TRANSVERSE EDGE EFFECT DURING FLOW OF A VISCOUS CONDUCTING LIQUID IN AN INDUCTION MACHINE WITH A HELICAL CHANNEL

L. M. Dronnik and A. I. Él'kin

UDC 621.313.333:538.4

With viscosity included in the analysis of a plane induction pump [1–4], it has been found that the velocity distribution over the channel width becomes very nonuniform and entails backstreaming at the walls, which results in a loss of head larger than according to [1]. The damping coefficient k_{d1} calculated in [1] is much lower than the damping coefficient k_d obtained in [5] on the assumption of a constant velocity.

The effect of a viscous liquid flowing through the channel on the characteristics of a helical machine has never been considered before.

Such a channel is represented here (Fig. 1) by 2n segments of width a_1 hydraulically connected in series and separated by infinitesimally thin infinitely conducting diaphragms, with the walls at the extreme ends insulated.

This analysis will be based on the approximation in [1], namely, on a plane-parallel laminar steady-state flow with $Re_M < 1$ and no adhesion at the lateral walls. On the basis of these assumptions, the equations

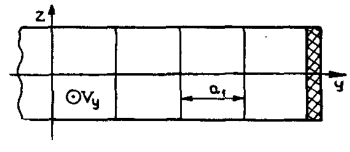

Fig. 1

Translated from Magnitnaya Gidrodinamika, No. 1, pp. 83-88, January-March, 1977. Original article submitted May 4. 1975; revision submitted August 2, 1976.

of motions and induction in [6] were averaged with respect to coordinates z, x, and time. A z profile of velocity as in the Hartmann problem will be assumed here, such a profile in a stream with a traveling magnetic field having been demonstrated in [7].

The system of equations (9b), (10) derived in [1] describes the distributions of velocity and magnetic field (intrinsic) intensity within one channel segment. For n segment pairs, considering the channel symmetry, we have n such systems of equations:

$$\frac{d^2h_{ak}}{dy^2}-\alpha^2h_{ak}-\sigma\mu\alpha(v_s-v_k)H_m=0;\ \frac{p_k-p_{k-1}}{2p\tau}+\frac{1}{2}\mu\alpha H_m h_{ak}+\frac{4\rho\nu\,\mathrm{Ha}^2\,\mathrm{th\,Ha}}{\delta^2(\mathrm{Ha}-\mathrm{th\,Ha})}v_k=0, \quad (1)$$

where k is the consecutive number of a segment and variable y ranges from $(k-1)a_1$ to ka_1.

Each of these systems contains, as the unknown quantities, the intensity h_{ak} (active component of the intrinsic magnetic field), the local velocity v_k, and the local pressure difference $p_k - p_{k-1}$ at a given half-channel inlet pressure p_0 and outlet pressure p_n.

The system of equations is supplemented with several conditions which yield the values of the constants:

1. condition of symmetry, i.e., dh_{a1}/dy at $y=0$;

2. condition of continuity with regard to the normal components of the magnetic field intensity and the tangential components of the electric field intensity, i.e., $h_{ak-1}=h_{ak}$; $dh_{ak-1}/dy=dh_{ak}/dy$ and $y=(k-1)a_1$;

3. condition of zero j_y component of the current density and, therefore, $h_{an}=0$ at $y=na_1$;

4. condition of equal flow rates across sections, i.e.,

$$\int_0^{a_1} v_1 dy=\int_{a_1}^{2a_1} v_2 dy=\cdots\int_{(k-1)a_1}^{ka_1} v_k dy=\cdots\int_{(n-1)a_1}^{na_1} v_n dy,$$

and the condition that the pressure at the exit from one segment is equal to the pressure at the entrance to the next one.

The restriction must be added here that the y profile of the v_x velocity component within one channel segment can be analyzed in the one-dimensional approximation, if the condition $a_1/2p\tau \ll 1$ is satisfied and thus the derivatives with respect to the x coordinate are small relative to the derivatives with respect to the y coordinate. This will make it permissible to assume that $\partial/\partial x=0$ and the flow is becoming steady.

