FREE ROTATING LAYER OF AN ELECTRICALLY CONDUCTING FLUID IN AN AXIAL MAGNETIC FIELD

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1. The greater part of the published theoretical and experimental investigations of rotating MHD flows pertains to conditions such that the cylindrical sidewalls of a vessel filled with an electrically conducting fluid strongly affect the velocity profiles and hydrodynamic stability. This is the case for flows between rotating cylinders in an axial or azimuthal magnetic field and in cylindrical vessels or annular ducts with crossed electric E and magnetic H fields. Considerably less attention has been given to the class of flows in which the rotation zone is situated at some distance from the sidewalls. Typical of this situation are a rotating layer removed from the axis and sidewall and, as a special case, rotational flow adjacent to the axis of rotation. These flows have free boundaries and are therefore jet-type flows. In the customary terminology of the theory of jets, it seems natural to call the given flow pattern a submerged free rotating layer.

A free rotating layer formed by the interaction of an axial magnetic field and the radial component of an electric field can serve as an important fluid-dynamic element in engineering devices. For example, research is being conducted in some countries in connection with the possibility of building a plasma centrifuge for the separation of isotopes and chemical elements. A bibliography on this topic may be found in [1-3]. Certain other engineering applications are noted in [4].

For investigations of the working process plasma machines can operate in the short-pulse regime, allowing contact between the rotating plasma and the wall [3]. In making the transition from physical investigations to the design of efficiently operating devices it is necessary to contain the rotating plasma at a certain distance from the wall, the temperature of which is limited by the heat resistance of the construction material. In this connection, a gas layer having a lower temperature is created between the wall and the rotating plasma, and the outer electrode of the current-conducting system is situated at a certain distance from the cylindrical wall [2]. One of the fundamental problems in such devices is to impart hydrodynamic stability to the free rotating layer in an axial magnetic field.

If the density of the plasma is not too small and the magnetic field induction is not too great, then the anisotropy of the electrical conductivity does not qualitatively influence the flow (for an assessment of the corresponding values of the pressure and induction, see, e.g., [5]). Many important MHD characteristics of such flows can be investigated in liquid-metal devices. Engineering devices in which a rotating layer is formed in a liquid metal are also possible. It is essential to note that in plasma and liquid-metal MHD devices gradients of the temperature and concentration can induce thermocurrents, which interact with the magnetic field to form, under definite conditions, a free rotating layer (problems of thermoelectric magnetohydrodynamics are discussed in [6]).

Any investigation of a free rotating layer or a free vortex flow must not be confined to the sphere of direct engineering applications, because it is closely allied with the study of the MHD turbulence problem. It is important to study the flow field in a stable regime, the stability limits, and the structure of the disturbance slightly below the critical point, along with the properties of the disturbed motion far above the critical point for free rotating layers and vortices of various geometries with various axial distributions of the circumferential velocity, ranging from uniform to strongly nonuniform and including an alternating distribution function.

2. In the classical experiment of Lehnert [7], a free rotating layer in an axial magnetic field is created in a vessel containing mercury with a rotating copper ring on its bottom. As the induction is increased the rotating layer elongates in the direction of the mag-

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netic field. For a high enough induction the experimental procedure makes it possible to obtain photographs of the surface of the mercury with a vortex street created by instability of the tangential velocity discontinuity. Discontinuities in the circumferential velocity profile are generated by discontinuities of the boundary conditions at the bottom of the vessel; stationary copper plates are placed inside and outside the rotating ring.

To increase the stability it is necessary to impart a smooth distribution to the circumferential velocity. Lehnert has proposed [2] a procedure for creating a free rotating layer by means of an axial magnetic field and a large number of coaxial electrodes mounted in the nonconducting bottom of the vessel. An adjustable potential distribution is applied to the electrodes, making it possible to control the circumferential velocity profile.

