear-viscosity media can be written in the form

$$s_{ij} = \varphi f_{ij}, \tag{5}$$

where $\varphi$ is a scalar function of the principal invariants of the rate of deformation tensor [9]. Accordingly, in the case of an incompressible nonlinear-viscosity medium, the equations of magnetorheology can be expressed in the form

$$\varrho\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u}\right] = -\nabla p + \varphi\nabla\mathfrak{f} + \mathfrak{f}\nabla\varphi + \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} + \mathbf{F}, \tag{6}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \frac{1}{\mu_0\sigma}\Delta\mathbf{B}, \quad \nabla\mathbf{u} = \nabla\mathbf{B} = 0.$$

If the magnetic Reynolds number $\mathrm{Re_m} \equiv \mu_0\sigma UL \ll 1$ (U and L denote the representative velocity and length), then the first term on the right side of the second equation of set (6) can be neglected, and when solving the equation of motion the magnitude of the magnetic induction can be regarded as prescribed (zero-induction approximation).

## 2. Steady MHD Flows of Viscoplastic Fluids

2.1. The existence in viscoplastic fluids of a limiting shear stress results in the formation in channel flows of such fluids of zones in which the viscoplastic medium moves in the manner of a solid body. In these quasisolid zones the stresses do not attain values sufficient to destroy the structure of the viscoplastic medium [see (1)].

The problem to be solved is one with unknown boundaries, it being required to determine the position and shape of the boundaries separating zones of viscous flow from zones of quasisolid motion. In solving the differential equations describing the motion of the fluid in regions of viscous flow one usually utilizes as additional boundary conditions the dynamic and kinematic conditions of motion of the quasisolid zone [10]. In the case of the simplest flows, the shape of surfaces separating zones of viscous flow from zones of quasisolid motion can be found from the symmetry conditions of the problem.

Evidently, in MHD flows of conducting viscoplastic fluids, a magnetic field, by altering the character of the stress distribution in the medium through the introduction of bulk (volume) electrodynamic forces, will affect the velocity profile of the fluid in the zones of viscous flow and also the position (and in some cases the shape as well) of the surfaces bounding the quasisolid zone.

The character of an MHD flow of viscoplastic fluid in a channel is determined by two dimensionless parameters: the Hartmann number Ha and the plasticity parameter S, where

$$\mathrm{Ha} = B_0 L\sqrt{\frac{\sigma}{\eta}}; \quad \mathrm{S} = \frac{\tau_0 L}{\eta U}. \tag{7}$$

2.2. The above-mentioned features of MHD flows of viscoplastic fluids we illustrate by the example of a steady flow in an annular channel $r_1 < r < r_2$ with conducting walls at $r = r_1 = \beta R$ $(0 < \beta < 1)$ and $r = r_2 = R$ under the action of a constant pressure gradient P in the presence of an external radial magnetic field with induction $B_r = B_0 R/r$.

By virtue of the symmetry of the problem, the quasisolid core region separating the regions of viscous flow will have the form of an annular sheet $\gamma_1 R < r < \gamma_2 R$, where $\beta \leq \gamma_1 < \gamma_2 \leq 1$.

As the characteristic length of the problem we introduce L = R, the outer radius of the annular channel, and as the characteristic velocity we introduce $U = PR^2/\eta$. The dimensionless equations determining the velocity distribution in the zones of viscous flow can then be written

$$\Delta_r u_\pm - \mathrm{Ha}^2 \frac{u_\pm}{r^2} \pm \frac{S}{r} = -1; \quad \Delta_r = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}, \tag{8}$$

where the plus sign corresponds to the region of viscous flow $\beta < r < \gamma_1$, where $(du/dr) > 0$, and the minus sign to the region $\gamma_2 < r < 1$, where $(du/dr) < 0$.