Introducing the same dimensionless parameters and criterial numbers as in [1],

$$\bar{y}=\alpha y;\quad \bar{a}_1=\alpha a_1;\quad \bar{v}=v/v_s;\quad \mathrm{Re}_{M_s}=\mu\sigma v_s/\alpha;$$

$$\bar{h}_a=h_a/H_m;\quad \alpha=\pi/\tau;\quad m=4\nu/\alpha v_s\delta^2;$$

$$c=m\,\mathrm{Ha}^2\,\mathrm{th\,Ha}/(\mathrm{Ha}-\mathrm{th\,Ha});\quad A^2=\mu H_m^2/\rho v_s^2;\quad \mathrm{Ha}=B\delta(\sigma/\rho\nu)^{1/2};$$

$$\Delta\bar{p}_{\mathrm{av}}=(p_n-p_0)/2cp\tau n\alpha\rho v_s^2;\quad \Delta\bar{p}_k=(p_k-p_{k-1})/2cp\tau\alpha\rho v_s^2;$$

$$g=\mathrm{Re}_{Ms}A^2/2c;\quad \beta=\sqrt{1+g},$$

where v_s is the velocity of the magnetic field and H_m is the intensity of the external magnetic field, then performing the identity transformations which include insertion of the velocity

$$v_k=-\Delta p_k-A^2\bar{h}_{ak}/2c \quad (2)$$

into the equation for the magnetic field intensity as well as integration of the momentum equation with respect to y and subsequent summation on the basis of condition (4), we will obtain a system of equations which contains, as the unknown variables, the intensity of the intrinsic magnetic field and the local pressure drop $\Delta\bar{p}_k$.

For the k-th segment pair we have

$$\frac{d^2h_{ak}}{d\bar{y}^2}-(g+1)\bar{h}_{ak}=\mathrm{Re}_{Ms}(1+\Delta\bar{p}_k);$$

$$\Delta\bar{p}_k=\Delta\bar{p}_{\mathrm{av}}+\frac{A^2}{2n\bar{a}_1c}\left[\int_0^{\bar{a}_1}\bar{h}_{a1}d\bar{y}+\int_{\bar{a}_1}^{2\bar{a}_1}\bar{h}_{a2}d\bar{y}+\cdots+\int_{(k-1)\bar{a}_1}^{k\bar{a}_1}\bar{h}_{ak}d\bar{y}+\cdots+\int_{(n-1)\bar{a}_1}^{n\bar{a}_1}\bar{h}_{an}d\bar{y}\right]-\frac{A^2}{2\bar{a}_1c}\int_{(k-1)\bar{a}_1}^{k\bar{a}_1}\bar{h}_{ak}d\bar{y}; \quad (3)$$

and n segment pairs are described by n such systems of equations.

The average velocity

$$\bar{v}_{av}=\frac{1}{\bar{a}_1}\int_{(k-1)\bar{a}_1}^{k\bar{a}_1}\bar{v}_k d\bar{y}$$

becomes, by virtue of relations (2) and (3),

$$\bar{v}_{av}=-\Delta\bar{p}_{av}-\frac{A^2}{2n\bar{a}_1 c}\left[\int_0^{\bar{a}_1}\bar{h}_{a1}d\bar{y}+\int_{\bar{a}_1}^{2\bar{a}_1}\bar{h}_{a2}d\bar{y}+\cdots+\int_{(k-1)\bar{a}_1}^{k\bar{a}_1}\bar{h}_{ak}d\bar{y}+\cdots+\int_{(n-1)\bar{a}_1}^{n\bar{a}_1}\bar{h}_{an}d\bar{y}\right]. \tag{4}$$

For any number of segment pairs the solution, a generalization of the exact solution for n = 2 and 4 segment pairs, reduces to relations between $\bar{h}_{ak}$ and local pressure drops $\Delta\bar{p}_k$ combined with a system of linear algebraic equations relating $\Delta\bar{p}_k$ to the average drop $\Delta\bar{p}_{av}$.

For the k-th segment pair we have

$$\bar{h}_{ak}=\frac{Re_{m\,s}}{g+1}\left\{1+\Delta\bar{p}_k-\sum_{i=0}^{k-1}\left(\operatorname{sh} i\beta\bar{a}_1\cdot\operatorname{sh}\beta\bar{y}-\frac{\operatorname{sh} i\beta\bar{a}_1\cdot\operatorname{sh} n\beta\bar{a}_1\cdot\operatorname{ch}\beta\bar{y}}{\operatorname{ch} n\beta\bar{a}_1}\right)\times\right.$$
$$\left.\times(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})-\sum_{i=k}^{n}\frac{\operatorname{ch}(n-i)\beta\bar{a}_1\cdot\operatorname{ch}\beta\bar{y}}{\operatorname{ch} n\beta\bar{a}_1}(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})-\frac{\operatorname{ch}\beta\bar{y}}{\operatorname{ch} n\beta\bar{a}_1}(\Delta\bar{p}_{n+1}+1)\right\}; \tag{5}$$

