FLOW OF AN ELECTRICALLY CONDUCTING LIQUID THROUGH A PERIODIC PIPE SYSTEM

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The subject of this investigation is the steady flow of a viscous, electrically conducting, incompressible liquid through the intertubular infinitely long space between a periodic system of nonconducting pipes (round or rectangular) while subjected to the action of perpendicular steady magnetic field $\mathbf{B}_0(0, 0, B_0)$. The system may be triangular, where the geometric pipe centers are located on the vertices of equilateral triangles or rectangular (quadratic), where the pipe centers lie on the vertices of the rectangles (squares). This problem is of interest in conjunction with various applications [1, 2]. A solution of the analogous problem but without the magnetic field has been presented in [3].

For the given magnetic field configuration, the flow field may consist of developed rectilinear flow [4], i.e., the streamlines of the liquid are parallel to the pipe axes, $\mathbf{u}(u(y, z), 0, 0)$. Based on the introduced assumptions, the resulting magnetic field has the form $\mathbf{B}(B(y, z), 0, B_0)$. In view of the periodicity of the problem, it is only necessary to examine the flow in a single control volume.

After the introduction of dimensionless parameters $\tilde{u}=u[k/\sigma B_0^2]^{-1}$; $\tilde{b}=b[\mu Lk/B_0]^{-1}$; $\tilde{j}=j[k/B_0]^{-1}$; $\tilde{z}=z/L$; $\tilde{y}=y/L$, the system of dimensionless equations describing the flow in this case (with curls being omitted henceforth) is:

$$1+\partial b/\partial z+\varepsilon\Delta u=0; \quad \Delta b+\partial u/\partial z=0; \qquad (1),\ (2)$$

$$j_y=\partial b/\partial z, \quad j_z=-\partial b/\partial y, \qquad (3)$$

where $k = -dp/dx$, $\varepsilon = \mathrm{Ha}^{-2}$, L is a characteristic pipe dimension and Ha is the Hartmann number.

The no-slip condition and $j_n = 0$ on the surface of the nonconducting pipe lead to the boundary conditions for velocity and the magnetic field:

$$u=b=0. \qquad (4)$$

If the system of round pipes considered is such that their radius is significantly larger than the spacing between pipes, it is possible to considerthe flow being in a "densely packed" system (Fig. 1). For Ha ≫ 1 (with the pipe radius being taken as the characteristic linear dimension) the presence of a small parameter in the higher derivative makes it possible to use the boundary layer methods in the solution of the system of equations (1) and (2) subject to boundary conditions (4).

For the case where the region is symmetrical with respect to the y axis (quadratic system) the velocity distribution, as derived in [5], is:

$$u(y, z)=\frac{1}{\sqrt{\varepsilon}}g(y)\left[1-\exp\left(-\frac{z-g(y)}{\sqrt{\varepsilon}}\cos^2\gamma\right)\right], \qquad (5)$$

where g(y) is the equation of the control volume boundary and $\mathrm{tg}\,\gamma = g'(y)$.

For the case where the region is not symmetrical with respect to the y axis (triangular system) the derived velocity distribution has the form:

$$u(y, z)=\frac{f(y)-f_1(y)}{2\sqrt{\varepsilon}}\left[1-\exp\left(\frac{z-f(y)}{\sqrt{\varepsilon}}\cos^2\theta\right)-\exp\left(-\frac{z-f_1(y)}{\sqrt{\varepsilon}}\cos^2\theta_1\right)\right], \qquad (6)$$

where f(y) and $f_1(y)$ are the equations of the upper and lower control volume boundaries and where $\mathrm{tg}\,\theta = f'(y)$ and $\mathrm{tg}\,\theta_1 = f_1'(y)$.

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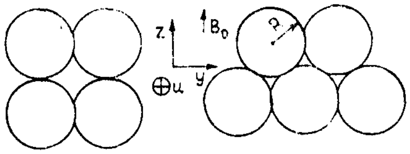

Fig. 1

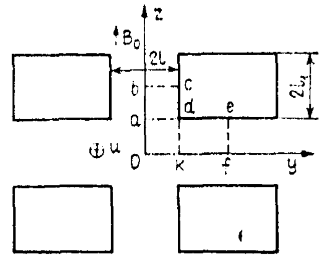

Fig. 2

When the parameter $\varepsilon^{-1/2}$ in Eqs. (5) and (6) is large and potitive, it is possible in the evaluation of $\langle u\rangle=(1/S)\iint u(y,z)\,dy\,dz$, where S is the flow-through surface area, to apply the Laplace method for the double integrals [6]. We then obtain the following expressions for the hydraulic friction coefficient in the control volume, $\lambda=k[\rho\langle u\rangle^2/2R]^{-1}$, for the case of the quadratic and triangular system, respectively:

$$\lambda_1=\frac{2\,\mathrm{Ha}}{0.446\,\mathrm{Re}}[1+O(\mathrm{Ha}^{-1/2})], \tag{7}$$

$$\lambda_2=\frac{2\,\mathrm{Ha}}{0.17\,\mathrm{Re}}[1+O(\mathrm{Ha}^{-1/2})]. \tag{8}$$

We next examine the rectangular system of rectangular nonconducting pipes arranged with a uniform spacing between pipes (as the characteristic linear dimension, we take the pipe half-width de, see Fig. 2).