Equations (8) must be solved subject to the boundary conditions that the liquid adheres to the walls and that the shear stresses at the boundaries of the quasisolid zone equal the limiting shear stress. These conditions can be expressed

$$u_+(\beta) = u_-(1) = 0; \quad \left.\frac{du_+}{dr}\right|_{r=\gamma_1} = \left.\frac{du_-}{dr}\right|_{r=\gamma_2} = 0. \qquad (9),\ (10)$$

Integrating Eqs. (8) subject to boundary conditions (9) and (10) gives

$$u_+(r) = -\frac{Sr}{1-\mathrm{Ha}^2} - \frac{r^2}{4-\mathrm{Ha}^2} + C_1\left(\frac{r}{\beta}\right)^{\mathrm{Ha}} + C_2\left(\frac{r}{\beta}\right)^{-\mathrm{Ha}} \equiv \Phi_1(\mathrm{Ha}, S, \beta, \gamma_1, r);$$
$$u_-(r) = \frac{Sr}{1-\mathrm{Ha}^2} - \frac{r^2}{4-\mathrm{Ha}^2} + C_3 r^{\mathrm{Ha}} + C_4 r^{-\mathrm{Ha}} \equiv \Phi_2(\mathrm{Ha}, S, \gamma_2, r), \tag{11}$$

where

$$C_1(\mathrm{Ha}, S, \beta, \gamma_1) = \frac{S\beta(4-\mathrm{Ha}^2)\left[\mathrm{Ha}\left(\frac{\gamma_1}{\beta}\right)^{-\mathrm{Ha}-1} + 1\right] + \beta^2(1-\mathrm{Ha}^2)\left[\mathrm{Ha}\left(\frac{\gamma_1}{\beta}\right)^{-\mathrm{Ha}-1} + 2\frac{\gamma_1}{\beta}\right]}{\mathrm{Ha}(1-\mathrm{Ha}^2)(4-\mathrm{Ha}^2)\left[\left(\frac{\gamma_1}{\beta}\right)^{-\mathrm{Ha}-1} + \left(\frac{\gamma_1}{\beta}\right)^{\mathrm{Ha}-1}\right]};$$

$$C_2(\mathrm{Ha}, S, \beta, \gamma_1) = \frac{S\beta(4-\mathrm{Ha}^2)\left[\mathrm{Ha}\left(\frac{\gamma_1}{\beta}\right)^{\mathrm{Ha}-1} - 1\right] + \beta^2(1-\mathrm{Ha}^2)\left[\mathrm{Ha}\left(\frac{\gamma_1}{\beta}\right)^{\mathrm{Ha}-1} - 2\frac{\gamma_1}{\beta}\right]}{\mathrm{Ha}(1-\mathrm{Ha}^2)(4-\mathrm{Ha}^2)\left[\left(\frac{\gamma_1}{\beta}\right)^{-\mathrm{Ha}-1} + \left(\frac{\gamma_1}{\beta}\right)^{\mathrm{Ha}-1}\right]};$$

$$C_3 = C_1(\mathrm{Ha}, -S, 1, \gamma_2); \quad C_4 = C_2(\mathrm{Ha}, -S, 1, \gamma_2).$$

As the position of the boundaries of the quasisolid zone is not prescribed in condition (10), it follows that to find $\gamma_1$ and $\gamma_2$ recourse must be made additionally to the kinematic condition that all points of the quasisolid core move with constant velocity V,

$$u_+(\gamma_1) \equiv \Phi_1(\mathrm{Ha}, S, \beta, \gamma_1, \gamma_1) = V,$$
$$u_-(\gamma_2) \equiv \Phi_2(\mathrm{Ha}, S, \gamma_2, \gamma_2) = V; \tag{12}$$

and the dynamic condition that the vector sum of all forces acting on the quasisolid core, including the Ampere force $\mathbf{j} \times \mathbf{B}$, equals zero. This condition also establishes a relationship between $\gamma_1$, $\gamma_2$ and V. It can be brought to the form

$$\gamma_2^2 - \gamma_1^2 = 2\mathrm{Ha}^2 V \ln \gamma_2/\gamma_1 + 2S(\gamma_1 + \gamma_2). \tag{13}$$

Simultaneous solution of (12) and (13) gives the values of $\gamma_1$, $\gamma_2$, and V for prescribed Ha, S, and $\beta$. Thus, for example, for Ha = 3, S = 0.1, and $\beta = 0.5$, we find that $\gamma_1 = 0.62$, $\gamma_2 = 0.85$, and V = 0.0081. An analysis of relationships (12) and (13) shows that the magnetic field increases the velocity of motion of the quasisolid core and simultaneously increases its dimensions. On making the transition $\tau_0 \to 0$ $(S \to 0)$, $\gamma_1 \to \gamma_2$ and the zone of quasisolid motion collapses to a point corresponding to the point in the MHD flow of a Newtonian fluid where the shear stresses equal zero.

