EXPERIMENTAL INVESTIGATION OF A FREE ROTATING LAYER IN AN AXIAL MAGNETIC FIELD. I. STABLE CONDITIONS

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1. A free rotating layer in an axial magnetic field can be an important element in a number of technical applications of hydrodynamics and in astrophysics [1, 2].

In the classical experiment of Lehnert [3] a free rotating layer was created with the help of a copper ring rotating near the bottom of a vessel with mercury. Lehnert [1] suggested a method of creating a rotating layer with the help of a large number of coaxial electrodes towhich a controllable potential distribution was supplied. It was discovered in connection with the experimental realization of this method [4] that above each electrode there is a step in the circumferential velocity distribution which produces a Kelvin–Helmholtz instability. A more detailed literature survey has been conducted in [5], where it is also shown that for a free rotating layer in an axial magnetic field the Kelvin–Helmholtz instability is more dangerous than is the Taylor instability, not only in the presence of velocity steps but also in the case of smooth velocity distributions. However, smooth distributions are more stable; therefore their realization and investigation are essential for applications.

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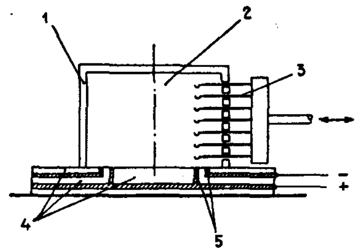

Fig. 1

Results which refer to the method of creating a free rotating layer with a smooth velocity distribution and to its characteristics in stable flow conditions are given in the first part of the experimental investigation published here. The results of an experimental investigation of the stability limit and supercritical flow conditions will be presented in the second part.

2. The experiments were performed with the apparatus described in [5]. Some changes have been introduced into the apparatus since then. An inner lower pair of electrodes and a Teflon ring 6 mm in width were used. The average radius of the current-supply system was equal to 35 mm, the width of the annular current-supply surfaces was 2 mm, and the total width of the current-supply system was b_0 = 10 mm (Fig. 1). The flow of a eutectic gallium–indium–tin alloy 2 inside a plastic cylinder with an inner diameter of 100 mm and height h = 70 mm was investigated. The upper current-supply system was insulated from the liquid metal by a plastic plate.

In order to provide a uniform current distribution around the circumference, massive copper electrodes 5 were welded to copper disks 3 mm thick on whose edge massive annular copper spines with 18 contacts uniformly spaced around the circumference for current supply from storage batteries. Special nonconductive inserts 4 were placed between the electrodes and the disks. The current, whose maximum value did not exceed 5 A, was controlled by a rheostat. The surface of the electrodes was amalgamated prior to filling the working part in order to provide for extended reliable contact of the electrodes with the eutectic.

The working part of the apparatus was mounted in the horizontal gap of an electromagnet which was supplied from a constant-current generator. The height of the gap was 150 mm and its width was 500 mm, which provided for a uniform magnetic field in the working volume whose maximum induction was equal to 1.4 T.

Interaction of the field **B** with the current **j** flowing in the liquid created a volume electromagnetic force $\mathbf{j} \times \mathbf{B}$ which resulted in motion of the liquid.

The azimuthal and radial components of the flow velocity were measured in the experiments. A conducting anemometer with a three-electrode detector was used for this purpose. The distance between the detector electrodes was 1.8 mm, and the electrodes themselves were made out of copper wire in polychlovinyl insulation 0.2 mm in diameter. There were seven detectors 3 in all (Fig. 1). They were arranged in the axial direction at a distance of 8 mm from each other and were attached in one holder. The first detector was located at the same distance from the plane of the electrodes. The detectors entered the holes on the lateral surface of the working cylinder and could be shifted along its diameter. Shifting of the holder with the detector rack was accomplished by a flange equipped with a coordinate device. This system permitted measuring the velocity distribution of the flow being investigated both radially and with respect to height.

Besides the detectors of the conduction anemometer seven more pairs of electrodes measuring the potential difference $\Delta\varphi_\Sigma$ between the wall and axis of the cylinder were mounted in the working volume of the apparatus in the same cross section along the cylinder axis and on its wall. These electrodes were also made out of copper wire 0.2 mm in diameter.

