DIRECT-CURRENT CONDUCTION-TYPE PUMP WITH IDEALLY SECTIONALIZED ELECTRODES

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With the building of direct-current conduction-type pumps, the electrodes must be sectionalized, due to the considerable vortical losses in the case of solid electrodes [1, 2].

The distribution of the density of the currents and stresses in the channel of a direct-current single-phase conduction-type pump with ideally sectionalized electrodes where the distribution of the current at the electrodes is assumed to be known has been obtained in [3-5]. Here it was assumed that the distribution of the current at the electrodes is uniform. This is possible only in the case where each pair of electrodes is fed from an independent regulated feed source. In practice, to ensure the work of the feed sources, due to the small voltages and large currents, an intermediate transformer must be used, whose low-voltage side feeds the channel of the machine. Thus, in each section of the electrodes, identical smf's are applied; the currents will be different as a result of edge effects.

§1. The present article considers a single-phase conduction-type pump having a channel with a height Δ and a width 2 h. One part of the walls of the channels $|x| > \lambda$ is made up of insulators, and the other of ideally sectionalized electrodes, each section of which is connected to its own turn of a multiwinding transformer (Fig. 1). Such a scheme of the external circuit leads to an inhomogeneous distribution of the current at the electrode, although the active resistances of each subcircuit are identical (the inductance of the dissipation of the secondary winding of the transformer is neglected). In each turn of the transformer the emf E is induced. It can be shown that [6], in the case where there is no dissipation scattering, the phase of E practically coincides with the phase of the perturbation of the primary field in the gap, set up by the winding.

A working body with the conductivity σ is moving in a longitudinal direction with a given velocity $\mathbf{v}(v(y), 0, 0)$. At the walls of the channel, the velocity is equal to zero.

The pulsating magnetic field set up by the perturbation winding is perpendicular to the plane of the flow and depends on the longitudinal coordinate in the following manner:

$$B(x, t) = B(x)e^{i\omega t};$$

$$B(x) = \begin{cases} -B_0 & \text{with } |x| \leqslant \lambda, \\ -B_0 e^{-\frac{|x|-\lambda}{T}} & \text{with } |x| > \lambda. \end{cases} \quad (1)$$

If the effective depth of the penetration of the field is greater than the height of the channel Δ, the plane-parallel problem can be considered.

It is further assumed that the "armature reaction" is compensated by a compensating busbar, that the demagnetizing action of the vortical currents is small, and that the resultant magnetic field on the channel is close to the external field, set up by the perturbation winding.

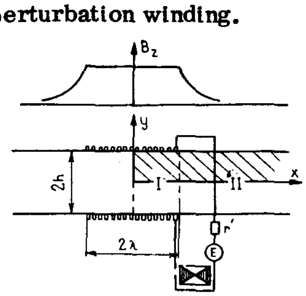

Fig. 1. Statement of problem.

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With the flow of an electrically conducting liquid in an alternating-sign magnetic field, analogously to [1] scalar and vector potentials can be introduced; the vector potential can be taken in the form

$$\mathbf{A}(0, A_y(x, t), 0),$$

where

$$A_y(x, t) = A(x)e^{i\omega t};$$

$$A(x) = \begin{cases} -\left[B_0\lambda + B_0T\left(1 - e^{-\frac{x-\lambda}{T}}\right)\right] & \text{with } x > \lambda, \\ -B_0x & \text{with } |x| \leqslant \lambda, \\ B_0\lambda + B_0T\left(1 - e^{\frac{x+\lambda}{T}}\right) & \text{with } x < -\lambda. \end{cases} \quad (2)$$

Further, all the fields are assumed to be sinusoidal. In what follows, the factor $e^{i\omega t}$ is omitted.

