FORCE ACTING FROM A NEEDLE ELECTRODE ON A POORLY CONDUCTING LIQUID DIELECTRIC AND THE FLOW EFFECTS IT PRODUCES

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UDC 538.3:532.538.4

Prebreakdown phenomena in liquid dielectrics are characterized by a nonuniformity of the electrical conductivity, which leads to the buildup of a space charge and resulting flow effects in the liquid [1-3]. The electrodynamic jet flow from a thin high-voltage needle electrode has been calculated in [3] approximately, on the basis of well-known solutions for plain jets (Fig. 1) and with the aid of the empirical dependence of the electrical conductivity on the electric field intensity

$$\sigma=\sigma_{M}\exp\{\beta(E-E_{M})\},\tag{1}$$

with the empirical constant β.

Intensive jets of liquid dielectrics from thin electrodes were observed by Pohl [4]. It has not yet been possible, however, to describe such flow patterns without the use of empirical constants. In this study the field dependence of the conductivity will be used; it has been derived analytically taking into account that dissociation is amplified by an increasing field intensity [1, 5-8].

A change in the conductivity of a liquid can be due a change in the temperature [9], but in this case the increases in the conductivity due to a higher field intensity [1, 10, 11] is more significant. The experiments in [12], during which the electrode heated up or cooled down appreciably, indicate that temperature changes hardly affect the flow pattern. An increase in the dissociation constant $\varkappa$ (a quantity which characterizes the ion concentration) with increasing field intensity has been established in [6] as follows. It is well known from statistical mechanics that $\varkappa = C\exp(-W/kT)$, where C is a constant, k is the Boltzmann constant, and W is the energy expended on separating an ion from a molecule. Action of the field in direction $\mathbf{r}$ ($\theta = 0$ in Fig. 2 for this case) yields the change in this energy

$$\Delta W=\frac{e^2}{4\pi\varepsilon r}+eEr,$$

with r determined from the equation

$$\frac{e^2}{4\pi\varepsilon r^2}=eE,$$

i.e.,

$$r=\frac{1}{2}\sqrt{\frac{e}{\pi\varepsilon E}},\quad \Delta W=\frac{e^{3/2}E^{1/2}}{(\pi\varepsilon)^{1/2}}.\tag{2}$$

Here e is the charge of an electron and ε is the dielectric permittivity in the SI units. From this we have

$$\varkappa(E)=\varkappa(0)\exp\frac{\Delta W}{kT}=\varkappa(0)\exp\frac{e^{3/2}E^{1/2}}{(\pi\varepsilon)^{1/2}kT}.\tag{3}$$

The exponential function in expression (3) contains not E, as does the empirical relation (1), but $E^{1/2}$ in-instead. For solid dielectrics expression (1) is also known as a generalization of the experimental data in [13], and its refinement on the basis of considerations analogous to those in [6] has been considered in [7]. Moreover, in SI units, the relation $\sigma(E)$ becomes

$$\sigma=\sigma_0\exp\frac{e^{3/2}E^{1/2}}{(\pi\varepsilon)^{1/2}kT},\tag{4}$$

where σ_0 is the conductivity in weak fields.

Translated from Magnitnaya Gidrodinamika, No. 4, pp. 73-80, October-December, 1977. Original article submitted January 3, 1977.

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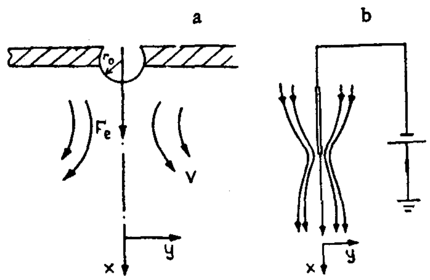

Fig. 1

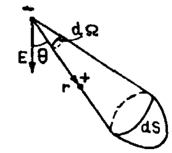

Fig. 2

Fig. 1. Patterns of jet flow from a needle electrode: (a) jet from the wall (Schlichting flow); (b) jet from the needle electrode into an unbounded volume of liquid (Landau–Slezkin flow).

Fig. 2. System of notation accounting for the direction of ion escape.

