EDDY LOSSES IN A DIRECT-CURRENT CONDUCTION-TYPE PUMP WITH FINITE SECTIONALIZATION

G. M. Gekht and I. M. Tolmach

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The present article proposes a method for calculation of the distribution of the current density and the eddy losses in a two-channel single-phase pump with a finite spacing of the sectionalization of the electrodes. Along the channels, connected in series electrically, and connected to the secondary winding of a step-down transformer, there flows a conducting liquid, in opposite directions (Fig. 1). Neglecting the conductance of the scattering of the secondary winding of the transformer and the magnetizing action of the eddy currents, and assuming (thanks to the arrangement of the channels one above the other) that the armature reaction is compensated, we can consider separately the problems of the distribution of the eddy currents and the currents due to the flow of the conducting liquid [1].

In [1] it is shown that, in an single-channel conduction-type machine with ideally sectionalized electrodes, the eddy currents are partially short-circuited through the secondary winding of the transformer, bringing about additional Joule losses. With the presence of two channels, arranged one above the other and connected in series electrically by a set of sections of the electrodes, closing of the eddy currents through the secondary winding is impossible, since the latter will be completely closed through the channel. This offers the possibility, in calculation of the eddy currents, of considering only one channel, whose walls consist of infinitely conducting electrodes and insulating gaps; opposite electrodes can be regarded as open. Since in real pumps the effective depths of the penetration of the field is usually considerably greater than the height of the channel Δ, we can consider the plane-parallel problem.

The amplitude of the alternating-sign magnetic field is assumed constant along the channel; the field has the form $B_z = -B_0 e^{i\omega t}$. In this case, with a large number of electrodes, the picture of the distribution of the eddy current will be repeated along the channel (with the exception of the end zones), which makes it possible to consider only one element of the period BB'D'D (Fig. 2).

The same figure, in another element, shows a qualitative picture of the lines of the eddy current, constructed from the experimentally found quantitative distribution of the current density at each point of the element. The solution of similar periodic problems, except for constant-current machines, was obtained in [2, 3].

Introducing the dimensionless quantities

$$x=\tilde{x}/\tilde{h};\quad y=\tilde{y}/\tilde{h};\quad \lambda=\tilde{\lambda}/\tilde{h};\quad \delta=\tilde{\delta}/\tilde{h};$$

$$j_x=\frac{\tilde{j}_x}{\sigma\omega\tilde{h}B_0};\quad j_y=\frac{\tilde{j}_y}{\sigma\omega\tilde{h}B_0};\quad \Phi=\frac{\tilde{\Phi}}{\omega\tilde{h}^2B_0};\quad A=\frac{\tilde{A}}{\tilde{h}B_0},$$

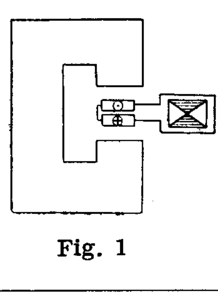

Fig. 1

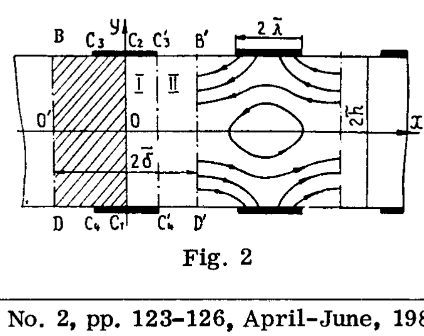

Fig. 2

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we obtain, analogously to [1], equations describing the distribution of the dimensionless density of the eddy current in the channel:

$$j_x = -\partial\Phi/\partial x - A_x; \quad j_y = -\partial\Phi/\partial y - A_y; \quad \Delta\Phi = 0; \tag{1), (2}$$

here A_x and A_y are the components of the vector potential. All the fields are assumed to be sinusoidal, and in what follows, the factor $e^{i\omega t}$ is everywhere omitted.