In accordance with [7] the electromagnetic fields in the channel are described by the equation

$$\Delta\dot{\varphi} = -B\partial v/\partial y. \quad (3)$$

Ohm's law has the form

$$j_x = -\sigma\partial\dot{\varphi}/\partial x; \quad j_y = -\sigma(vB + \partial\dot{\varphi}/\partial y) - i\sigma\omega A. \quad (4)$$

Here $\dot{\varphi}(x, y) = \varphi_1(x, y) + i\varphi_2(x, y)$, $j_x = j_{1x}(x, y) + ij_{2x}(x, y)$, $j_y = j_{1j}(x, y) + ij_{2y}(x, y)$ are the complex amplitudes of the potential and the currents.

Let us consider the boundary conditions. At the insulator walls there is no normal current; therefore, from (4) we obtain

$$\partial\dot{\varphi}/\partial y = -i\omega A \quad \text{with} \quad |x| > \lambda, \ y = \pm h. \quad (5)$$

At ideally sectionalized electrodes, Ohm's law can be written for the "external" circuit of each section [6]:

$$j_y(x) = -\frac{\dot{U}(x) + E}{2\lambda\Delta r}. \quad (6)$$

Here r is the resistance of the secondary winding of the transformer; $\dot{U}(x) = \dot{\varphi}(x, -h) - \dot{\varphi}(x, h)$ is the potential difference between the electrodes. Condition (6) at the electrodes is analogous to the third boundary condition in [8]. From (6), taking account of (4), the following boundary conditions are obtained:

$$\dot{\varphi} = -\lambda\sigma\Delta r\partial\dot{\varphi}/\partial y - i\lambda\sigma\Delta r\omega A + E/2 \quad \text{with} \quad |x| \leqslant \lambda, \ y = h;$$
$$\dot{\varphi} = \lambda\sigma\Delta r\partial\dot{\varphi}/\partial y + i\lambda\sigma\Delta r\omega A - E/2 \quad \text{with} \quad |x| \leqslant \lambda, \ y = -h. \quad (7)$$

With $x \to \pm\infty$, the current density must tend toward zero; therefore, from (4) taking account of (1) and (2) we obtain

$$\frac{\partial\dot{\varphi}}{\partial x} = 0, \quad \frac{\partial\dot{\varphi}}{\partial y} = -i\omega A(x \to \pm\infty) \text{ with } x \to \pm\infty. \quad (8)$$

Introducing the dimensionless quantities $\tilde{x} = x/h$, $\tilde{y} = y/h$, $\tilde{\varphi} = \varphi/v_0B_0h$, $\tilde{j} = j/\sigma v_0B_0$, $\tilde{A} = A/B_0\lambda$, $\tilde{B} = B/B_0$, $\tilde{v} = v/v_0$, $\tilde{E} = E/2hv_0B_0$, $\tilde{T} = T/h$, $\tilde{\lambda} = \lambda/h$, $q = \omega\lambda/v_0$ is the parameter of quasi-steady-state conditions, $k = r\Delta\sigma\lambda/h$ is the coefficient of the load (in what follows the tilde is omitted), and substituting the complex amplitudes of the potential and the current in Eqs. (3) and (4) and the boundary conditions (5), (7), and (8), for $\varphi_1(x, y)$ and $\varphi_2(x, y)$ individually we obtain equations with boundary conditions.

For $\varphi_1(x, y)$ we obtain

$$\Delta\varphi_1 = -B\partial v/\partial y; \quad (9)$$

$$\partial\varphi_1/\partial y = 0 \quad \text{with} \quad |x| > \lambda, \ y = \pm 1;$$

$$\varphi_1 = -k\partial\varphi_1/\partial y + E \quad \text{with} \quad |x| \leqslant \lambda, \ y = 1, \quad (10)$$

$$\varphi_1 = k\partial\varphi_1/\partial y - E \quad \text{with} \quad |x| \leqslant \lambda, \ y = -1; \quad (11)$$

$$\partial\varphi_1/\partial x = \partial\varphi_1/\partial y = 0 \quad \text{with} \quad x \to \pm\infty. \quad (12)$$