With the direction of ion escape (Fig. 2) taken into account, the change in energy ΔW within an infinitesimally small solid angle $d\Omega$ can be calculated as [1]

$$\Delta W = \frac{e^{3/2} E_r^{1/2}}{(\pi\varepsilon)^{1/2} kT}.$$

The number of ions contained within a spherical sector of unit radius and bounded by the solid angle $d\Omega$ is then proportional to

$$\frac{1}{3}\varkappa(0)\left[\exp\left\{\frac{e^{3/2}E_r^{1/2}}{kT(\pi\varepsilon)^{1/2}}\right\}\right] dS.$$

Therefore,

$$\varkappa(E) = \frac{1}{4\pi}\varkappa(0)\int_S \exp\left\{\frac{e^{3/2}E_r^{1/2}}{kT(\pi\varepsilon)^{1/2}}\right\} dS.$$

Since $dS = 2\pi \sin\theta d\theta$, the integration will yield a correction factor for expression (3)

$$\frac{kT(\pi\varepsilon)^{1/2}}{2e^{3/2}E^{1/2}} \to \infty \quad \text{when } E \to 0.$$

The expression

$$\sigma = \sigma_0 \frac{kT(\pi\varepsilon)^{1/2}}{2e^{3/2}E^{1/2}} \exp\frac{e^{3/2}E^{1/2}}{kT(\pi\varepsilon)^{1/2}} \tag{5}$$

is thus inapplicable to weak fields.

The Wien effect in weak electrolytes is described by the expression [8]

$$\frac{\varkappa(E)}{\varkappa(0)} = \frac{J_1\{4(-\beta_0 q)^{1/2}\}}{2(-\beta_0 q)^{1/2}}, \tag{6A}$$

where J_1 is the Bessel function of the first order,

$$0 < q = -\frac{e_1 e_2}{8\pi\varepsilon kT}; \quad 2\beta_0 = \frac{|E(e_1\omega_1 - e_2\omega_2)|}{kT(\omega_1+\omega_2)},$$

e_j ($j = 1, 2$) is the charge of each respective kind of ions, and $e_j\omega_j$ is the mobility of each. Expression (6A), which has been derived mathematically with great precision in [10], is independent of ω_j when $e_1 = -e_2 = e$

(according to the diagram in Fig. 2). Changing to a cylindrical function of the imaginary argument $J_1(iz) = iI_1(z)$ and applying the asymptotic relation $I_1(z) \approx (2\pi z)^{1/2} \exp z$ at $z \to \infty$ (p. 650 in [4]) yields for strong fields

$$\frac{\sigma}{\sigma_0}=\frac{2^{1/2}\varepsilon^{3/4}(kT)^{3/2}\pi^{1/4}}{e^{9/4}E^{3/4}}\exp\frac{e^{3/2}E^{1/2}}{(\pi\varepsilon)^{1/2}kT} \tag{6B}$$

with the same exponential relation $\sigma(E)$ as in expression (4) and with approximately the same correction factor as in expression (5).

Approximation of expression (5) with an exponential relation of the form (1) on the interval $[E_M, E_M + \Delta E]$ (ΔE is defined as the approximation error), taking into account that not only functions (1) and (5) but also their derivatives must be respectively equal at $E = E_M$, yields

$$\beta=\frac{1}{2E_M}\left[\frac{e^{3/2}E_M^{1/2}}{kT(\pi\varepsilon)^{1/2}}-1\right]. \tag{7}$$

Calculations according to expression (7) (with $E_M = 1$ kV/mm, $T = 300°K$, and $\varepsilon = 2.5\,\varepsilon_0$) yield $\beta \approx 0.3$ mm/kV, which is close to the experimental value of this constant for transformer oil used in [3].

We will now calculate the total electric force from a thin needle electrode, following the procedure in [11, 12]. The following will be assumed:

a. The conduction current is much larger than the convection current, i.e., $\sigma E \gg \rho_e v$. The equation for the volume charge density ρ_e, derived in [12] from the equations div $\mathbf{v} = 0$ (law of mass conservation), div $\varepsilon\mathbf{E} = \rho_e$ (Gauss' law), and div $(\sigma\mathbf{E} + \rho_e\mathbf{v} + \partial\varepsilon\mathbf{E}/\partial t) = 0$ (law of charge conservation combined with Ohm's law), yields

$$\frac{d\rho_e}{dt}=-\frac{1}{\tau}(\rho_e-\sigma\mathbf{E}\nabla\tau), \tag{8}$$

where $\tau = \varepsilon/\sigma$ is the relaxation time. When $\sigma E \gg \rho_e v$, the convective terms $\tau\mathbf{v}\nabla\rho_e$ in Eq. (8) may be disregarded. For steady flow we then have

$$\rho_{e,y}=\sigma\mathbf{E}\nabla\tau=-\varepsilon\mathbf{E}\nabla\ln\frac{\sigma}{\sigma_0}. \tag{9}$$

b. The field intensity E' due to charges in the liquid is much lower than intensity of the external field, i.e., $E' \ll E_0$.

c. The length of the flow path L_v and the range of the Coulomb force L_E are incommensurable, namely, $L_v \gg L_E$. This assumption is based on observations of the flow pattern and it allows us to treat the flow from a needle electrode along the x axis as an axisymmetric jet. A point source of a jet pulse is produced by the Coulomb force near the tip of a thin needle electrode of dimensions negligibly smaller than its distance from the counterelectrode.

