FLOW OF AN ELECTRICALLY CONDUCTING FLUID IN A DISTRIBUTING COLLECTOR IN A MAGNETIC FIELD

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§1. In [1], in a one-dimensional statement, a detailed study was made of the flow of a fluid, in various systems of collectors. It is of interest to make an analogous qualitative analysis for the case where the fluid moving in the collector is electrically conducting and the collector is situated in an external magnetic field. Typical schemes of collectors — scheme Z (Fig. 1a) and scheme Π (Fig. 1b,c) — contain inlet (distributing) collectors, outlet (collecting) collectors, and an active section located between them, consisting of tubes parallel to the magnetic field B. The collector is regarded as a long and narrow channel of variable cross section with electrically conducting walls, along whose length, perpendicular to the walls, there is blowing or suction of an electrically conducting fluid; here $\mathrm{Re}_m \ll 1$ and $\tilde{\mathrm{Ha}} \gg 1$. In this case, the equations of a quasi-one-dimensional approximation in the following form are valid [3]:

$$\rho v \frac{dv}{dx} = -\frac{dp}{dx} - c_1 v B^2 - \frac{v}{F}\frac{dG}{dx}, \quad \rho v F = G, \tag{1}$$

where

$$c_1 = \sigma\alpha(1+\alpha)^{-1}; \quad \alpha = 2\delta_w\sigma_w/\sigma a.$$

In equations (1), friction was not taken into consideration, which is obviously valid for large values of $\tilde{\mathrm{Ha}}$ and not too small values of α, and thus is admissible in the present qualitative analysis.

We write equations (1) in the form

$$\frac{2G}{\rho F}\frac{dG}{dx} - \frac{G^2}{\rho F^2}\frac{dF}{dx} = -F\frac{dp}{dx} - \frac{c_1 B^2 G}{\rho}. \tag{2}$$

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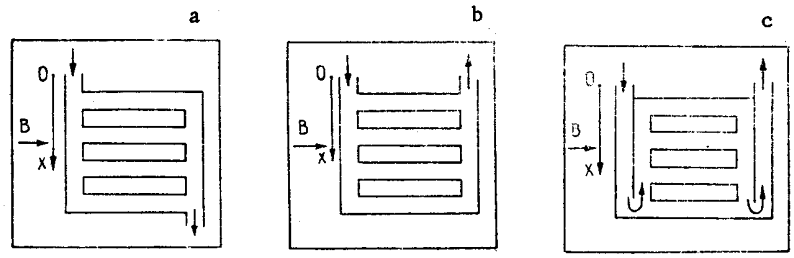

Fig. 1. Scheme of collectors.

In the section parallel to the magnetic field, assuming the flow to be laminar (or laminarized), we write

$$p-p^*=c\,\mathrm{d}G/\mathrm{d}x, \tag{3}$$

where the superscript asterisk (*) relates to the outlet collector. After a transition to dimensionless quantities, we obtain

$$-\frac{2\,\mathrm{Re}}{\bar F}\bar G\frac{\mathrm{d}\bar G}{\mathrm{d}\bar x}+\frac{\mathrm{Re}\,\bar G^2}{\bar F^2}\frac{\mathrm{d}\bar F}{\mathrm{d}\bar x}=-\bar F\frac{\mathrm{d}\bar p}{\mathrm{d}\bar x}+\frac{\mathrm{Ha}^2}{2}\bar G; \tag{4}$$

$$\bar p-\bar p^*=\mathrm{d}\bar G/\mathrm{d}\bar x. \tag{5}$$

We denote

$$\bar x=x/l;\quad \bar F=F/F_0;\quad \bar G=G/G_0;\quad \bar p=pl/cG_0;$$
$$\widetilde{\mathrm{Ha}}^2=-2c_1B^2l^2/\rho F_0c;\quad \mathrm{Re}=-G_0l/\rho F_0^2c.$$

Here the subscript "zero" relates to a section with a maximal mass flow rate; l is the length of the collector; the dimensionless complexes $\widetilde{\mathrm{Ha}}$ and Re have the sense of the Hartmann and Reynolds numbers, where the electromagnetic forces and the forces of inertia are taken in the collector section, and the force of friction is calculated in the section parallel to the magnetic field.

§2. Let us consider the motion of the liquid in scheme Z for a constant cross section of the collectors.