From (4) follows

$$j_{1x} = -\frac{\partial \varphi_1}{\partial x}, \quad j_{1y} = -\frac{\partial \varphi_1}{\partial y} - vB. \tag{13}$$

For $\varphi_2(x, y)$ we obtain

$$\Delta\varphi_2 = 0; \tag{14}$$

$$\partial\varphi_2/\partial y = -qA \quad \text{with} \quad |x| > \lambda, \; y = \pm 1; \tag{15}$$

$$\varphi_2 = -k\partial\varphi_2/\partial y - kqA \quad \text{with} \quad |x| \leq \lambda, \; y = 1,$$
$$\varphi_2 = k\partial\varphi_2/\partial y + kqA \quad \text{with} \quad |x| \leq \lambda, \; y = -1; \tag{16}$$

$$\partial\varphi_2/\partial x = 0, \quad \partial\varphi_2/\partial y = -qA \quad \text{with} \quad x \to \pm\infty. \tag{17}$$

From (4) follows:

$$j_{2x} = -\frac{\partial \varphi_2}{\partial x}, \quad j_{2y} = -\frac{\partial \varphi_2}{\partial y} - qA. \tag{18}$$

Thus, the distribution of the current density in the channel is obtained from the superposition of the solutions of two independent boundary-value problems. One of them (9) describes the distribution of the currents with the flow of an electrically conducting liquid with a velocity v in the channel of a pump with ideally sectionalized electrodes in a constant magnetic field with a given three-dimensional distribution; the second (14) describes the distribution of the density of the vortical current in a channel with ideally sectionalized electrodes in a fixed liquid in a pulsating magnetic field.

§2. We solve Eq. (9) with the boundary conditions (10)-(12). From the symmetry of the problem it follows that $\varphi_1(x, y) = \varphi_1(-x, y)$; this makes it possible to consider half the channel $x \geq 0$. By virtue of the continuity of the normal derivative at the y axis

$$\partial\varphi_1/\partial x = 0 \quad \text{with} \quad x = 0, \; |y| \leq 1. \tag{19}$$

We take the profile of the velocity in the form

$$v(y) = \frac{s(\operatorname{ch} s - \operatorname{ch} sy)}{s \operatorname{ch} s - \operatorname{sh} s}. \tag{20}$$

Here s is a parameter characterizing the filling-out of the profile.

We represent the solution of Eq. (9) in the form of a Fourier series; in the electrode region (region I; $x \leq \lambda$) satisfying the boundary conditions (10) and (19):

$$\varphi_{11}(x, y) = \sum_{n=1}^{\infty} \left[C_n \frac{\operatorname{ch} \beta_n x}{\operatorname{ch} \beta_n \lambda} - g_n \right] \sin \beta_n y + \frac{E}{1+k} y, \tag{21}$$

and, in the region of insulators (region II, $x > \lambda$) satisfying the boundary conditions (11) and (12):

$$\varphi_{12}(x, y) = \sum_{n=1}^{\infty} \left[A_n e^{-\alpha_n (x-\lambda)} + L_n e^{-\frac{x-\lambda}{T}} \right] \sin \alpha_n y. \tag{22}$$

Here φ_{11} and φ_{12} are the values of the potential φ_1 in the regions I and II; β_n are the roots of the equation $\tan \beta_n = -k\beta_n$; $\alpha_n = \pi(2n-1)/2$;

$$g_n = \frac{2s^3 \cos \beta_n}{(s - \operatorname{th} s)(\beta_n - \cos \beta_n \sin \beta_n)(s^2 + \beta_n^2)} \left(k + \frac{\operatorname{th} s}{s} \right);$$
$$L_n = \frac{T^2}{1 - T^2 \alpha_n^2} L'_n; \quad L'_n = \frac{2(-1)^n s^3}{(s^2 + \alpha_n^2)(s - \operatorname{th} s)}.$$