$$\Delta\bar{p}_k+g\left\{\sum_{i=0}^{k-1}\frac{n[\operatorname{ch}(n-k)\beta\bar{a}_1-\operatorname{ch}(n+1-k)\beta\bar{a}_1]\operatorname{sh} i\beta\bar{a}_1-\operatorname{sh} i\beta\bar{a}_1}{n\beta\bar{a}_1\operatorname{ch} n\beta\bar{a}_1}(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})+\right.$$
$$+\sum_{i=k}^{n}\frac{n\operatorname{ch}(n-i)\beta\bar{a}_1[\operatorname{sh} k\beta\bar{a}_1-\operatorname{sh}(k-1)\beta\bar{a}_1]-\operatorname{sh} i\beta\bar{a}_1}{n\beta\bar{a}_1\operatorname{ch} n\beta\bar{a}_1}(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})+$$
$$\left.+\frac{n[\operatorname{sh} n\beta\bar{a}_1-\operatorname{sh}(n-1)\beta\bar{a}_1]-\operatorname{sh} n\beta\bar{a}_1}{n\beta\bar{a}_1\operatorname{ch} n\beta\bar{a}_1}(\Delta\bar{p}_{n+1}+1)\right\}=\Delta\bar{p}_{av}.$$

Here n is the number of segment pairs, k is the consecutive number of a segment, and i is the summation index.

Solving the system with respect to $\Delta\bar{p}_k$, one can now calculate the intensity of the intrinsic magnetic field and the distribution of local velocity.

On the basis of the definitions in [1], we determine the following integral characteristics of the machine operating as a pump:

a) average velocity

$$\bar{v}_{av}=-\Delta\bar{p}_{av}+(1+\Delta\bar{p}_{av})\bar{v}_0, \tag{6}$$

where

$$\bar{v}_0=\frac{g}{g+1}\left\{1-\frac{\sum_{i=1}^{n-1}\operatorname{sh} i\beta\bar{a}_1(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})+\operatorname{sh} n\beta\bar{a}_1(1+\Delta\bar{p}_n)}{n\beta\bar{a}_1\operatorname{ch}\beta\bar{a}_1(1+\Delta\bar{p}_{av})}\right\} \tag{7}$$

is the velocity in a zero-gradient stream found from the conditions $\Delta\bar{p}_{av}=0$;

b) pressure drop at zero flow rate ($\bar{v}_{av}=0$)

$$\Delta\bar{p}_{av0}=\bar{v}_0/(1-\bar{v}_0); \tag{8}$$

c) damping coefficient

$$k_{dt}=\Delta\bar{p}_{av0}/g,$$

which in our case becomes

$$k_{dt}=\frac{n\beta\bar{a}_1\operatorname{ch} n\beta\bar{a}_1(1+\Delta\bar{p}_{av})-\sum_{i=1}^{n-1}\operatorname{sh} i\beta\bar{a}_1(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})-\operatorname{sh} n\beta\bar{a}_1(1+\Delta\bar{p}_n)}{n\beta\bar{a}_1\operatorname{ch} n\beta\bar{a}_1(1+\Delta\bar{p}_{av})+g\sum_{i=1}^{n-1}\operatorname{sh} i\beta\bar{a}_1(\Delta\bar{p}_i-\Delta\bar{p}_{i+1})+g\operatorname{sh} n\beta\bar{a}_1(1+\Delta\bar{p}_n)}. \tag{9}$$

d) efficiency

$$\eta_{\text{p}}=\Delta\bar{p}_{\text{av}}\left[1-\frac{\Delta\bar{p}_{\text{av}}}{(1+\Delta\bar{p}_{\text{av}})\bar{v}_0}\right]. \tag{10}$$

Calculating the distributions of local and integral characteristics reduces to solving a system of equations of the n-th order and this is difficult even for n=4 only. For this reason, in order to indicate the trends which these characteristics follow as the number of segments is increased, we will consider a few specific cases.