The application of the conduction method of measurement requires its own justification in specific cases, since it is necessary to take account of the effect of the induced currents, whose distribution is usually unknown. In our case it is necessary to take account also of the external currents creating motion in addition to the induced ones. Therefore we will estimate the effect of currents on measurements of velocity by the conduction method.

One can assume from the geometrical conditions of the problem that the flow will be of the jet type. In addition, for this conductivity and low viscosity of the working liquid the Hartmann effect should form a sufficiently uniform velocity distribution along the field even for moderate values of the magnetic field. Therefore we will neglect the dependence of the velocity on the coordinate along the field in making the estimate.

For simplicity we will discuss a plane-parallel flow $\mathbf{u} = \{0, u, 0\}$ in a uniform external field $\mathbf{B} = \{0, 0, B\}$.

In the first approximation we will assume the velocity distribution to be parabolic

$$u(x)=u_0+c(x-x_0)^2,$$

where

$$u_0=u(x_0)=u_m,\ c=0{,}5(\mathrm{d}^2u/\mathrm{d}x^2)\big|_{x=x_0}.$$

We find from the condition $u(x_1) = u(x_2) = 0$ that $x_0 = (x_2 - x_1)/2$ and $c = -4u_0/b^2$, where x_1 and x_2 are the boundaries of the layer, and $b = x_2 - x_1$ is the width of the layer.

We have from the Navier–Stokes equation in the form

$$(j_0+j_i)B=-\mu\frac{\mathrm{d}^2u}{\mathrm{d}x^2},$$

where j_0 is the current density produced by the external emf and j_i is the density of the induced Hartmann currents, that

$$u_0=\frac{(j_0+j_i)Bb^2}{8\mu}.$$

Let us set up the relationship

$$\frac{\sigma u(x)B}{j_0+j_i}=\frac{\mathrm{Ha}^2}{8}\left[1+4\left(\frac{x-x_0}{b}\right)^2\right], \tag{1}$$

where $\mathrm{Ha} = Bb\sqrt{\sigma/\mu}$ is the Hartmann number based on the flow width. If the quantity $\sigma uB/(j_0 + j_i) >> 1$, one can calculate the velocity from the formula

$$u=\frac{\mathrm{grad}_x(\varphi_0+\varphi_i)}{B}, \tag{2}$$

which follows from Ohm's law

$$\mathbf{j}_0+\mathbf{j}_i=\sigma(\mathbf{E}_0+\mathbf{E}_i+\mathbf{u}\times\mathbf{B}).$$

If one sets $\sigma uB/(j_0 + j_i) = 100$, then by solving Eq. (1) we find the dimensions of that region $x_1' \leqslant x \leqslant x_2'$ in which one can neglect the effect of external and induced currents with an accuracy of 1% and calculate the velocity from Eq. (2)

$$\frac{\Delta x}{b}=\frac{x'_2-x'_1}{b}=\sqrt{1-\frac{800}{\mathrm{Ha}^2}}.$$

Thus, with $\Delta x/b = 0.99$ the value of the Hartmann number turns out to be equal to 140.

One can estimate the value of the induction B corresponding to this value of Ha for our case. Let us set the number Ha_0 based on the distance between electrodes b_0 equal to 140. Since for finite Ha the true width of the layer b is always larger than b_0, the number Ha for $\mathrm{Ha}_0 = 140$ based on the true width of the layer will be larger than this value, and the ratio $\Delta x/b$ will certainly be larger than 0.99. Consequently, for the given distance between electrodes $b_0 = 10$ mm we obtain

$$B=\frac{140}{b_0}\sqrt{\frac{\mu}{\sigma}}\approx 0{,}47\ \mathrm{T}.$$

Our measurements of the width of the velocity distribution have given values of Ha = 140 for $B \simeq 0.3$ T. Thus, when $B \geqslant 0.3$ T one can neglect currents in the liquid and determine the velocity from Eq. (2) in practically the entire flow region. Measurements of the potential difference $\Delta\varphi_\Sigma$ noted above have permitted determining the average velocity v_{av} in the experiments. Using Eq. (2), we obtain:

$$v_{\mathrm{av}}=\frac{1}{b}\int_{x_1}^{x_2}u\,\mathrm{d}x=\frac{\Delta\varphi_\Sigma}{Bb}. \tag{3}$$

The extent of the effect on the flow of a rack of seven detectors creating resistance to the liquid flow was also estimated in our work on measurements of $\Delta\varphi_\Sigma$. The measurements showed that when the detectors are placed in the middle of the flow $\Delta\varphi_\Sigma$ decreases by about 5%, and when they are placed on the inner boundary of the layer, it decreases by approximately 10%.

