ANALOGY BETWEEN TURBULENT TRANSFER PROCESSES IN MHD-STREAM AND IN STRATIFIED STREAM

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1. One of the timely problems in hydrodynamics nowadays is the study of anisotropy of transfer processes in turbulent streams, a determination of the causes of which should make it possible to explain the common characteristics of such seemingly different kinds of flow as that of electrically conducting media in a magnetic field, of a rotating fluid, of a stratified stream, of a fluid with polymer additives, etc. An analysis reveals that in such kinds of flow the anisotropy of turbulent transfer of hydrodynamic quantities is caused by the existence of external body forces acting on the stream. The specific geometry of this anisotropy is, moreover, determined by the direction in which these forces act.

In this study we will compare data on MHD shear flow and data on stratified ground flow. As the principal physical characteristics for this study will serve eddy friction and generation of turbulence energy, which, on the one hand, well describe the flow and, on the other hand, are not connected with specific properties of the fluids in motion.

It is well known that both kinds of flow must be describable by the complete system of Navier—Stokes equations

$$\mathbf{u}_t+(\mathbf{u}\nabla)\mathbf{u}=-\operatorname{grad} p+\nu\Delta\mathbf{u}+\mathbf{F}; \tag{1}$$

$$\operatorname{div}\mathbf{u}=0;\quad \rho_t=D\Delta\rho-c\frac{\partial\rho}{\partial z}, \qquad (2),\ (3)$$

where **u** is the velocity; ρ, density; p, pressure, ν, kinematic viscosity; and **F**, external force. The force **F** in these equations will have different physical meanings in MHD flow and in flow of a stratified stream. In the former case the force is produced by the external magnetic field and in the latter case it is produced by the earth's gravitational field.

2. In the first part of the experiment the object was free turbulent shear flow of a conducting liquid in a uniform magnetic field. Flow was produced by means of an electromagnetic force resulting from interaction of the magnetic field and the radial component of external electric current, the latter spreading over the liquid between two ring electrodes located at the bottom of a cylindrical vessel. As the active medium served the eutectic In—Ga—Sn alloy.

In the experiment the longitudinal and transverse (radial) average and fluctuational components of velocity were measured. These measurements were made with a conduction-type anemometer using a four-electrode probe. The experimental apparatus and procedure have been described in another report [1]. The main measurements were made in h/2-sections, h denoting the stream dimension parallel to the field. Here we will report the results obtained at a magnetic induction of 1.4 T and a maximum flow velocity of 35 cm/sec.

Measurements of the average flow characteristics have revealed that in a sufficiently strong magnetic field there forms a jet which is two-dimensional in a plane perpendicular to the field and has a velocity profile as shown in Fig. 1a. Measurements of longitudinal and transverse velocity fluctuations have established that the coefficient of two-point correlation in the direction of the field is high (Fig. 1b, where z is the coordinate in the direction of the field and b_0 is the distance bwtween the electrodes) and in the other two directions is low, which indicates a high degree of anisotropy of the energy-carrying large-scale turbulent perturbations. This result agrees with the results of previous structural studies of plane-parallel jet flow [2].

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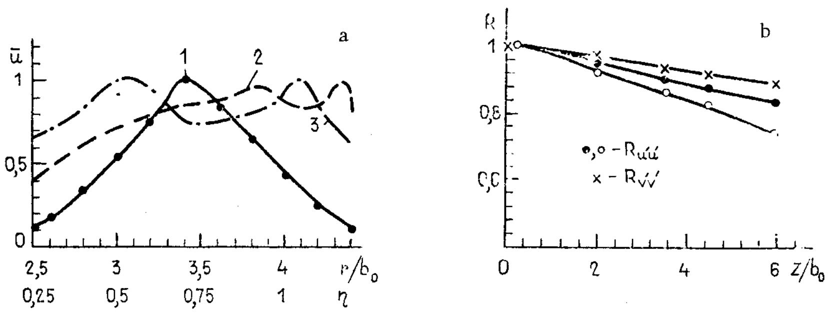

Fig. 1. (a) Velocity profiles: 1) MHD stream, 2) stratified stream, 3) [4]; (b) correlation coefficients for longitudinal and transverse velocity fluctuations: r/b_0 = 3.7 (center curve), 3.1 (lower curve), 3.5 (upper curve).