An experiment involving the rotation of a mercury layer by means of twelve electrodes is described in [8]. The outer electrode is situated on a cylindrical wall having a radius of 15 cm. The axial extent (height) of the mercury layer is ∿1.5 cm. The circumferential velocity profile near the surface of the mercury and the stability limit are determined with a conduction anemometer. To obtain the limiting stable profile the potential difference between each pair of electrodes is increased to a value at which a near-critical regime is attained. In the measured circumferential velocity profiles each electrode pair corresponds to a velocity step. As the stability limit is crossed, each step generates a disturbed annular zone. The experimental results are processed in accordance with the theory of Chandrasekhar for Taylor flow instability between rotating cylinders. A separate stability limit is obtained for each step of the velocity profile.

Hunt and Malcolm [9] have conducted a theoretical and experimental investigation of a free rotating layer created in a vessel containing mercury by the interaction of a uniform external magnetic field with current flowing between two disk-shaped electrodes. The planes of the electrodes are parallel, their axes coincide, and the magnetic field is directed along the axis. The electrodes are mounted flush in nonconducting plates. For large Hartmann numbers the flow is concentrated in a free rotating layer situated close to the hypothetical cylindrical surface running through the edges of the disks. The width of the free rotating layer is proportional to $Ha^{-1/2}$. Hartmann layers with a thickness proportional to Ha^{-1} are formed at the end planes. The circumferential velocity in the median plane between the electrodes is equal to zero, the rotations on either side of this plane are oppositely directed and at each value of the radius the modulus of the velocity is a maximum at the boundary of the Hartmann layer.

3. We do not know of any theoretical investigations of the stability in an axial magnetic field of a free rotating layer bounded by Hartmann layers at the end surfaces. Some estimates are given in [10]. Certainly the investigation of this flow by the methods of stability theory entails considerable difficulties. In order to set up experimental work it is useful to know the dimensionless criteria governing the stability limit, because the design of the working section of the apparatus and the experimental procedure must provide for variations of these parameters between fairly wide limits. It is first of all necessary to establish the most dangerous type of instability. In Lehnert's experimental work [2], Kelvin–Helmholtz instability is determined by tangential velocity discontinuities. In Brahme's experiment [8] it is related to steps in the velocity profile. If the ratio of the width b of the rotating layer to the mean radius r_m of the layer is not too great and if the wall is situated at a certain distance from the rotating layer, then Kelvin–Helmholtz instability is more dangerous than Taylor instability, even without any influence of a magnetic field on the stability. Moreover, Taylor instability is more strongly suppressed by an axial magnetic field than Kelvin–Helmholtz instability. Thus, in the latter case the disturbed motion is close to planar in the plane perpendicular to the field direction, so that the influence of the magnetic field on it is small and is realized through the Hartmann layers at the insulating end planes. According to linear stability theory, Taylor vortices in a magnetic field are also elongated in the direction of the field, but their vorticity remains perpendicular to the induction vector, so that the stabilizing action of the magnetic field on Taylor instability turns out to be much stronger than in the case of jet-type instability. Consequently, Kelvin–Helmholtz instability is the most dangerous for the flow investigated here.

It is useful to determine the postulated form of the criterial relation corresponding to the critical flow regime. To do so we analyze the energy equation for the vortex that evolves after transition across the stability limit. Vortex energy is generated as a result of interaction of the Reynolds stresses $-\rho\overline{u'_\varphi u'_r}$ with the average velocity profile u(r). The energy generated per unit volume is $-\rho\overline{u'_\varphi u'_r}\,du/dr$, and in the entire vortex it is proportional to $\rho u'^2 u_m d^2 h/b$, where d is the diameter; h, length of the vortex; u_m, maximum velocity of the profile u(r); b, characteristic width of that profile; and u', characteristic value of the velocity in the vortex. The interaction of the circumferential velocity of the vortex with the axial magnetic field induces a radial emf. The corresponding radial current is closed through the axial current sheets and Hartmann layers. For large Hartmann numbers the thickness of the Hartmann layers is very small, and the main losses of mechanical energy are determined by the Joule and viscous dissipation in those layers. The total dissipation in the Hartmann layers of the investigated vortex is proportional to $\sigma B^2 u'^2 d^2 h/Ha_h$, where Ha_h is the Hartmann number evaluated with respect to the length h [10, 8]. For very large values of h/d viscous dissipation outside the Hartmann layers can become significant. The viscous dissipation per unit volume is proportional to $\rho\nu u'^2/d$, and for the entire vortex it is $\sim\rho\nu u'^2 h$. In the given semiempirical approach the vortex energy equation is written in the form