We note that the liquid moves in the channel only provided $P > P_*$, where

$$P_* = \frac{2\tau_0}{R(1-\beta)}. \tag{14}$$

It can be seen from (14) that the critical pressure gradient $P_*$ at which the fluid in the channel is brought into motion does not depend on the magnetic induction. It can be shown that this conclusion is general for MHD flows of viscoplastic fluids in any channel.

2.3. The above features of steady MHD flows of a viscoplastic fluid have been studied in a number of papers. In [11] an investigation is made of a gradient Hartmann flow in a planar channel under various electrical conditions, and [12] discusses a Couette flow in an annular channel with a radial external magnetic field (the flow being induced by the axial displacement of the inner wall of the channel). We mention that in a transverse magnetic field a stationary quasisolid core adjacent the channel wall can also be formed in a Couette flow of a viscoplastic fluid in a planar channel [2].

Steady rotational motion in an annular channel under the action of the Ampere force in the presence of a radial or axial external magnetic field was considered in [13]. We also note paper [14], in which the results of [11] were used to investigate heat exchange in a planar steady MHD flow of a viscoplastic medium.

The problem of a steady-state viscoplastic boundary layer in a transverse magnetic field is considered in [15, 16], where some approximate solutions are found that describe the flow in the boundary layer at large values of the plasticity parameter.

## 3. Unsteady MHD Flows of Viscoplastic Media

3.1. To date, studies of unsteady MHD flows of viscoplastic media have been made only in planar channels and in the zero-induction approximation. However, the analysis of even these simplest cases runs up against serious difficulties. The calculation of such unsteady flows involves solving an equation of parabolic type with an unknown (sought) moving boundary. It is necessary, of course, to determine simultaneously the solution of the differential equation describing flow in the viscous zone and the law of motion of the sought boundary of the quasisolid core. It is only in exceptional cases that a simple analytic solution can be found [17, 18]; in the general case the solution is described with the aid of complex functional equations [17, 19, 20]. This relates to all analytic methods provided they are correctly applied [21], which is not the case in [22].

3.2. The known analytic methods do not lend themselves to solving problems on unsteady MHD flows of viscoplastic media, and accordingly considerable importance attaches to approximate and numerical methods of solution. In particular, Kosachevskii used numerical methods to discuss in [23] an unsteady MHD flow of a viscoplastic medium in a half-space bounded by a plane surface and in [24] an unsteady Couette flow. Of the various numerical methods we mention in particular the Monte Carlo method, by means of which the law of motion of the sought boundary can be found without having to determine the velocity distribution over the entire channel. The volume of computer calculations is thereby greatly reduced [25]. The results of a Monte Carlo solution of some problems on unsteady MHD flows of viscoplastic media are cited in [8].

## 4. Steady MHD Flows of Fluids with a Power Rheological Law

4.1. The study of MHD flows of fluids with a power rheological law has shown the qualitatively different character of flows of dilatant and pseudoplastic media. This difference is observed even in steady flows, when, in the motion of a dilatant fluid in a transverse magnetic field, zones where the medium moves with a constant velocity can be formed. The formation of quasisolid zones in MHD flows of conducting dilatant fluids is, naturally, called the magnetic plasticity effect [26]. This effect is absent in the case of pseudoplastic and Newtonian fluids.

The magnetic plasticity effect is interesting from the point of view of the analytic nature of the result when the solution of the nonlinear differential equation has different forms in the different regions. In fact, in these eases the solution of the problem is the generalized solution of the differential equation, the integral curve of the generalized solution consisting of a particular solution and a singular solution [27] fitted together on the boundary of the quasisolid zone in accordance with the requirements of continuity of velocity and stress. The zone of quasisolid motion corresponds to the singular solution.