1. Parameter $n\beta\bar{a}_1 \gg 1$ with n such that all terms of the $\sinh n\beta\bar{a}_1/\cosh n\beta\bar{a}_1$ kind may be assumed equal to unity and all terms of the $\sinh (n-1)\beta\bar{a}_1/\cosh n\beta\bar{a}_1$ kind may be assumed equal to zero. This corresponds to the actual relation between the parameters of a model AMG-5 MHD machine [6] and greatly simplifies the system of algebraic equations (5) to the form

$$\begin{aligned}
&(1+\gamma)x_1-x_2+0+0+\cdots+0+0+0-2n^{-1}x_n=1;\\
&-x_1+(2+\gamma)x_2-x_3+0+\cdots+0+0+0-2n^{-1}x_n=1;\\
&0-x_2+(2+\gamma)x_3-x_4+\cdots+0+0+0-2n^{-1}x_n=1;\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&0+0+0+0+\cdots-x_{n-2}+(2+\gamma)x_{n-1}-x_n-2n^{-1}x_n=1;\\
&0+0+0+0+\cdots+0-x_{n-1}+(3+\gamma)x_n-2n^{-1}x_n=1,
\end{aligned} \tag{11}$$

where

$$x_i=\frac{\Delta\bar{p}_i+1}{\Delta\bar{p}_{\text{av}}+1};\qquad \gamma=\frac{2\beta\bar{a}_1}{g}. \tag{12}$$

Moreover,

$$\bar{v}_0=\frac{g}{g+1}\left[1-\frac{\Delta\bar{p}_n+1}{n\beta\bar{a}_1(\Delta\bar{p}_{\text{av}}+1)}\right]; \tag{13}$$

$$k_{\text{dl}}=\frac{\gamma(\Delta\bar{p}_{\text{av}}+1)-2(\Delta\bar{p}_n+1)/gn}{\gamma(\Delta\bar{p}_{\text{av}}+1)+2(\Delta\bar{p}_n+1)/n}. \tag{14}$$

Let us solve system (12) for n=2:

$$\Delta\bar{p}_1+1=\frac{\gamma+4}{\gamma+3}(\Delta\bar{p}_{\text{av}}+1);\qquad \Delta\bar{p}_2+1=\frac{\gamma+2}{\gamma+3}(\Delta\bar{p}_{\text{av}}+1) \tag{15}$$

and, retaining only the quadratic polynomials, for n=4:

$$\begin{aligned}
\Delta\bar{p}_1+1&=\frac{16+20\gamma+8\gamma^2}{11+17\gamma+7.5\gamma^2}(\Delta\bar{p}_{\text{av}}+1);\\
\Delta\bar{p}_2+1&=\frac{14+20\gamma+8\gamma^2}{11+17\gamma+7.5\gamma^2}(\Delta\bar{p}_{\text{av}}+1);\\
\Delta\bar{p}_3+1&=\frac{10+18\gamma+8\gamma^2}{11+17\gamma+7.5\gamma^2}(\Delta\bar{p}_{\text{av}}+1);\\
\Delta\bar{p}_4+1&=\frac{4+10\gamma+6\gamma^2}{11+17\gamma+7.5\gamma^2}(\Delta\bar{p}_{\text{av}}+1).
\end{aligned} \tag{16}$$

The local pressure drops, according to relations (13) and (14), satisfy the inequalities $\Delta\bar{p}_1>\Delta\bar{p}_{\text{av}}>\Delta\bar{p}_2$ for n=2 and $\Delta\bar{p}_1>\Delta\bar{p}_2>\Delta\bar{p}_3>\Delta\bar{p}_4$ for n=4, which indicate that with n > 1 the pressure drop becomes redistributed so as to increase at the center and decrease at the ends, i.e., approach the profile of the electrodynamic force.

2. $n=\infty$. In this case the system of segments hydraulically connected in series is equivalent to the one with a uniform-over-the-width velocity considered in [5].

The pressure drop according to relation (2)

$$\Delta\bar{p}=-\bar{v}-\frac{A^2}{2c}\bar{h}_a$$

agrees, except for the constant term ($\bar{v}$=const) due to friction, with the distribution of the electrodynamic force according to [5].