$$\rho h d^2 \frac{du'^2}{dt} = a_1 \rho h d^2 u'^2 \frac{u_m}{b} - a_2 h d^2 B^2 \sigma \frac{u'^2}{Ha_h} - a_3 \rho\nu h u'^2. \quad (1)$$

In the critical regime $du'/dt = 0$. We assume that the critical regimes for different values of the number Ha correspond to constant values of the ratios of coefficients a_2/a_1, a_3/a_1 and of the ratios d/b, u'/u_m. Then for the critical regime we obtain

$$Re_{cr} = Re_{cr,0} + \alpha_1 Ha_b \frac{b}{h}, \quad (2)$$

where $Re = u_m b/\nu$, $Ha_b = Bb\sqrt{\sigma/\mu}$, and $Re_{cr,0}$ is the critical Reynolds number for the jet in the absence of the magnetic field. The coefficient a_1 must be determined experimentally. Because of the large number of underlying assumptions, the given semiempirical estimate is only of methodological significance in the setting up of an experiment and the processing of its results. Inasmuch as $Re_{cr,0} \approx 10$, for Re > 200 we have

$$\left(\frac{Ha}{Re}\right)_{cr} = \text{const}\,\frac{h}{b}. \quad (3)$$

We remark once again that the estimates obtained above refer to large Hartmann numbers. Unlike free flows (such as a jet or mixing layer) in a longitudinal magnetic field, where in the case of large Hartmann numbers linear stability theory [11] yields for the stability limit the expression $(Ha/Re)_{cr}$ = const, which does not depend on the geometrical length, in the investigated flow the geometrical parameter h/b plays an important role. The given estimate refers to flow that is uniform in the axial direction. If the rotating layer and, together with it, the vortices generated as a result of stability loss are strongly nonuniform in the axial direction, then along with dissipation in the Hartmann layers it is also necessary to take into consideration dissipation along the entire length of the vortex and the fact that this dissipation grows concomitantly with the departure of the motion from planarity. Due to the increased dissipation, the critical value of the parameter Ha/Re in this case is smaller than in planar flow. The departure of the investigated flows from planarity can be characterized by the parameter du_φ/dz. The foregoing remark applies to flows for which $du_\varphi/dz < du_\varphi/dr$, since otherwise a much more complex form of stability loss than Kelvin–Helmholtz instability is realized. The refinements imparted to relation (2) by the consideration of Joule dissipation due to the axial current and by the possible influence of the current from an external source as it partially flows through the given vortex are not too advantageous. Considering the approximate character of the semiempirical method, we must determine the actual stability limit on the basis of experimental data obtained over a wide range of variation of the governing criteria Re, Ha, and h/b.

4. The most far-reaching possibilities for the experimental investigation of a free rotating layer are afforded by the procedure proposed by Lehnert in [2]. Brahme's work [8] is interesting as the first experimental realization of that procedure. A schematic diagram of the working section and the measured profile of the circumferential velocity close to the stability limit are taken from [8] and reproduced in Fig. 1. The following aspects of the cited work are important: 1) The small axial length of the rotating layer (h/b ≈ 0.1) prevents the display and investigation of an important property of MHD flows, namely the growth with Ha of the axial (along the field) nonuniformity of the average and disturbed motions; 2) steps in the circumferential velocity profile lower the stability and complicate the processing of the results; 3) the processing of the results is inconsistent with the Kelvin–Helmholtz instability produced in the experiment; 4) the problem of the influence of the