It will be sufficient to calculate the projection of the force on the x axis, as has been done before [15], by converging to the limit of an infinitesimally thin edge. The field intensity around this edge is approximately the same as in the case of spherical symmetry

$$E_r=\frac{Ur_0}{r^2};\quad U=-\int_{r_0}^{\infty}E_r dr, \tag{10}$$

where U is the voltage applied to the electrode and r_0 is the characteristic dimension of the edge.

According to [11], the projection of the force on the x axis is

$$F_e=p\pi r_0^2=\int_{r_0}^{\infty}dr\int_0^{\pi/2}f\pi r_0^2 2\sin\vartheta\cdot\cos\vartheta d\vartheta, \tag{11A}$$

where $\mathbf{f} = \rho_e\mathbf{E} + (\varepsilon - \varepsilon_0)\nabla(E^2/2)$ is the force density; p is the pressure at the edge; and r, ϑ are spherical coordinates x, y (Fig. 1). Calculating of the pressure and the Coulomb force on the basis of relation (5) reduces to an evaluation of $\int_{E(r_0)}^{E(\infty)} E^2 d\ln\frac{\sigma(E)}{\sigma_0}$ by inserting E from relation (10) here. Then

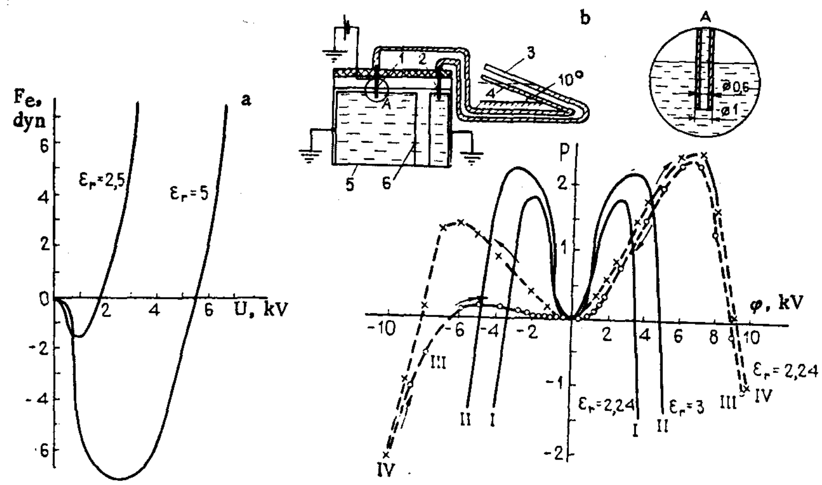

Fig. 3. Force as a function of the electric field intensity (a) and a comparison between pressure calculated according to relation (11) and measured in [16] (b): I) calculation according to relations (11) with $\varepsilon_r = 2.24$; II) calculation with $\varepsilon_r = 3$; III) measurement [16] with decreasing voltage V; IV) measurement with increasing voltage V.

$$\ln\frac{\sigma}{\sigma_0}=\frac{\Delta W}{kT}-\ln\frac{2\Delta W}{kT}.$$

The integral of the first term is

$$\int_{r_0}^{\infty}-\varepsilon E^2\frac{d}{dr}\left(\frac{\Delta W}{kT}\right)dr=\frac{-\varepsilon^{1/2}e^{3/2}}{\pi^{1/2}kT}\int_{E(r_0)}^{E(\infty)}E^{3/2}dE=-\frac{2}{5}\frac{\varepsilon^{1/2}e^{3/2}}{\pi^{1/2}kT}E^{5/2}\Big|_{r_0}^{\infty}=\frac{2}{5}\frac{\varepsilon^{1/2}e^{3/2}}{\pi^{1/2}kT}\frac{U^{5/2}}{r_0^{5/2}}.$$

The second term is integrated analogously so that

$$F_e=\frac{2\sqrt{\pi}}{5kT}\sqrt{\frac{\varepsilon e^3U^5}{r_0}}-\frac{\pi}{2}(2\varepsilon-\varepsilon_0)U^2. \tag{11B}$$

This is the sum of the Coulomb force and the polarization force, both acting on the liquid. The Coulomb force repels the liquid, inasmuch as the volume charge is of the polarity as the electrode charge, while the polarization component of the force attracts the liquid to the electrode (so as to increase the field intensity). A graph of the $F_e(V)$ relation is shown in Fig. 3a for various values of ε.