Due to the periodicity and symmetry of the problem, it is sufficient to consider only the flow in the region Obcdef. The following boundary conditions will apply for the magnetic induction and velocity over the boundary section cbOfe:

$$j_z=\partial u/\partial n=0. \tag{9}$$

Within the region Obcdef the solution of the system of equations (1) and (2) subject to the boundary conditions (4) and (9) can be obtained by the Schwartz [7] method; consequently, both in the region Oaef (region I) and Obck (region II) the solutions can be represented in terms of trigonometric expansions.

We seek a solution in region I in the form:

$$u_1(y,z)=\sum_{n=0}^{\infty}u_{1n}(z)\cos\beta_n y;\quad b_1(y,z)=\sum_{n=0}^{\infty}b_{1n}(z)\cos\beta_n y, \tag{10}$$

where $\beta_n=\pi n/(1+l)$ and $2l$ is the dimensionless distance between pipes.

The functions $u_m(z)$ and $b_m(z)$ are determined from a solution of a system of ordinary differential equations which is obtained by substituting (10) into the system of equations (1) and (2) with the values of $u_1(y, a)$ and $b_1(y, a)$ over the boundary section ad being taken from a solution of the boundary-value problem for the region II.

We seek the solution in region II in the form:

$$u_2(y,z)=\sum_{n=0}^{\infty}u_{2n}(y)\cos\lambda_n z;\quad b_2(y,z)=\sum_{n=1}^{\infty}b_{2n}(y)\sin\lambda_n z, \tag{11}$$

where $\lambda_n=\pi n/(l+l_1)$ and $2l_1$ is the dimensionless pipe height.

Analogously to the procedure employed in the solution for region I, we find $u_{2n}(y)$ and $b_{2n}(y)$ from a system of ordinary differential equations which are obtained by substituting Eq. (11) into Eqs. (1) and (2), while the boundary values $u_2(a, z)$ and $b_2(a, z)$ on the boundary section kd are obtained from the solution of the problem for region I. Once the distribution of the magnetic induction is known, it becomes possible with the aid of Eq. (3) to find the distribution of the current density. The stated solution algorithm was performed

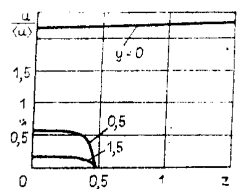

Fig. 3

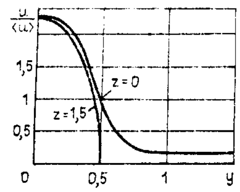

Fig. 4

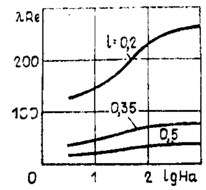

Fig. 5

with the EC-1022 electronic computer. Depending on the number of terms in the series, the computation of one case required between 8 and 15 min.

The velocity distribution, nondimensionalized by the average through-flow velocity <u>, is shown for vertical control volume cross sections in Fig. 3 and for horizontal cross sections in Fig. 4 for Ha = 50, l = 0.5, and l_1 = 1.

The presence of the magnetic field leads to an essential rearrangement in the hydrodynamic flow on account of the Hartmann effect between pipe boundaries that are perpendicular to the magnetic field. In fact, even for Ha = 50 the velocity distribution in the cross section y = 1.5 is close to the Hartmann profile (Fig. 3). The flow rearrangement results in an increase in flow across the cross section Obck and to retardation of liquid in the region kdef. This effect becomes more accentuated with increasing Hartmann number.

Shown in Fig. 5 is the dependence of the product λRe, where $\lambda = k[\rho\langle u\rangle^2/2L]^{-1}$ is the flow resistance coefficient and Re = <u>L/ν, on the Hartmann number for l_1 = 1 and various values of l.

The outcome of liquid retardation in the region kdef is that at large Hartmann numbers (Ha > 10^3) the flow regime approximates Poiseuille flow through an opening formed by planes which touch pipe boundaries and are parallel to the magnetic field.

In fact, if the obtained friction coefficient and Re are recomputed from flow parameters in a gap defined by planes passing pipe boundaries that are parallel to the magnetic field for the condition of conservation of flow rate, it is found that already for Ha ~ 10^3 for arbitrary l values λRe ~ 6, i.e., a relationship is obtained which practically coincides with the known formula for Poiseuille flow between two parallel walls.

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