The difference in the pressures between the inlet and outlet collectors of the system is represented in the form

$$\bar p(0)-\bar p^*(1)=[\bar p(0)-\bar p(x)]+[\bar p(x)-\bar p^*(x)]+[\bar p^*(x)-\bar p^*(1)], \tag{6}$$

where the total pressure drop does not depend on the coordinate $\bar x$. Taking account of the connection between the mass flow rates of the collectors $\bar G^*=1-\bar G$, using (4) we obtain

$$\bar p(0)-\bar p(\bar x)=\int_x^0\left(\frac{\widetilde{\mathrm{Ha}}^2}{2}\bar G+2\mathrm{Re}\,\bar G\frac{\mathrm{d}\bar G}{\mathrm{d}\bar x}\right)\mathrm{d}\bar x; \tag{7}$$

$$\bar p^*(\bar x)-\bar p^*(1)=\int_1^0\left(\frac{\widetilde{\mathrm{Ha}}^2}{2}(1-\bar G)-2\,\mathrm{Re}(1-\bar G)\frac{\mathrm{d}\bar G}{\mathrm{d}\bar x}\right]\mathrm{d}x. \tag{8}$$

Combining expressions (5), (7), and (8), we obtain

$$\bar p(0)-\bar p^*(1)=\int_{\bar x}^0\left[\frac{\widetilde{\mathrm{Ha}}^2}{2}\bar G+2\,\mathrm{Re}\,\bar G\frac{\mathrm{d}\bar G}{\mathrm{d}\bar x}\right]\mathrm{d}\bar x+\frac{\mathrm{d}\bar G}{\mathrm{d}\bar x}+\int_1^{\bar x}\left[\frac{\widetilde{\mathrm{Ha}}^2}{2}(1-\bar G)-2\,\mathrm{Re}(1-\bar G)\frac{\mathrm{d}\bar G}{\mathrm{d}\bar x}\right]\mathrm{d}\bar x. \tag{9}$$

Differentiating Eq. (9) with respect to $\bar x$, we obtain the equation of motion in the inlet collector:

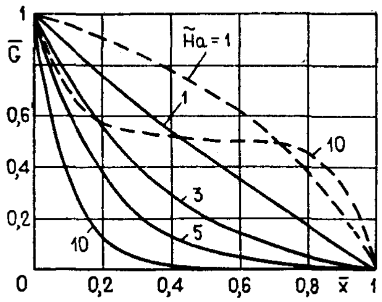

Fig. 2

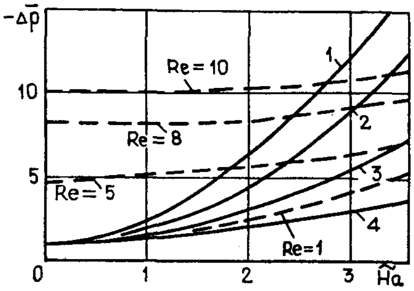

Fig. 3

Fig. 2. Distribution of mass flow rates along the length of the collectors. Solid lines relate to scheme Π and dashed lines to scheme Z.

Fig. 3. Dependence of pressure drop on $\tilde{\text{Ha}}$ number. Solid lines relate to scheme Π: 1) with nonuniformity, set up according to the scheme of Fig. 1c; 2) with nonuniformity artificially set up using a throttling grid; 3) with a uniform distribution of the mass flow rates using a throttling grid; 4) with the natural flow of an electrically conducting fluid. Dashed lines relate to scheme Z.

$$\frac{d^2\bar{G}}{d\bar{x}^2}-2\,\text{Re}\frac{d\bar{G}}{d\bar{x}}-\tilde{\text{Ha}}^2\bar{G}+\frac{\tilde{\text{Ha}}^2}{2}=0 \tag{10}$$

with the boundary conditions

$$\bar{G}(0)=1 \text{ and } \bar{G}(1)=0. \tag{11}$$

We write its solution in the form

$$\bar{G}=\frac{1}{2}+c_1e^{(\text{Re}+a)\bar{x}}+c_2e^{(\text{Re}-a)\bar{x}} \quad \left(a=\sqrt{\text{Re}^2+\tilde{\text{Ha}}^2}\right), \tag{12}$$

where

$$c_1=\frac{e^{\text{Re}-a}+1}{2(e^{\text{Re}-a}-e^{\text{Re}+a})};\quad c_2=-\frac{1+e^{\text{Re}+a}}{2(e^{\text{Re}-a}-e^{\text{Re}+a})}.$$

For the pressure drop, setting $\bar{x}=1$ in (9), using (12) we obtain

$$\bar{p}(0)-\bar{p}^*(1)=\frac{\tilde{\text{Ha}}^2}{2}\left[-\frac{1}{2}+\frac{c_1}{\text{Re}+a}(1-e^{\text{Re}+a})+\frac{c_2}{\text{Re}-a}(1-e^{\text{Re}-a})\right]+ \\ +\text{Re}+c_1(\text{Re}+a)e^{\text{Re}+a}+c_2e^{\text{Re}-a}(\text{Re}-a). \tag{13}$$