We find the unknown coefficients A_n and C_n using the methods of successive approximations analogous to the method of Schwarz [9], making use of the continuity of the potential and its normal derivative at the boundary between the electrode region and the region of insulators

$$\partial\varphi^{(i+1)}{}_{11}/\partial x = \partial\varphi^{(i)}{}_{12}/\partial x; \tag{23}$$
$$\hat{\varphi}^{(i+1)}{}_{12} = \varphi^{(i+1)}{}_{11}; \quad \varphi^{(i+1)}{}_{12} = (1-t)\hat{\varphi}^{(i+1)}{}_{12} + t\varphi^{(i)}{}_{12}.$$

As calculations have shown, the the values of the parameter t ($0 < t < 1$), ensuring convergence of the iterational process, depend essentially on the loading coefficient.

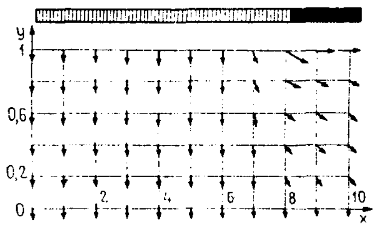

Fig. 2. k = 1, λ = 8, T = 2, E = 1.5, s = 50.

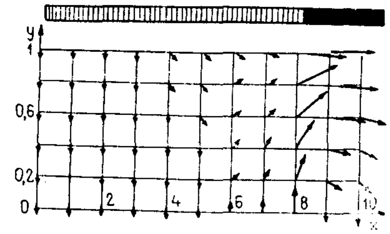

Fig. 3. k = 100, λ = 8, T = 2, E = 15, s = 50.

Knowing the distribution of the potential (21) and (22), using (13) we can find the distribution of the current densities j_{1x} and j_{1y}. The results of these calculations are given in Figs. 2 and 3. As a result of the symmetry of the problem (j_{1y} is an even function with respect to x and even with respect to y; j_{1x} is an odd function with respect to x and odd with respect to y), the distributions of the density are given only in the fourth of the channel hatched in Fig. 1. For clarity, the scale of Fig. 3 was enlarged by 5 times in comparison with Fig. 2. From Fig. 3 it can be seen that, with large values of k, in the end zone of the magnetic field, vortical currents are brought about by the longitudinal inhomogeneity of the field.

§3. We now solve the problem of the distribution of the density of a vortex current: Eq. (14) with the boundary conditions (15)-(17). By virtue of the symmetry $\varphi_2(x, y) = -\varphi_2(-x, y)$, only half of the channel is considered, $x \geq 0$. From the condition of the continuity of the potential at the axis follows

$$\varphi_2(y) = 0 \quad \text{with} \quad x = 0, \ |y| \leqslant 1. \tag{24}$$

Analogously to the preceding section, we represent the solution in the form of Fourier series, different in region I and in region II, and each in its own region satisfying the boundary conditions (15)-(17), (24).

In region I ($x \leq \lambda$)

$$\varphi_{21}(x, y) = \sum_{n=1}^{\infty} C'_n \frac{\operatorname{sh} \beta_n x}{\operatorname{sh} \beta_n \lambda} \sin \beta_n y + \frac{kq}{\lambda(1+k)} xy. \tag{25}$$

In region II ($x > \lambda$)

$$\varphi_{22}(x, y) = \sum_{n=1}^{\infty} \left[A'_n e^{-\alpha_n (x-\lambda)} + D_n e^{-\frac{x-\lambda}{T}} \right] \sin \alpha_n y - qA(x) y. \tag{26}$$

Here φ_{21} and φ_{22} are the values of the potential φ_2 in regions I and II;

$$D_n = \frac{2(-1)^{n+1} qT}{\lambda \alpha_n^2 (1 - T^2 \alpha_n^2)}.$$

The unknown coefficients C'_n and A'_n are found, as in the preceding section, using the method of successive approximations, where use is made of the continuity of the potential and the normal derivative of the potential at the boundary between the regions. As in Sec. 2, the parameter t depends essentially on the loading coefficient k. For the values of the parameter $k \geq 1$, the calculating time for the solution of problems (9) and (14) was 10-20 min in an M-222 digital computer.