The presence of highly anisotropic field-oriented turbulent perturbations in a two-dimensional shear flow creates the condition for reverse energy transfer from fluctuating to average motion [2, 3]. Direct measurements of the one-point cross-correlation $\overline{u'v'}$ have revealed that the distribution of eddy friction τ over the stream cross section is negative with respect to its distribution in the case of isotropic turbulence, when eddy perturbations draw energy from the average stream. Both curve 1 in Fig. 3, which describes generation of kinetic turbulence energy $\Pi = -\rho\overline{u'v'}(\partial u/\partial r - u/r)$ and the space correlations, indicate that energy is transferred from large-scale perturbations to the average stream and back to perturbations within the region with velocity gradients. The influx of energy to these perturbations comes, evidently, from medium-scale perturbations, which are still sufficiently anisotropic, as well as from turbulence energy generated at velocity gradients and near the walls perpendicular to the field.

3. In the second part of the study the object is a density ground stream flowing along an inclined bottom driven by a component of the gravity force, this part of the study involving numerical experiments. The flow is assumed to be nearly plane-parallel and isothermal so that the relations

$$u=\bar{u}+u', \quad v=v', \quad \rho=\rho_0+\bar{\delta}+\delta' \tag{4}$$

apply, where u and v are the velocity components along the x axis parallel to the inclined plane and the z axis perpendicular to the inclined plane, respectively, ρ_0 is the density of the "light" fluid, and $\delta = \rho - \rho_0$ is the "excess" density directly proportional to the impurity concentration in the ground (bottom) layer.

Starting from the general system of equations of thermohydrodynamics, with incompressibility of the fluid taken into account but with the derivatives with respect to the x-coordinate disregarded, we obtain the dimensionless system of equations

$$N_{Sh}\frac{\partial\bar{\sigma}}{\partial\tau}=\frac{S}{N_{Re}}\frac{\partial^2\bar{\sigma}}{\partial\eta^2}-N_B\frac{\partial\bar{\sigma}}{\partial\eta}-El\frac{\partial\bar{u}}{\partial\eta}\frac{\partial}{\partial\eta}l\frac{\partial\bar{\sigma}}{\partial\eta}, \tag{5}$$

$$N_{Sh}(1+\bar{\sigma})\frac{\partial\bar{u}}{\partial\tau}=\frac{1+b\bar{\sigma}}{N_{Re}}\frac{\partial^2\bar{u}}{\partial\eta^2}+\frac{\partial\bar{\sigma}}{\partial\eta}\frac{\partial\bar{u}}{\partial\eta}\left(\frac{b}{N_{Re}}-El^2\frac{\partial\bar{u}}{\partial\eta}\right)+\frac{\bar{\sigma}}{N_{Fr_1}}, \tag{6}$$

where $N_{Sh} = h_0/t_0u_0$, $N_{Re} = u_0h_0\rho_0/\mu_0$, $N_{Fr_1} = u_0^2/gh_0i$, $N_S = D\rho_0/\mu_0$, $N_B = \gamma gDj/u_0$, $b = a\rho_0/\mu_0$, $\eta = z/h_0$, $u=u/u_0$, $\sigma=\delta/\rho_0$, h_0 is the initial thickness of the ground layer, u_0 is the characteristic velocity, t_0 is the characteristic time, μ_0 is the dynamic viscosity of the "light" fluid", D is the diffusion coefficient, γ is a parameter representing the effect of the gravitational field on diffusion of the impurity [4], g is the acceleration of gravity, i = sin α, j = cos α, and α is the inclination angle of the bottom to the horizontal. The viscosity of the fluid is, furthermore, assumed to depend linearly on the density: $\mu=\mu_0+a\delta$.

We note that the equations of the model have been derived from the complete Navier–Stokes equations, without any assumptions about the nature of eddy friction and about the generation of turbulence energy during development of a density ground stream.