Let us consider the boundary conditions. At the sections of the boundary BD, B'D', and C_2C_1, from the symmetry and periodicity of the problem it follows that:

$$j_y = 0 \quad \begin{matrix} \text{with} & x=0, & |y| \leqslant 1; \\ \text{with} & x=\pm\delta, & |y| \leqslant 1. \end{matrix} \tag{3}$$

At infinitely conducting electrodes $E_x = 0$, i.e.

$$j_x = 0 \quad \text{with} \quad |x| \leqslant \lambda, \quad y = \pm 1. \tag{4}$$

At the insulated gaps

$$j_y = 0 \quad \text{with} \quad \lambda \leqslant |x| \leqslant \delta, \quad y = \pm 1. \tag{5}$$

The solution of Eq. (2) with the given boundary conditions (3)-(5) can be obtained either using the Keldysh–Sedov formula [4] or by an iteration method, analogous to the Schwarz method.

Calculation of the distribution of the current density using a solution obtained by the first of the above methods leads to an increase in the volume of calculations, in view of which, in what follows, for numerical calculations the second method for solution of the problem posed was used.

In this case within the limits of an element it is convenient to represent the vector potential in the form A(0, A(x), 0), where A(x) = −x. In view of the symmetry of the problem $\Phi(x, y) = -\Phi(-x, y)$, it is sufficient to consider only half of the element $x \geq 0$.

From (3)-(5) the following boundary conditions can be written for $\Phi(x, y)$:

$$\Phi = 0 \quad \text{with} \quad x=0, \quad |y| \leqslant 1; \tag{6}$$

$$\Phi = \delta y \quad \text{with} \quad x=\delta, \quad |y| \leqslant 1. \tag{7}$$

At ideally conducting electrodes

$$\Phi = 0 \quad \text{with} \quad x \leqslant \lambda, \quad y = \pm 1. \tag{8}$$

At insulated gaps

$$\frac{\partial\Phi}{\partial y} = x \quad \text{with} \quad \lambda < x < \delta. \tag{9}$$

For solution of Eq. (2) with the boundary conditions (6)-(9), we use an iteration method analogous to the Schwarz method [5]. For this purpose, a half-element of the periods $C_2B'D'C_1$ is divided into two regions $I(C_2C_3'C_4'C_1)$ and $II(C_3'B'D'C_4')$. We represent the solution of Eq. (2) in the form of series, satisfying the boundary conditions (6) and (8) in the electrode zone (region I)

$$\Phi_I(x, y) = \sum_{n=1}^{\infty} C_n \frac{\operatorname{sh} \beta_n x}{\operatorname{sh} \beta_n \lambda} \sin \beta_n y, \tag{10}$$

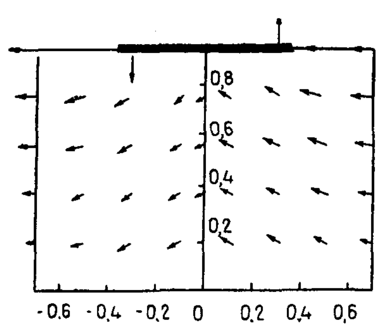

Fig. 3

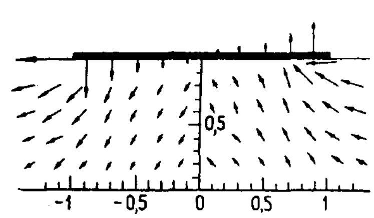

Fig. 4

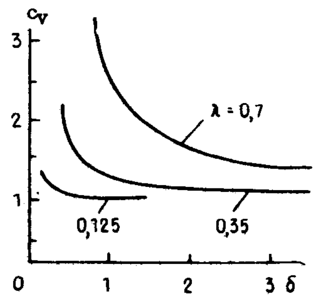

Fig. 5

and, in the region of the insulators (region II), satisfying the boundary conditions (7) and (9)

$$\Phi_{II}(x, y)=\sum_{n=1}^{\infty} A_n \frac{\operatorname{sh} \alpha_n(\delta-x)}{\operatorname{sh} \alpha_n(\delta-\lambda)} \sin \alpha_n y+x y. \tag{11}$$

Here Φ_I and Φ_{II} are the values of the potential $\Phi(x, y)$ in regions I and II; $\beta_n = \pi n$; $\alpha_n = (2n-1)\pi/2$.