The graph of the p(U) relation follows an analogous trend. Approximately the same expressions are also obtained on the basis of relations (4) and (6B).

The pressure at the edge was measured in [16] in the following manner (Fig. 3b). The needle electrode (syringe) 1 was connected through a tube with a manometer 4 and mounted in the cover of a vessel 5 made of acrylic glass and filled with transformer oil, containing also the counterelectrode 6. Another needle electrode 2, without applied voltage, was used for control (having it connected to an identical manometer 3). Both positive and negative potentials $\varphi(U = |\varphi|)$ were applied to electrode 1. The results of these measurements are shown on the same diagram in Fig. 3b (curves III and IV).

As the voltage was increased (U > 6 kV), the level in manometer 4 dropped to below the zero mark. No changes occurred in manometer 3 at this time. The pressures were also different at potentials of different polarities and, moreover, hysteresis was observed at $\varphi < 0$. For comparison, curves I and II have been calculated according to relations (11) for the relative dielectric permittivity $\varepsilon_r = 2.24$ and $\varepsilon_r = 3$, respectively. The trend of these curves agrees qualitatively with the occurrence of the pressure peak. Many quantitative aspects of the experiment and of the hysteresis effect cannot, of course, be explained by our theory.

When the voltage becomes sufficiently high, then the second term in expression (11B) becomes negligible and for the repelling force (Coulomb) we may use the expression

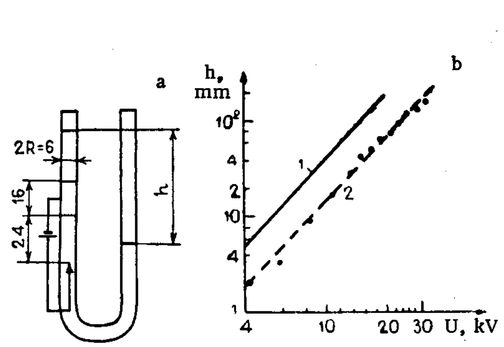

Fig. 4

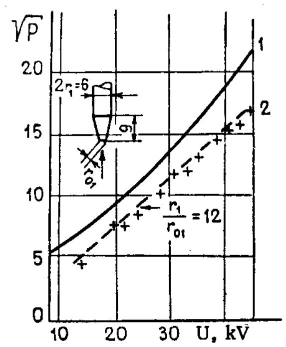

Fig. 5

Fig. 4. Comparison between calculated and measured (by the authors) liquid lift height: a) schematic diagram of the experiment; b) calculated and measured curves; 1) calculation according to relation (13); 2) measurement.

Fig. 5. Comparison between calculation and experiment [17]: 1) calculated curve; 2) measured curve.

$$F_{e0}=\frac{2\sqrt{\pi}}{5kT}\sqrt{\frac{\varepsilon e^3U^5}{r_0}}. \qquad (12)$$

We have verified the validity of relation (12) in an experiment with a vertical needle electrode inside one of the connecting tubes (Fig. 4a), where the level of the transformer oil was found to rise. Also jets were observed originating from the tip of the needle electrode above the liquid surface and other flow patterns as well. Equating the electric force to the weight of lifted liquid yields

$$h=\frac{2}{5\sqrt{\pi}kT}\sqrt{\frac{\varepsilon e^3U^5}{r_0}}\frac{1}{\rho g R^2}. \qquad (13)$$

The tube radius R and the lift height h are indicated in Fig. 4a in millimeters. The experimental points in Fig. 4b lie close to a straight line obtained by shifting the straight line 1, the latter one corresponding to relation (13).

Tubes narrowing down toward the needle (Fig. 5) were used so as to eliminate internal vortices which had appeared in the well known experiments by Stuetzer [17] with grade D-550 silicone oil. Our test points deviated not so very much from the straight line 2. For comparison, curve 1 has been calculated according to relation (13). The discrepancy by a factor of 1.5-2 between calculation and measurement is in both cases due, of course, to the approximateness of the theory and the departure of the test conditions from assumptions which have been made. Nevertheless, there is an obvious overall agreement between curves calculated and measured by various authors. It seems that the discrepancies can be reduced by taking into account the actual flow pattern within the column of lifted liquid, the apparent lift height being in effect smaller.