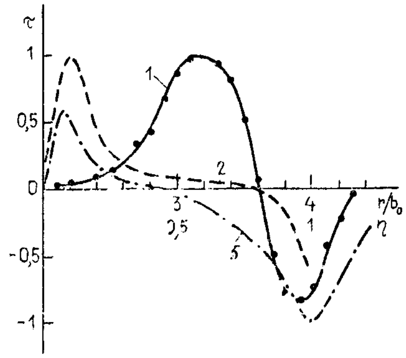

Fig. 2

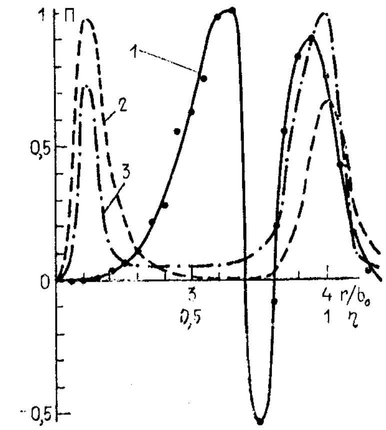

Fig. 3

Fig. 2. Distribution of eddy friction over stream cross section: 1) MHD stream $(\tau=-\rho\overline{u'v'})$; 2) stratified stream $(\tau=\varrho l^2|\partial u/\partial\eta|\partial u/\partial\eta)$; 3) [4].

Fig. 3. Distribution of generated turbulence energy over stream cross section: 1) MHD stream $[\Pi=\rho\overline{u'v'}(\partial u/\partial r-u/r)]$; 2) stratified stream $(\Pi=\varrho\overline{u'v'}\partial u/\partial\eta)$; 3) [4].

On the basis of experimental data [4], we impose on the fluctuation components the constraints

$$u'=l\frac{\partial\bar{u}}{\partial\eta},\quad \sigma'=El\frac{\partial\bar{\sigma}}{\partial\eta},\quad E=\text{const}\sim 1, \tag{7}$$

where $l(\eta,\tau)=L_1(\eta)\,L_2(\tau)$. The function $L_1(\eta)$ becomes zero at the bottom and has a minimum within the zone of density inflection, while $L_2(\tau)=1-\exp(-\lambda\tau)$, which makes it possible to describe the process of turbulence stabilization in time.

The initial and boundary conditions for system (5)-(6) are stipulated as

$$u|_{\tau=0}=u|_{\eta=0}=u|_{\eta\to\infty}=0; \tag{8}$$

$$\bar{\sigma}|_{\tau=0}=\begin{cases}\sigma_0, & 0\leqslant\eta\leqslant 1,\\ 0, & \eta>1,\end{cases}\qquad \bar{\sigma}|_{\eta=0}=\sigma_0;$$

$$\bar{\sigma}|_{\eta\to\infty}=0.$$

This model was simulated numerically according to ideas and methods shown in reference [5].

The numerical solution of the system of equations describing the behavior of a density ground stream has yielded its vertical density and velocity profiles as well as their evolution in time. The graph in Fig. 1a indicates that the flow of a stratified ground stream is a shear flow. The graphs in Figs. 2 and 3 depict typical vertical profiles of eddy friction and of generation of turbulence energy at various instants of time. In the analysis of eddy friction profiles it is worth noting that both kinds of flows are characterized by zones of positive and negative maxima, somewhat staggered along the vertical in a ground stream and directly passing from one to another in an MHD stream, which can apparently be explained by the existence of a sufficiently large region with small density and velocity gradients inside the stream in the first case and by the jet mode of flow in the second case. The existence of such zones has also been recorded under natural conditions [4, 6].

The profiles of generation of turbulence energy in a density ground stream indicate a transfer of energy from fluctuating to average motion, which ensures a long duration of such streams according to numerical as well as physical experiments [4, 6].

This comparison of the results of numerical and laboratory experiments described here, as well as of the results of measurements under natural conditions [4], indicates, therefore, that the fluctuation components of the various hydrodynamic quantities concentrate in a plane which is determined by the vectors of average velocity and external force. The observed anisotropy of transfer processes is the principal condition for existence of the given kinds of flow and is responsible for their basic characteristics.

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