The unknown coefficients A_n and C_n are found by the method of successive approximations, using the continuity of the potential and its normal derivative at the boundary of regions I and II, using the relationships

$$\frac{\partial \Phi^{(i+1)}{}_{I}}{\partial x}=\frac{\partial \Phi^{(i)}{}_{II}}{\partial x}, \quad \hat{\Phi}^{(i+1)}{}_{II}=\Phi^{(i+1)}{}_{I}, \quad \Phi^{(i+1)}{}_{II}=(1-t) \hat{\Phi}^{(i+1)}{}_{II}+t \Phi^{(i)}{}_{II}. \tag{12}$$

The parameter t $(0 < t < 1)$ assures the convergence of the iteration process. The iteration process is continued until the conditions $\left|\frac{A^{(i)}{}_n-A^{(i+1)}{}_n}{A^{(i)}{}_n}\right|<\varepsilon$ and $\left|\frac{C^{(i)}{}_n-C^{(i+1)}{}_n}{C^{(i)}{}_n}\right|<\varepsilon$ are satisfied. In the calculations it was assumed that $\varepsilon = 10^{-3}$. The calculation of one variant in an M-222 digital computer took 10-20 min.

Knowing the distribution of the potential (10) and (11), using (1) it is possible to obtain the distribution of the density of the eddy current. The results of these calculations are given in Fig. 3 with $\delta = 0.7$ and $\lambda = 0.35$, and in Fig. 4 with $\delta = 1.5$, $\lambda = 1$ (scale; 1 mm corresponds to 0.0333 units of current density). As a result of the periodicity and symmetry of the problem (j_x is an odd function with respect to y; j_x is an even function with respect to y), Figs. 3 and 4 show the distribution of the current density only in the upper half of the element. From Figs. 3 and 4 it can be seen that part of the eddy current flows through the electrode; the density of the current rises sharply near the ends of the electrodes, and part is closed in the central zone of the element, forming a current eddy.

The relative eddy losses in one element are determined from the overall relationship

$$Q=\frac{1}{8\lambda} \int_{-1}^{1} \int_{-\delta}^{\delta} j^2(x, y)\, dx dy, \tag{13}$$

obtained by dividing the dimensional expression by the value of $\sigma\omega^2 B_0^2 4\tilde{h}^3 \tilde{\lambda}\, \tilde{\Delta}$. Taking account of (1) from (13) an identical expression can be obtained for the given problem

$$Q=-\frac{1}{8} \int_{-1}^{1} \int_{-\delta}^{\delta} j_y(x, y) \cdot A(x)\, dx dy-\frac{1}{4\lambda} \int_{-1}^{1} \Phi(\delta, y) \cdot j_x(\delta, y)\, dy. \tag{14}$$

Substituting into (14), in place of j_y and Φ, their expressions from (1), (10), and (11) and integrating, we obtain:

$$Q=Q_{id}\left[1+\frac{\lambda^3}{\delta}+\frac{3}{\delta} \sum_{n=1}^{\infty} \frac{(-1)^n A_n}{\alpha_n{}^2}(1+\lambda \gamma_n)\right]. \tag{15}$$

Here $\gamma_n = \alpha_n \operatorname{cth} \alpha_n(\delta-\lambda)$; $Q_{id} = \delta/6\lambda$ are the dimensionless eddy losses in one element of the channel with a linear distribution of the density of the eddy current $j_x = -y$; $j_y = 0$ [2].

Figure 5 gives the dependence of the correction coefficient $c_v = Q/Q_{id}$ on the dimensionless length of an element δ with different values of λ. Consequently, with a search for the eddy losses in an element of the channel with finite sectionalization, the values of the losses Q_{id} must be multiplied by the correction coefficient c_v.

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