Substituting the force (12) for the total impulse in the well known Schlichting solution and Landau–Slezkin solution (p. 108 in [19] and p. 150 in [20]) for laminar jets will yield an approximate description of an EHD jet without the empirical constant in [3]:

1. Jet from a needle electrode in a nonconducting wall

$$v_x=\frac{3}{4\pi^{1/2}}\frac{\varepsilon^{1/2}e^{3/2}U^{5/2}}{\rho\nu x r_0^{1/2}5kT(1+{}^1\!/_4\xi^2)^2};$$

$$v_y=\frac{3^{1/2}\varepsilon^{1/4}e^{3/4}U^{5/4}(\xi-{}^1\!/_4\xi^3)}{2^{3/2}\pi^{1/4}\rho^{1/2}x r_0^{1/4}(5kT)^{1/2}(1+{}^1\!/_4\xi^2)^2}; \qquad (14)$$

$$\xi=\gamma\frac{y}{x};\quad \gamma=\frac{1}{4\nu}\sqrt{\frac{3\pi}{\rho}}\sqrt{F_e}=\frac{3^{1/2}\pi^{3/4}\varepsilon^{1/4}e^{3/4}U^{5/4}}{2^{3/2}\nu r_0^{1/4}(5kT)^{1/2}};$$

where γ is a dimensionless electrical parameter [3, 12] and ν is the kinematic viscosity.

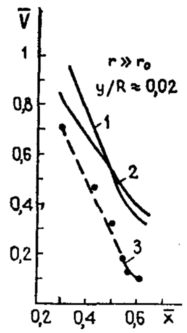

Fig. 6

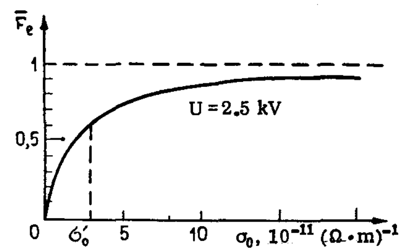

Fig. 7

Fig. 6. Comparison between calculations and experiment [22]: 1) calculation according to relation (15); 2) calculation according to relation (14); 3) determination from photographs taken during the experiment in [22].

Fig. 7. Calculated Coulomb force, taking into account the transfer of volume charge by a weak jet in liquid dielectrics, as a function σ_0; $\sigma_0' = 3\cdot 10^{-11}$ $(\Omega\cdot m)^{-1}$ is the electrical conductivity as measured in the experiments [16, 22].

2. Jet from a needle electrode immersed in a boundless volume of liquid

$$v_x = \frac{2\nu L}{x\sqrt{1+\eta^2}}\left\{1+\frac{1-L^2}{(\sqrt{1+\eta^2}-L)^2}\right\};$$
$$v_y = \frac{2\nu L\eta}{x\sqrt{1+\eta^2}}\,\frac{1-L\sqrt{1+\eta^2}}{(\sqrt{1+\eta^2}-L)^2};\quad \eta=\frac{y}{x}, \tag{15}$$

with constant L being a parametric function of γ in (14)

$$\gamma^2 = 3\left(\frac{1}{L}+\frac{4}{3}\,\frac{L}{1-L^2}-\frac{1}{2L^2}\ln\frac{1+L}{1-L}\right).$$

Expressions (14) and (15) are applicable only at $r \gg r_0$ (p. 32 in [21]). The velocity calculated according to these expressions at U = 4 kV is compared in Fig. 6 with the velocity determined from photographs taken during experiments [22] with a cylindrical counterelectrode of a height $h \gg r_0$ and a radius $R \gg r_0$. Along the axes of coordinates in Fig. 6 are plotted the dimensionless velocity $\bar{v} = v/v_0$ (v_0 = 1 cm/sec) versus the dimensionless distance $\bar{x} = x/h$. The deviation of calculated curves 1 and 2 from the experimental values is relatively large and, as before, the calculated values are approximately twice as high.