We note with respect to secondary flows that the estimate of the velocity of radial flows made in [2] as well as our measurements have shown that it is smaller by at least an order of magnitude than the velocity of the main flow.

3. The distributions of azimuthal velocity measured radially for different values of the magnetic field and normalized to the maximum velocity are shown in Fig. 2. As is evident from the figure, the main flow is concentrated in the region between the electrodes, and free shear flow is created with a pronounced velocity maximum for field values B > 0.2T.

As has already been noted, a magnetic field acting on the flow tends to decrease the gradients along its direction. The results of measurements of the distributions of the maximum azimuthal velocity in the direction of the magnetic field are presented in Fig. 3. It is evident that as the field increased the uniformity of the flow with respect to height increases appreciably. Even at a comparatively small field B = 0.34 T the ratio u_{m7}/u_{m1} amounts to 0.7. For B = 1.39 T it is equal to 0.93.

The width of the shear layer was determined in tests from the distributions of the velocity u/u_m. The distance between the two points of the distribution at which the velocity amounts to 5% of the maximum value was taken as the width b of the shear layer.

Investigations of the velocity structure have shown that variation of the supplied current with B = const has no effect on the shape of the distribution, while an increase in the field induction results in a decrease in its width, whose limiting value is equal to the distance between electrodes. The dependence of the width of the layer averaged over height on the field is shown in Fig. 4a. Since the shape of the distribution for B = const was not altered with a change of the current supplied, the width of the shear layer also did not depend on the current supplied. The emf induced in the core of the flow in the region above the electrodes should lead entirely to a decrease in the nonuniformity of the potential distribution in this region as the field increases, and consequently to a decrease in the shear layer with respect to z. The measurements have shown (Fig. 5) that if with B = 0.16 T the ratio b_1/b_7 is 0.6, then with B = 1.39 T it is equal to 0.92.

The dependence of the potential difference $\Delta\varphi_\Sigma$ measured in the fourth (middle) cross section on the quantity IB is presented in Fig. 4b. It is evident from the figure that this dependence is linear over a wide range of field and current values with a proportionality coefficient a equal to 140 μV/A·T.

The results of measurements of the potential difference $\Delta\varphi_\Sigma$ together with the results of measurements of the width of the shear layer (Figs. 4a, b) have permitted determining the average discharge velocity in the experiments. Since $\Delta\varphi_\Sigma$ is proportional to IB, it follows from Eq. (3) that the product $v_{av}b$ is equal to aI and does not depend on the field. Experiment has shown that for B $\geqslant$ 0.5 T independence of the field is observed as well as a similar dependence on current for the product $u_m b$. As should be expected, the ratio of the maximum velocity to the average discharge velocity remains constant as the field increases, when the flow acquires a pronounced two-dimensional nature. For a field B $\geqslant$ 0.5 T this ratio is approximately 1.4 (Fig. 6). We note that it is equal to 1.5 for a parabolic distribution.