Using Eqs. (7), (8), and (12), we obtain expressions for the distribution of the pressures in the collectors:

$$\bar{p}(0)-\bar{p}(x)=\frac{\tilde{\text{Ha}}^2}{2}\left\{-\frac{\bar{x}}{2}+\frac{c_1}{\text{Re}+a}\left[1-e^{(\text{Re}+a)\bar{x}}\right]+\frac{c_2}{\text{Re}-a}[1-e^{(\text{Re}-a)\bar{x}}]\right\}+\text{Re}(1-\bar{G}^2); \tag{14}$$

$$p^*(x)-\bar{p}^*(1)=\frac{\tilde{\text{Ha}}^2}{2}(\bar{x}-1)-\frac{\tilde{\text{Ha}}^2}{2}\left\{\frac{\bar{x}-1}{2}+\frac{c_1}{\text{Re}+a}[e^{(\text{Re}+a)\bar{x}}-e^{\text{Re}+a}]+ \right. \\ \left. +\frac{c_2}{\text{Re}-a}[e^{(\text{Re}-a)\bar{x}}-e^{\text{Re}-a}]\right\}+\text{Re}[(\bar{G}-1)^2-1]. \tag{15}$$

The distribution of the mass flow rate along the length of the inlet collector for scheme Z is shown in Fig. 2 by the dashed lines. The flow is distributed nonuniformly over the tubes parallel to the magnetic field. With small magnetic fields, the main flow of fluid issues from a collector removed from the inlet part of the collector. With large fields, a stagnant zone arises in the tubes coming out of the middle part of the collector.

The dependence of the pressure drop on the $\tilde{\mathrm{Ha}}$ number is shown in Fig. 3. The drop increases with a rise in $\tilde{\mathrm{Ha}}$ and Re. As $\tilde{\mathrm{Ha}} \to \infty$, from (13) it follows that

$$\bar{p}(0)-\bar{p}^*(1)=-{}^1/_4\widetilde{\mathrm{Ha}}^2. \qquad (16)$$

In the limiting cases $\tilde{\mathrm{Ha}} \to 0$, $\mathrm{Re} \to \infty$ we have $\bar{p}(0) - \bar{p}^*(1) \to -\mathrm{Re}$ and, in the limit $\tilde{\mathrm{Ha}} \to 0$ and $\mathrm{Re} \to 0$,

$$\bar{p}(0)-\bar{p}^*(1)\to -1. \qquad (17)$$

§3. We consider now the motion of the liquid in scheme Π, also with a constant cross section.

We represent the total pressure drop in the form

$$\bar{p}(0)-\bar{p}^*(0)=[\bar{p}(0)-\bar{p}(\bar{x})]+[\bar{p}(\bar{x})-\bar{p}^*(\bar{x})]+[\bar{p}^*(\bar{x})-\bar{p}^*(0)]. \qquad (18)$$

Taking account of $\bar{G}^* = -\bar{G}$ in (18) and using (4) we write

$$\bar{p}^*(\bar{x})-\bar{p}^*(0)=\int_0^{\bar{x}}\left[-\frac{\widetilde{\mathrm{Ha}}^2}{2}\bar{G}+2\,\mathrm{Re}\,\bar{G}\frac{d\bar{G}}{d\bar{x}}\right]d\bar{x}. \qquad (19)$$

Combining expressions (5), (7), and (19), we obtain

$$\bar{p}(0)-\bar{p}^*(0)=-\int_0^{\bar{x}}\widetilde{\mathrm{Ha}}^2\,\bar{G}\,d\bar{x}+\frac{d\bar{G}}{d\bar{x}}. \qquad (20)$$

Differentiating Eq. (20) with respect to $\bar{x}$, we obtain an expression for the motion in the inlet collector in the form

$$d^2\bar{G}/d\bar{x}^2-\widetilde{\mathrm{Ha}}^2\,\bar{G}=0 \qquad (21)$$

with boundary conditions (11).