The heat emitted by a moderately hot electrode to the liquid [12] at a constant thermal power Q_0 can be calculated when $\Delta T \ll T$. The expressions for the dimensionless heat-transfer coefficient (Nusselt number) applicable to a flow according to relation (14) [3, 12] or according to relation (15) are, respectively,

$$\mathrm{Nu}=\frac{2\sqrt[4]{12}\gamma^{3/2}\,\mathrm{Pr}^{3/2}}{32\,\mathrm{Pr}+\sqrt{3}\gamma};\quad \mathrm{Nu}=\frac{\sqrt{\mathrm{Re}\cdot\mathrm{Pr}}}{\mathrm{Re}\cdot\mathrm{Pr}+1}\,\frac{\sqrt{1+\mathrm{Re}\cdot\mathrm{Pr}}+(2\mathrm{Pr}-1)L\sqrt{\mathrm{Re}\cdot\mathrm{Pr}}}{\sqrt{\mathrm{Re}\cdot\mathrm{Pr}+1}-L\sqrt{\mathrm{Re}\cdot\mathrm{Pr}}} \tag{16), (17}$$

with $\mathrm{Re} = 8\gamma/\sqrt{3}$ not containing any empirical constants. Both main drawbacks of our theory – first, that the calculated values are approximately twice as high as the experimentally determined ones and, second, that the dependence of the force as well as the velocity on σ_0 has not been taken into account in expressions (11B), (12), (14), and (15) – can evidently be overcome by taking into account the drift of charges by the flowing liquid. This has been done, approximately, by using the series expansion in [3]

$$\rho_e=\sigma\mathbf{E}\cdot\nabla\tau+\sum_{n=1}^{\infty}(-1)^n[\tau\mathbf{v}\underbrace{\nabla\tau\mathbf{v}\ldots\nabla\tau\mathbf{v}}_{n-1}\nabla(\sigma\mathbf{E}\cdot\nabla\tau)] \tag{18}$$

for a weak jet (according to Landau) with the impulse P → 0 and the velocities (p. 110 in [19])

$$v_r = \frac{P}{4\pi\nu\rho}\frac{\sin\vartheta}{r}, \quad v_\vartheta = -\frac{P}{8\pi\nu\rho}\frac{\cos\vartheta}{r}. \tag{19}$$

The series (18) is a convergent one, if the condition $|\tau\mathbf{v}\cdot\nabla\rho_e| < |\rho_e|$ is satisfied for any infinite number of times differentiable function ρ_e.

When the components of **v** have been calculated according to expressions (19), the common term in this series is an infinitesimally small quantity of order P^n. Disregarding the terms of order P^2 and higher orders in expression (18), we calculate the Coulomb force according to expression (11A) and taking into account relation (4)

$$\overline{F}_e = \frac{F_e}{F_{e0}} = \frac{\sigma_0}{\sigma_0 + c_0}, \tag{20}$$

where F_{e0} is the force according to expression (12),

$$c_0 = \frac{\sigma_0}{P}\int_{r_0}^{\infty} dr \int_0^{\pi/2} \tau\mathbf{v}\cdot\nabla(\sigma\mathbf{E}\cdot\nabla\tau)E\pi r_0^2 2\sin\vartheta\cdot\cos\vartheta d\vartheta = \frac{2}{3}\frac{\varepsilon^2 E_0^2}{\nu\rho}\times$$

$$\times\exp(-\alpha E_0^{1/2})\left[1 - \frac{6}{\alpha E_0^{1/2}} + \frac{30}{\alpha^2 E_0} - \frac{120}{\alpha^3 E_0^{3/2}} + \frac{360}{\alpha^4 E_0^2} - \frac{720}{\alpha^5 E_0^{5/2}} + \frac{720}{\alpha^6 E_0^3}\right];$$

$$\alpha = \frac{e^{3/2}}{(\pi\varepsilon)^{1/2}kT}; \quad E_0 = \frac{U}{r_0}.$$

With the next approximation of order P^2 we obtain

$$F_e = \frac{\sigma_0}{\sigma_0 + c_0}F_{e0} + 0(F_{e0}^2),$$

where $0(F_{e0}^2)$ is an infinitesimally small quantity of order F_{e0}^2.

A graph of relation (20) is shown in Fig. 7. According to this curve, the electric current through the liquid as well as the Coulomb force vanish at very low values of σ_0, and at sufficiently high values of σ the force becomes almost independent of σ_0 [it is to be noted that expression (9) has been derived for exactly the condition of a small $\tau \leq \varepsilon/\sigma_0$]. At the realistic value $\sigma_0 = \sigma_0'$, as measured in the experiments [16, 22], the force remains at almost only half its asymptotic level, which removes the discrepancy between measured and calculated curves.

The authors thank V. V. Gogosova, V. A. Naletov, and Yu. K. Stishkova for their helpful comments.

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