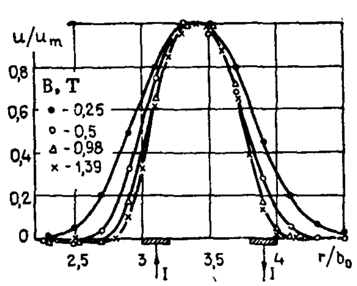

Fig. 2

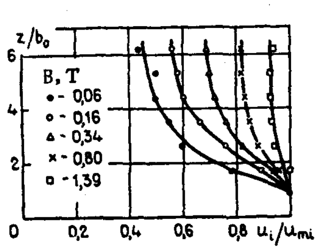

Fig. 3

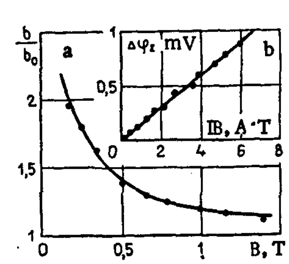

Fig. 4

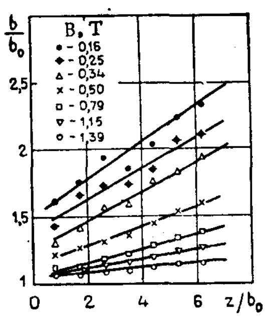

Fig. 5

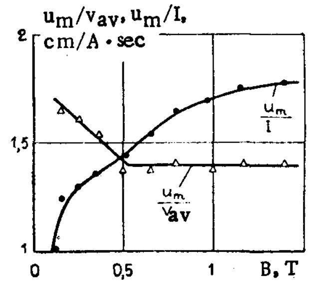

Fig. 6

It has also been obtained that the maximum velocity u_m varies proportionally to the current supplied I, and for sufficiently large magnetic fields $B \gtrsim 1$ T, for which the width of the layer is changed little, the ratio u_m/I evidently becomes independent of the field (Fig. 6).

The fact that as the field increases Hartmann layers with a thickness of the order of Ha^{-1} are formed on the end walls and free boundary layers with thickness of the order of $Ha^{-1/2}$ are formed on the outer and inner flow boundaries is a characteristic peculiarity of the flow being discussed [6, 7]. A potential core is contained between these layers where the action of viscosity forces is small and the total current $j_0 + j_i$ becomes equal to zero. Proceeding on the basis of this fact, we will make approximate calculations of the velocity.

Let us distinguish within the flow a cylindrical surface with radius R equal to the radius on which the velocity distribution u(r) has a maximum. The total current flowing through this surface with area $2\pi Rh$ is equal to the total value of the current supplied to the electrodes I.

The currents in the Hartmann layers are determined by the potential difference induced in the core of the flow. Therefore, the current density is proportional to the quantity $\sigma u_m B$ at the radius $r = R$ in the Hartmann layer with a thickness $b_H \sim 1/B\sqrt{\sigma/\mu}$, and the total current in this layer is equal to $I_H = \alpha \cdot 2\pi R u_m \sqrt{\sigma/\mu}$, where the proportionality coefficient α is of the order of unity. Taking account of the fact that $h \gg I_H$, the total current in the core of the flow is equal to $I - 2I_H$, and the current density is

$$j_0 + j_i = \frac{I - 2I_H}{2\pi Rh}.$$

Since a potential core is formed in the flow as the Hartmann number increases, where $(j_0 + j_i) \to 0$, we obtain the limiting expression $u_{m0} \approx I/4\pi R_0\sqrt{\sigma\mu}$, where R_0 is the position of the velocity maximum as $Ha \to \infty$. The velocity distribution in the core becomes proportional to $1/r$ and takes the form $u = u_{m0}R_0/r$.

If one sets the radius of the inner electrode equal to r_1, then to an accuracy out to a correction for finiteness of the electrode width the limiting value of the average velocity has the form

$$v_{av.0} = \frac{1}{b_0}\int_{r_1}^{r_1+b_0} u_{m0}\frac{r_1}{r}\,dr = \frac{r_1 u_{m0}}{b_0}\ln\frac{r_1+b_0}{r_1},$$

where $R_0 = r_1$ has been adopted in the limit.

Knowing the limiting value of the average velocity, one can estimate the proportionality coefficient a mentioned in the description of Fig. 4b. Actually, since the quantity $v_{av}b$ does not depend on the field, it is equal to its own limiting value $v_{av0}b_0$, and then

$$a = \frac{1}{4\pi\sqrt{\sigma\mu}}\ln\frac{r_1+b_0}{r_1}.$$

As the estimates have shown, the value of a lies within the limits of 120-200 μV/A·T, depending on the choice of the value of r_1 — out to the outer or inner boundary of the annular electrodes. The average value of this quantity determined from the two extremes is equal to 160 μV/A·T, which is larger by 8% than the value obtained experimentally.

A peculiarity of the flow under discussion is the fact that the average velocity and width of the flow for I = const are related by an inversely proportional dependence. An increase in the average velocity occurs upon a decrease in the discharge cross section. Therefore the amount of liquid flowing through a cross section per unit time remains unchanged and is completely determined by the expression

$$Q=\rho v_{\text{av}}bh=\rho v_{\text{av},0}\, b_0h=\frac{\rho hI}{4\pi\sqrt{\sigma\mu}}\ln\frac{r_1+b_0}{r_1}.$$

In conclusion we note that the analytic solution and numerical calculation of the model problem have shown good agreement with the experimental data given here.

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