Its solution has the form

$$\bar{G}=(e^{-2\widetilde{\mathrm{Ha}}}-1)^{-1}[e^{\widetilde{\mathrm{Ha}}(\bar{x}-2)}-e^{-\widetilde{\mathrm{Ha}}\bar{x}}]. \qquad (22)$$

For the pressure drop, setting $\bar{x} = 0$ in (20), using (22), we obtain

$$\bar{p}(0)-\bar{p}^*(0)=\frac{\widetilde{\mathrm{Ha}}\,(e^{-2\widetilde{\mathrm{Ha}}}+1)}{e^{-2\widetilde{\mathrm{Ha}}}-1}. \qquad (23)$$

Using (7), (19), and (22), we obtain expressions for the distribution of the pressures in the collectors:

$$\Delta\bar{p}=(1-\bar{G}^2)\,\mathrm{Re}\pm\frac{\widetilde{\mathrm{Ha}}}{2}(e^{-2\widetilde{\mathrm{Ha}}}-1)^{-1}[e^{\widetilde{\mathrm{Ha}}(\bar{x}-2)}-e^{-2\widetilde{\mathrm{Ha}}}+e^{-\widetilde{\mathrm{Ha}}\bar{x}}-1]. \qquad (24)$$

Here $\Delta\bar{p} = \bar{p}(0) - \bar{p}(\bar{x})$ for the inlet collector and $\Delta\bar{p} = \bar{p}^*(0) - \bar{p}^*(\bar{x})$ for the outlet collector; the "plus" sign relates to the outlet collector and the "minus" sign to the inlet collector. The distribution of the mass flow rate along the length of the inlet collector for the scheme under consideration is shown in Fig. 2 by the solid lines. With a magnetic field, the fluid spreads out uniformly over tubes parallel to the magnetic field. In a strong field, the fluid flows out of the collector mainly near its inlet. In distinction from scheme Z, the pressure drop depends only on the $\tilde{\mathrm{Ha}}$ number and increases with a rise in the latter. In limiting cases, where $\tilde{\mathrm{Ha}} \to 0$,

$$\bar{p}(0)-\bar{p}^*(0)\to -1, \qquad (25)$$

while for $\tilde{\mathrm{Ha}} \to \infty$,

$$\bar{p}(0)-\bar{p}^*(0)\to -\widetilde{\mathrm{Ha}}. \qquad (25a)$$

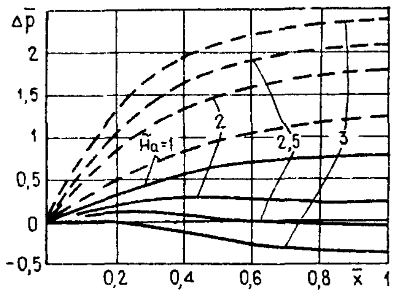

Fig. 4. Distribution of pressure along length of distributing collectors of scheme П. Solid lines relate to the inlet collector and dashed lines to the outlet collector.

From Fig. 3 and from a comparison of the limiting cases (16), (17) and (25), (25a), it follows that the pressure drop in scheme П is less than that in scheme Z.

Without a magnetic field, the pressure rises downstream in the inlet collector and falls in the outlet collector. In moderate fields, the pressure in the inlet collector at first rises, attains a maximum, and then starts to drop. In sufficiently strong fields, the pressure falls along the whole length of the collector. In the outlet collector, a magnetic field leads to a steeper fall of the pressure along its length (Fig. 4).

§4. Let us consider some ways to equalize the mass flow rates. In the case of a uniform distribution of the mass flow rates

$$\bar{G}_{\Pi}=\bar{G}_{z}=1-\bar{x} \tag{26}$$

and

$$\bar{G}^{*}_{\Pi}=\bar{x}-1; \quad \bar{G}^{*}_{z}=\bar{x}. \tag{27}$$

a) The distribution of the mass flow rates will obviously be uniform if the collectors are so shaped that the following condition will be satisfied:

$$\mathrm{d}(\bar{p}^{*}-\bar{p})/\mathrm{d}\bar{x}=0. \tag{28}$$

In the special case $d\bar{p}^{*}/d\bar{x} = d\bar{p}/d\bar{x} = 0$, from Eq. (4) we can obtain a shaping law which holds for both schemes:

$$(\mathrm{St}-1)(\mathrm{St}-\bar{F}^{-1})=\bar{G}^{2}. \tag{29}$$

Here the dimensionless Stewart number $\mathrm{St} = \tilde{\mathrm{Ha}}^{2}/4\mathrm{Re}$ for the inlet collector and $\mathrm{St} = -\tilde{\mathrm{Ha}}^{2}/\mathrm{Re}$ for the outlet collector. The value of the quantity $\bar{G}$ in formula (29) is taken from expressions (26) and (27). From Eq. (29) it follows that, in distinction from the case where there is no field, such shaping is possible only for $|\mathrm{St}| < 1$. The pressure drop for scheme П with a uniform distribution of the mass flow rates is minimal and equal to

$$\bar{p}(0)-\bar{p}^{*}(0)=-1. \tag{30}$$

b) Now let, in scheme П, a uniform distribution of the mass flow rates be created by the use of a throttling grid, installed at the inlet and parallel to the magnetic field of the tube. In this case

$$p-p^{*}=[c+\bar{\delta}(x)]\mathrm{d}\bar{G}/\mathrm{d}\bar{x} \quad (\bar{\delta}=\delta/c), \tag{31}$$

where δ is the resistance introduced by the throttling grid. In dimensionless form

$$\bar{p}-\bar{p}^{*}=(1+\delta)\,\mathrm{d}\bar{G}/\mathrm{d}\bar{x}. \tag{32}$$