From (18) we can find the distribution of the density of the vortex current. Figure 4 gives the densities of the vortex current in the channel of a single-phase conduction-type pump with ideally sectionalized electrodes. The symmetry of the problem (j_{2x} is an even function with respect to x and an odd function with respect to y; j_{2y} is an odd function with respect to x and an even function with respect to y) makes it possible to show in Fig. 4 only a fourth of the channel.

From Fig. 4 it can be seen that only part of the current is closed inside the channel (in its central zone), forming a current vortex, while part flows into the electrode, short-circuited through the windings of the transformer. These currents, flowing through the windings of the transformer, bring about additional vortex currents which, like the density of the vortex current, depend mainly on the loading coefficient. With large values of k, the vortical current, as was to be expected, is almost completely closed inside the channel; here, in the central zone the distribution of the current is linear.

§4. Let us consider the solutions (21), (22), (25), and (26) with $k \to \infty$ (no-load conditions). Since with ideal sectionalizing of the electrodes there are no short-circuiting currents through the electrodes, the present case corresponds to the problem with insulated walls and $j_{1y}(x, \pm 1) = 0$, $j_{2y}(x, \pm 1) = 0$ with $|x| < \infty$.

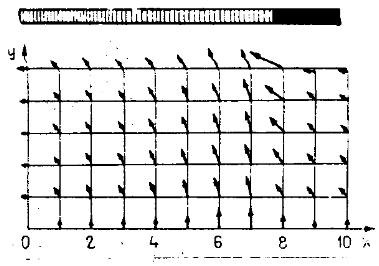

Fig. 4. k=1, λ=8, T=2, q=3.

The roots of the equation $\tan\beta_n = -k\beta_n$ with $k\to\infty$ go over into $\alpha_n=\pi(2n-1)/2$; the systems of eigenfunctions in the region of the electrodes and the region of the insulators coincide and, from the condition of the equality of the potential and its normal derivative at the boundary between the regions, the coefficients of the Fourier coefficients can be found directly:

$$C_n=L_n\frac{1-\alpha_nT}{\alpha_nT^2(\gamma_n+\alpha_n)};\quad A_n=-L_n\frac{\alpha_n{}^2T+\gamma_n}{\alpha_n{}^2T^2(\gamma_n+\alpha_n)};$$
$$C'_n=-D_n\frac{1-\alpha_nT}{T(\gamma'_n+\alpha_n)};\quad A'_n=-D_n\frac{1+\gamma'_nT}{(\gamma'_n+\alpha_n)T}. \tag{27}$$

Here $\gamma_n=\alpha_n\tanh\alpha_n\lambda$, $\gamma'_n=\alpha_n\coth\alpha_n\lambda$.

In the case of an infinite homogeneous field $(T\to\infty)$ $C_n=A_n=C'_n=A'_n=0$ and the current density $\mathbf{j}_1$ in the channel will be equal to zero. The distribution of the density of the vortical current in this case will have the form

$$j_{2x}=-\frac{q}{\lambda}y;\quad j_{2y}=0 \tag{28}$$

and coincides with that obtained in [6].