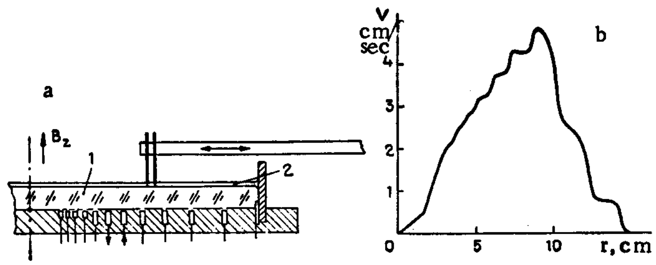

Fig. 1. Schematic view of the working section (a) and circumferential velocity profile (b) from [8]; 1) mercury; 2) nitric acid.

axial flow scale on the stability limit is ignored. Consequently, the published experimental results refer only to the particular device used in the experiment.

Making use of the results of [8] and the semiempirical estimate (2), we have developed a procedure for the experimental investigation and design of a working section of the apparatus [12]. It is taken into consideration here that, according to expression (3), with the presence of a velocity step Δu of width Δb the assurance of stability requires a field $B_{cr} \sim \Delta u/\Delta b$, i.e., one that is considerably stronger than for smooth profiles.

Smooth circumferential velocity profiles close to symmetrical jet profiles can be obtained by means of two annular electrodes. For the creation of strongly asymmetrical profiles, clearly, three electrodes are adequate. For the exclusion of abrupt velocity gradients the width of the electrodes must not be too small in comparison with the interelectrode spacing. If we place electrodes at both rather than at one of the end planes bounding the cylindrical vessel containing the conducting fluid, we greatly expand the experimental possibilities. When equal potentials are applied to the corresponding electrodes, rotation takes place at the end planes in the same direction and with identical profiles. The reduction of the circumferential velocity with increasing distance from the plane of the electrodes is mutually compensating in this case, and the axial uniformity of the flow is greatly enhanced. If the potential difference across corresponding electrode pairs is opposite in sign, another limiting case can be realized, namely a sharply nonuniform axial distribution of the circumferential velocity and opposite directions of rotation near the end planes, a situation that proves important for investigation of the dependence of the flow stability on the parameter du_φ/dz characterizing the departure of the flow from planarity. Intermediate regimes are also possible between these two extremes.

The axial length h of the working section must be much greater than the width b of the rotating layer. By conducting the experiment with a lower system of electrodes and allowing metal to drain from the working section it is a simple matter to vary h and to investigate the influence of the parameter h/b on the stability. It is also desirable to be able to vary the width b of the layer in order to analyze the direct influence of this dimension on the Kelvin–Holmnoltz stability and the influence of centrifugal forces as characterized by the parameter b/r_m. It is also interesting to determine for what distance Δ of the cylindrical sidewall from the boundary of the rotating layer it significantly affects the stability by inducing in the outer part of the layer the transition from Kelvin–Helmholtz to Taylor instability. It appears obvious that for $\Delta \approx d \approx b$ the sidewall will still not suppress the Kelvin–Holmholtz instability, whereupon the most important experiment is conducted in the range $\Delta/b = 0$ to 1.

With a given magnetic field and the abrupt application of voltage to the lower or upper system of electrodes, an uncoiling wave propagates from the electrode plane in the axial direction at a velocity that can be determined by means of a system of conduction anemometer probes placed along the magnetic field.