The determinative dimensionless parameter for MHD channel flows of a power-law fluid is the generalized Hartmann number $\mathrm{Ha}_n$:

$$\mathrm{Ha}_n{}^2 = \frac{\sigma B_0{}^2}{k} L^{n+1} U^{1-n}. \qquad (15)$$

We note that as $n \to 1$ this parameter goes over into the ordinary Hartmann number, while for $n \neq 1$ it depends on the characteristic velocity of the flow.

4.2. By way of example we consider the steady shear flow of a conducting dilatant fluid occupying the half-space $z > 0$ when the nonconducting plane at $z = 0$ moves with constant velocity $u_0$. The external transverse magnetic field $B_0$ is then uniform; there is no electric field. The appropriate boundary problem describing a flow of this sort can be formulated as follows:

$$\begin{cases} k\dfrac{d}{dz}\left[\left|\dfrac{du}{dz}\right|^{n-1}\dfrac{du}{dz}\right]-\sigma B_0^2 u=0 \quad (z>0,\ n>1); \\ u(0)=u_0=\text{const}, \quad u(\infty)=0. \end{cases} \tag{16}$$

The generalized solution of this problem has the following characteristic form [28]:

$$u(z)=\begin{cases} u_0\left(1-\dfrac{z}{z_0}\right)^{\frac{n+1}{n-1}} & \text{for} \quad 0<z<z_0, \\ 0 & \text{for} \quad z_0\leqslant z<\infty, \end{cases} \tag{17}$$

where

$$z_0=\left[\frac{2n(n+1)^n k u_0^{n-1}}{(n-1)^{n+1}\sigma B_0^2}\right]^{\frac{1}{n+1}} \tag{18}$$

It follows from (17) that the plate brings into motion not all layers of fluid (as in the case of Newtonian and pseudoplastic media), but only a sheet of finite thickness near the plate. In other words, in the case under consideration, the MHD boundary layer is spatially localized. As $n \to 1$ or $B_0 \to 0$, $z_0 \to \infty$ and the effect of the moving plate is extended to all layers of the fluid.

4.3. For media with a power rheological law solutions are known for steady MHD Hartmann and Couette flows in a planar channel [29] and for the flow in a boundary layer with a transverse magnetic field [30]. As was shown in [29], zones of quasisolid motion are formed for channel flows of a dilatant fluid only provided the generalized Hartmann number exceeds a certain critical value. In [31] a study is made of heat exchange in a steady planar gradient MHD flow of a power-law fluid. We note that in some cases the steady distributions can be obtained as limiting solutions of unsteady problems [28].

## 5. Unsteady MHD Flows of Fluids with Power Rheological Law

5.1. Studies of the characteristics of unsteady MHD flows of power-law fluids have helped to establish the physical interpretation of the magnetic plasticity effect. It was found that this effect is closely linked with the property of dilatant fluids whereby shear disturbances in such media propagate with a finite velocity, in contrast to the case of Newtonian and pseudoplastic fluids, in which the velocity of propagation of shear disturbances is infinite.

Shear disturbances in dilatant fluids propagate with a finite velocity even in zero magnetic field, but when a magnetic field is present in conducting media the velocity of motion of the shear-wave front is slowed down and in some cases it is stopped, having penetrated into the magnetized fluid by only a finite distance from the source of the disturbances (plate, channel walls, and so on). In a steady flow we then, in effect, have a situation in which the shear-wave front has stopped at a finite distance from the source of the disturbances, the zone of a quasisolid motion being the region into which the shear disturbances have not penetrated. The depth to which disturbances penetrate the medium decreases with increasing induction of the external magnetic field. Accordingly, for steady channel flows, a value of the generalized Hartmann number exists at which disturbances from the channel walls no longer interfere and, in the center of the channel, a zone of quasisolid motion is formed [29] into which these disturbances do not penetrate.

We note that even the study of the simplest unsteady problems of the MHD of power-law fluids involves solving boundary problems for nonlinear differential equations of parabolic type. The mathematical difficulties are very considerable [32]. It has thus been possible to obtain exact solutions only in the rare cases of self-similar flows. In general one must resort to approximate or numerical methods.