If the magnetic field is cut off outside the electrode zone $(T=0)$, from (27) we obtain

$$A_n=-\frac{L'_n}{\alpha_n{}^2}\operatorname{sh}\alpha_n\lambda e^{-\alpha_n\lambda},\quad C_n=\frac{L'_n}{\alpha_n{}^2}\operatorname{ch}\alpha_n\lambda e^{-\alpha_n\lambda}.$$

It can be shown that the distribution of the potential $\varphi_1(x, y)$ in this case coincides with that given in [7] for a channel with insulated walls in a constant magnetic field:

$$B(x)=-B_0 \text{ with } |x|\leqslant\lambda,\quad B(x)=0 \text{ with } |x|>\lambda.$$

§5. For the assumptions made, analogously to [2], after averaging with respect to the time we can write the energy balance for an alternating-current pump individually for the currents $\mathbf{j}_1$ and the vortical currents $\mathbf{j}_2$:

$$Q'_1+Q''_1-P_{el}=N;\quad Q'_2+Q''_2=P_{pg}, \tag{29}$$

where

$$Q'_1=\frac{1}{8\lambda}\int_{-1}^{1}\int_{-\infty}^{\infty}\mathbf{j}_1{}^2(x, y)\,dx\,dy;\quad Q''_1=\frac{k}{4\lambda}\int_{-\lambda}^{\lambda}j_{1y}{}^2(x, 1)\,dx;$$
$$P_{el}=-\frac{1}{8\lambda}\int_{-1}^{1}\int_{-\infty}^{\infty}v(y)B(x)j_{1y}(x, y)\,dx\,dy;$$
$$N=-\frac{E}{4\lambda}\int_{-\lambda}^{\lambda}j_{1y}(x, 1)\,dx;\quad Q'_2=\frac{1}{8\lambda}\int_{-1}^{1}\int_{-\infty}^{\infty}\mathbf{j}_2{}^2(x, y)\,dx\,dy;$$
$$Q''_2=\frac{k}{4\lambda}\int_{-\lambda}^{\lambda}j_{2y}{}^2(x, 1)\,dx;\quad P_{pg}=-\frac{q}{8\lambda}\int_{-1}^{1}\int_{-\infty}^{\infty}j_{2y}(x, y)A(x)\,dx\,dy.$$

Here Q'_1 are the Joule losses from the currents $\mathbf{j}_1$ in the channel; Q''_1 the Joule losses from the currents $\mathbf{j}_1$ in the secondary winding of the transformer; P_{el}, the electric power, which, in the absence of friction, is equal to the hydraulic power of the pump; N, power required by the pump from the primary circuit of the transformer; Q'_2, Joule losses from the vortex currents in the channel; Q''_2, Joule losses in the secondary winding of the transformer from the vortex currents short-circuited through this winding; P_{pg}, power required from the perturbation grid.

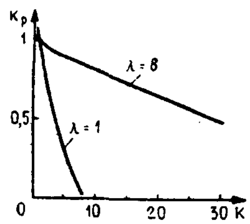

Fig. 5. E = 1.5, T = 2, S = 50.

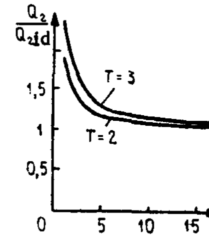

Fig. 6. k = 1000; q = 3.

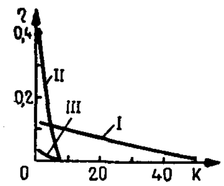

Fig. 7. E = 1.5, T = 2. Curve I) λ = 8, q = 3; II) λ = 1, q = 3/8; III) λ = 1, q = 3.

All the energy characteristics are brought into dimensionless form by dividing the corresponding expressions by the dimensinal quantity $\sigma v_0^2 B_0^2 4\lambda h\Delta$. Knowing the distribution of the current density in the channel, we can calculate the dimensionless head from the pressure developed by the pump:

$$\Delta p = -\frac{1}{4\lambda}\int_{-\infty}^{\infty} j_{1y}(x) B(x)\, dx,$$

where $j_{1y}(x)$ is the current density averaged with respect to y.

The expressions for the separate kinds of energy losses make it possible to obtain the characteristics of a single-phase conduction-type pump as a function of the different parameters.