This system of probes is also necessary in order to investigate the structure of the disturbances in the subcritical regime. In a slightly subcritical regime it is necessary to investigate the axial nonuniformity of the disturbances, as is characteristic of Kelvin–Helmholtz instability. It is to be expected that with increasing subcriticality (it is practical to increase the Reynolds number while leaving the Hartmann number constant) the geometry of the vortex system will become unstable at first, then with increasing instability

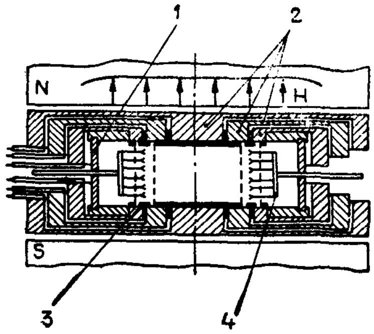

Fig. 2. Schematic view of the working section for investigation of the flow field, stability limit, and structure of the perturbations of a free rotating layer in an axial magnetic field; 1) sidewall of vessel containing liquid metal; 2) coaxial electrodes; 3) interchangeable Teflon ring; 4) finger array of seven conduction anemometer probes.

a motion similar to two-dimensional turbulence will set in, and eventually the motion will acquire a three-dimensional character. Thus, the investigation of the stability of the rotating layer comes close to the investigation of the plane turbulence problem.

The given experimental program is embodied in the design of the working section of the apparatus. The working section is designed for insertion in the horizontal gap of an electromagnet. It is shown schematically in Fig. 2.

The liquid-metal-filled vessel is in the shape of a cylinder with a radius of 37 cm, with a stainless steel sidewall 1. The bottom and cover of the vessel are formed by identical plane systems of current leads comprising three coaxial copper electrodes 2 with annular gaps of width 2 mm separating them. For operation with one pair of electrodes an interchangeable Teflon ring 3 is mounted in the gap between them. A Teflon or plastic ring and disk are attached to the electrodes in such a way as to leave a gap between them and the ring 3 for contact of the liquid metal with the electrodes. This configuration makes it possible to readily vary the width of the rotating layer and to work with two values of the median radius of that layer (35 and 70 mm). Figure 2 shows the insulating system corresponding to operation with the outer pair of electrodes. The dashed lines schematically indicate the boundaries of the rotating layer. Tubes with seven conduction anemometer probes 4 mounted on them are inserted into the cylinder through packing glands. The spacing between the upper and lower systems of leads is 70 mm, and the distance between probes is 8 mm. The design permits interchangeable plastic cylinders of different diameters to be placed inside the vessel for investigation of the influence of the distance from the wall to the boundary of the rotating layer on the structure of the disturbances and the stability limit. The measurement procedure is discussed in [13]. A brief description of the first series of experiments carried out with the use of the inner lower pair of electrodes is given in [14].

APPENDIX

We now examine in detail the relations used to derive Eqs. (1) and (2). We invoke the familiar approximation expressions for the distributions of the velocity and current density in a Hartmann layer for large numbers Ha, along with the expressions

$$u_\varphi(z)=u_{\varphi 0}[1-\exp(-B(\sigma/\mu)^{1/2}z)];\quad j(z)=-u_{\varphi 0}\sigma B\exp(-B(\sigma/\mu)^{1/2}z)$$

for viscous and Joule dissipation in the Hartmann layer, referred to unit surface of the insulation wall, whereupon we obtain

$$\int_0^\infty \mu\left(\frac{du}{dz}\right)^2 dz=\int_0^\infty \frac{j^2}{\sigma}\,dz=\frac{(\sigma\mu)^{1/2}Bu_0^2}{2}.$$

In the investigated vortex that evolves after loss of stability, the velocity at the outer boundary of the Hartmann layer is a function of the radial coordinate, so that the total dissipation in the Hartmann layer of this vortex is proportional to $(\sigma\mu)^{1/2}Bu'^2$, where u' is a characteristic value of the vortex velocity.

In the investigation of the generation of vortex energy we used the relation $\overline{u'_\varphi u'_r}\sim u'^2$, which is sufficiently accurate for large Reynolds numbers, and to estimate the gradient du/dr we used the natural assumption of self-similarity on the part of the velocity profiles. Under conditions such that the influence of longitudinal current sheets results in appreciable violation of self-similarity, we obtain the following condition for the critical regime at large values of the Hartmann number:

$$\frac{(\sigma\mu)^{1/2}B}{\rho h du/dy}=\text{const.}$$

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