5.2 Exact analytic solutions have been obtained in the zero-induction approximation for unsteady shear flows of a conducting fluid with a power rheological law occupying a half-space bounded by a nonconducting plate, movement of which induces flow of the fluid in a transverse magnetic field. In [33] a study was made of self-similar flows in a constant magnetic field for certain special laws of motion of the plate. In [34] the problem of the motion of the fluid under the action of an impulse communicated to the plate at the initial moment of time was generalized to the case of a magnetic field arbitrarily varying in time. The authors of [35] and [36] considered unsteady shear flows when $B \sim t^{-1/2}$, the velocity of the plate being taken in [35] as constant for $t > 0$, and in [36] as an arbitrary power-law function of time. In [35] the solution was constructed as an expansion in the MHD interaction parameter; in [36] the solution was obtained as a power series in the self-similar variable.

It was shown in [37] that transverse injection in shear hydrodynamic flows of a dilatant fluid affects the velocity of motion of the shear-wave front, in some cases causing the front to stop at a finite distance from the source of the shear disturbances. The authors of [38] and [39] studied the effect of transverse efflux in unsteady MHD flows of a dilatant fluid. They found that a magnetic field and injection (efflux) of the fluid through a porous plate had a joint effect on the velocity of motion of the shear-wave front.

## 6. Hydrodynamic Stability of Laminar MHD Flows of Nonlinear-Viscosity Media

6.1. The investigation of the transition from laminar modes of flow into turbulent modes is of fundamental significance in the MHD of non-Newtonian fluids, particular interest attaching in practice to cases corresponding to motion in transverse magnetic fields. As in the case of Newtonian fluids, most of the progress made in this direction relates to the theory of hydrodynamic stability of laminar flows. As a rule, a magnetic field stabilizes the motion, creating a distinctive "magnetic braking" in the conducting fluid and thereby imparting to the fluid a definite rigidity. Nevertheless, a magnetic field can sometimes be a destabilizing factor, showing up in a strongly deforming action on the velocity profile of the main undisturbed flow.

6.2. Poiseuille flow of a nonconducting viscoplastic medium, even at vanishingly small values of the limiting shear stress $\tau_0$, remains stable to infinitely small disturbances [40]. The explanation of this fact is that, for any $\tau_0$, a quasisolid zone is always located at the center of the channel, which has an important effect on the development of disturbances in a flow of viscoplastic fluid. Hartmann flow of a viscoplastic medium is also stable to infinitely small disturbances for Hartmann numbers $Ha < 6.5$. When $Ha > 6.5$ the flow can be unstable, the critical value of the Reynolds number first of all decreasing to $5.4 \cdot 10^5$ at $Ha = 9.1$ and then monotonically increasing again [41].

Thus, a magnetic field proves to be a destabilizing factor in the case of a gradient flow of a conducting viscoplastic medium. At the same time it always stabilizes a flow of a conducting fluid with a power rheological law. The stability of a shear MHD flow of a fluid with a power rheological law was considered in [42]; the case of a gradient flow is discussed in [43]. We note that the formation, in the case of dilatant fluids, of zones of quasisolid motion does not entail any appreciable change in the stability of flow.

## 7. Future Development of the Magnetohydrodynamics of Non-Newtonian Fluids

7.1. The future trend of the magnetohydrodynamics of non-Newtonian fluids will probably be determined to a considerable extent by the appearance of MHD devices in which the working media possess complex mechanical properties. Even today the practical application of magnetohydrodynamics in metallurgy, foundry manufacture, power engineering, and certain industrial processes has given rise to a pressing need for the development of the magnetohydrodynamics of non-Newtonian media. The number of such applications must increase. It should be noted that the tempo at which this region of magnetohydrodynamics develops is intimately linked with the progress of the hydrodynamics of nonconducting non-Newtonian media and with the subsequent development of the mathematical apparatus of this theory, including the latest achievements of contemporary mathematical physics.

7.2. Among the specific problems being studied at the present time in the MHD of non-Newtonian fluids we mention:

I. The extension of the classes of non-Newtonian media in magnetohydrodynamics.

II. The investigation of flows of non-Newtonian media with characteristics dependent on the magnitude of the electric and magnetic fields.

We note also that allowing for the non-Newtonian properties of the medium can be of interest in ferro- and electrohydrodynamics. The first steps in this direction are contained in [44-47].

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