Figure 5 shows the dependence $k_p = \Delta p/\Delta p_{id}$ on the loading coefficient; $\Delta p_{id} = (1-E)/2(1+k)$ is the pressure developed by a section of length 2λ of an "ideal" pump, i.e., an alternating current pump with ideally sectionalized electrodes, without end effects. The distribution of the current density j_{1y} in this pump will be homogeneous. With small values of k, the value of k_p is found to be greater than unity, since the end zones may set up a useful head; with large values of k, due to the appearance of vortex currents in the end zones k_p is less than unity.

Figure 6 shows the ratio of the total vortical losses $Q_2 = Q'_2 + Q''_2$ with $k = 10^3$ with different values of the parameter T to the vortical losses $Q_{2id} = \frac{1}{6}q^2/\lambda^2$ of a section of length 2λ of an "ideal" pump, in which the distribution of the vortical currents is given by expression (28). As can be seen from Fig. 6, with an increase in the length of the electrode zone, the effect of the end zones decreases and the ratio Q_2/Q_{2id} tends toward unity.

For an alternating-current single-phase pump, fed from a grid through an intermediate transformer, it is expedient to introduce the efficiency in the form

$$\eta = \frac{|P_{el}|}{P_{pg} + N}.$$

The dependence of the efficiency on the loading coefficient k with different values of the relative length of the electrode λ and the parameter of quasi-steady-state conditions q is given in Fig. 7. The appearance of current vortices in the end zones with an increase in the value of k leads to a situation in which the channel starts to work under braking conditions. This effect comes out more strongly the lower the value of λ, since, with a small length of the electrode, the effect of the end zones increases. With a decrease in the parameter of steady-state conditions the vortical losses decrease and the efficiency rises.

The low efficiency of a single-channel conduction-type pump with ideally sectionalized electrodes is explained by the significance of the vortical losses, bound up with the fact that the vortex currents are closed not only through the liquid metal in the channel of the pump, but also through the secondary winding of the intermediate transformer. Closing of the vortical currents through the secondary winding is eliminated in pumps with two channels with an opposite direction of the flow, arranged one above the other and connected in series electrically.

The above-described method for calculating the distribution of the current density may be used both with an analysis of the work of direct-current conduction-type pumps with ideally sectionalized electrodes, and also for the calculation of vortical losses in conducting busbars in a pulsating magnetic field.

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LITERATURE CITED

1. G. M. Gekht and I. M. Tolmach, "Distribution of the current density in the channel of an alternating-current conduction-type pump," Magn. Gidrodin., No. 1, 122-128 (1973).
2. G. M. Gekht, "Integral characteristics of an alternating-current conduction-type machine," Magn. Gidrodin., No. 3, 125-130 (1974).
3. N. M. Nadezhnikov, "A single-phase conduction-type MHD pump with sectionalized electrodes," Magn. Gidrodin., No. 2, 117-123 (1973).
4. Yu. A. Birzvalk, N. M. Nadezhnikov, and A. S. Petrazhitskii, "Calculation of the stresses in a single-phase conduction-type MHD pump with sectionalized electrodes," Magn. Gidrodin., No. 3, 85-89 (1973).
5. Ya. V. Briedis, "Calculation of the electrical field in conduction-type pumps with a given normal arbitrary potential at the electrodes," Magn. Gidrodin., No. 3, 139-143 (1968).
6. E. I. Yantovskii and I. M. Tolmach, Magnetohydrodynamic Generators [in Russian], Nauka, Moscow (1972).
7. A. B. Vatazhin, G. S. Lyubimov, and S. A. Regirer, Magnetohydrodynamic Flows in Channels [in Russian], Nauka, Moscow (1970).
8. G. W. Sutton, "End losses in magnetohydrodynamic channels with tensor electrical conductivity and segmented electrodes," J. Appl. Phys., 34, No. 2, 396-403 (1963).
9. V. É. Katsnel'son and V. V. Men'shikov, "An analog of the alternating Schwarz method," in: The Theory of Functions, Functional Analysis and Their Applications No. 17 [in Russian], Khar'kov State Univ., Khar'kov (1973), pp. 206-215.