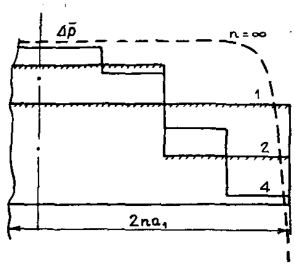

Fig. 2

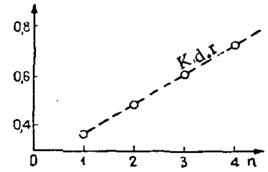

Fig. 3

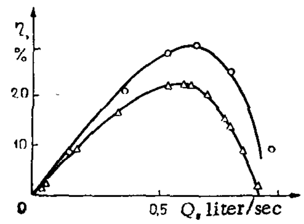

Fig. 4

Fig. 2. Distribution of pressure drop across the channel width.

Fig. 3. Referred damping coefficient as a function of the number of channel segment pairs.

Fig. 4. Comparison between experimental and theoretical curves of efficiency versus flow rate.

Calculations made for a helical induction machine in one of its operating modes [8] indicate that, when its parameters are $n\beta a_1 = 30.3$; $g = 82$; $\beta = 9.1$; $\Delta\bar{p}_{av} = 1.5$, and $Rm_s = 0.9$, with $n = 4$ segment pairs, the distribution of pressure drop may be approximated by the distribution in an infinitely segmented channel (Fig. 2).

The damping coefficient is then much higher than in the case $n = 1$ (Fig. 3), its limiting value being k_d [5].

Theoretical curves of efficiency versus flow rate are compared in Fig. 4 with those based on experimental data in [9], but there the flow was turbulent. An approximate correction for higher friction in a turbulent stream has been introduced in terms of the effective Hartmann number for a turbulent flow, just as in [1], with the friction coefficient calculated according to the expressions in [6] and thus taking into account the very nonuniform velocity profile in a helical channel.

CONCLUSIONS

1. With the ratios a/τ equal, the damping coefficient is higher in a machine with a helical channel than in one with a plane channel.

2. The electrodynamic approximation becomes valid with a sufficiently large number of diaphragms.

LITERATURE CITED

1. E. I. Yantovskii, "Transverse edge effect during flow of a conducting liquid in a traveling magnetic field," in: Transactions of the Fifth Conference on Electromagnetic Flowmeters, Tallin [in Russian], Vol. 1, Izd. Nauch.-Tekh. Otd. Priboroprom ÉstSSR, Tallin (1971), pp. 33-54.
2. R. A. Petrovicha and L. Ya. Ulmanis, "Laminar flow of an electrically conducting liquid through a rectangular channel in a plane-parallel traveling magnetic field," in: Transactions of the Fourth Conference on Electromagnetic Flowmeters, Tallin [in Russian], Vol. 3, Izd. Nauch.-Tekh. Otd. Priboroprom ÉstSSR, Tallin (1970), pp. 183-197.
3. A. Ya. Kalnyn', I. K. Mikryukova, R. A. Petrovicha, and V. A. Rupeneit, "Characteristics of a plane induction pump with a nonuniform distribution of electromagnetic forces over the channel width," Magnitn. Gidrodinam., No. 4, 94-98 (1971).
4. E. I. Yantovskii, "Role of friction against lateral walls in the transverse edge effect during flow of a conducting liquid in a traveling magnetic field," in: Transactions of the Fifth Conference on Electromagnetic Flowmeters, Tallin [in Russian], Vol. 5, Izd. Nauch.-Tekh. Otd. Priboroprom ÉstSSR, Tallin (1971), pp. 53-66.
5. A. I. Vol'dek, "Currents and forces in a layer of liquid metal in plane linear induction pumps," Izv. Vyssh. Uchebn. Zaved., Élektromekh., No. 1, 3-10 (1959).
6. E. I. Yantovskii and I. M. Tolmach, Magnetohydrodynamic Generators [in Russian], Izd. Nauka, Moscow (1972).
7. L. Harris, Magnetohydrodynamic Flow through Channels [Russian translation], Izd. Inostr. Lit., Moscow (1963).
8. L. M. Dronnik, "Combined effect of diaphragms and a nonuniform velocity profile in a helical induction channel," in: Transactions of the Fifth Conference on Electromagnetic Flowmeters, Tallin [in Russian], Vol. 4, Izd. Nauch.-Tekh. Otd. Priboroprom ÉstSSR, Tallin (1971), pp. 14-32.

9. Yu. A. Bakanov, L. M. Dronnik, M. N. Levin, V. K. Makarevich, L. M. Reshet'ko, V. E. Strizhak, I. M. Tolmach, S. R. Troitskii, and E. I. Yantovskii, "Experimental study of a liquid-metal induction machine operating as a pump," Magnitn. Gidrodinam., No. 1, 153-155 (1973).