Combining expressions (32), (7), and (19), we obtain

$$\bar{p}(0)-\bar{p}^{*}(0)=\int_{0}^{\bar{x}}(-\tilde{\mathrm{Ha}}^{2}\,\bar{G})\,\mathrm{d}\bar{x}+\frac{\mathrm{d}\bar{G}}{\mathrm{d}\bar{x}}(1+\bar{\delta}). \tag{33}$$

Substituting $\bar{G} = 1 - \bar{x}$ in (33), we write

$$\bar{p}(0)-\bar{p}^{*}(0)=\tilde{\mathrm{Ha}}^{2}(^{1}/_{2}\bar{x}^{2}-\bar{x})-1-\bar{\delta}(\bar{x}). \tag{34}$$

Assuming that $\bar{\delta}(1) = 0$, i.e., no resistance is introduced at the inlet to the tube with a minimal mass flow rate, using (34) we obtain

$$\bar{p}(0) - \bar{p}^*(0) = -{}^1/_2\widetilde{\mathrm{Ha}}^2 - 1. \tag{35}$$

We obtain the resistance of the throttling grid using (34) and (35) in the form

$$\bar{\delta}(\bar{x}) = {}^1/_2\,\widetilde{\mathrm{Ha}}^2\,(\bar{x}-1)^2. \tag{36}$$

The dependence of the pressure drop on the $\widetilde{\mathrm{Ha}}$ number for the case in question is shown in Fig. 3. The use of a throttling grid, as in the absence of a magnetic field [1], is found to be less effective for equalizing the mass flow rates than shaping of the collectors.

§5. Let us consider the example of the creation of an artificial nonuniformity in scheme П. Let it be required to set up a considerable mass flow rate in the longitudinal tubes with $\bar{x} = 1$, and let the fluid be supplied and removed as before, with $\bar{x} = 0$. This can be achieved either by installing a barrier in the direction of the flow (Fig. 1c), or, for example, by using a throttling grid. In the first case, using (22) we write

$$\bar{G} = (e^{-2\widetilde{\mathrm{Ha}}} - 1)^{-1}[e^{-\widetilde{\mathrm{Ha}}(1-\bar{x})} - e^{-\widetilde{\mathrm{Ha}}(1+\bar{x})}]. \tag{37}$$

For the total pressure drop, using expression (23), we obtain

$$\bar{p}(0) - \bar{p}^*(0) = \frac{\widetilde{\mathrm{Ha}}\,(e^{-2\widetilde{\mathrm{Ha}}}+1)}{e^{-2\widetilde{\mathrm{Ha}}} - 1} - \widetilde{\mathrm{Ha}}^2. \tag{38}$$

In the case where the distribution (37) is achieved by the use of a throttling grid, from expressions (33) and (37) for $\bar{\delta}(1) = 0$ it follows that

$$\bar{p}(0) - \bar{p}^*(0) = -\widetilde{\mathrm{Ha}}^2 + \frac{2\widetilde{\mathrm{Ha}}\,e^{-\widetilde{\mathrm{Ha}}}}{e^{-2\widetilde{\mathrm{Ha}}} - 1}.$$

A comparison of the pressure drops for both cases (see Fig. 3) shows that the use of a throttling grid is more effective here.

CONCLUSIONS

1. The flow of an electrically conducting fluid in schemes Z and П of the collectors has been discussed in a quasi-one-dimensional statement.

2. The distribution of the fluid is nonuniform in both cases. In a strong magnetic field, in scheme Z a stagnant zone is formed in the tubes coming out of the middle part of the feeding collector and, in scheme П, in the tubes coming out of its end zone. The required head is greater in scheme Z than in scheme П.

3. Means of achieving a uniform distribution of an electrically conducting fluid have been discussed and the example of the creation of artificial nonuniformity has been given. In particular, in distinction from the case where there is no magnetic field, a uniform distribution can be achieved using shaping, with an unchanged pressure in the collectors, only for St < 1.

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