MOTION OF A VISCOUS, ELECTRICALLY CONDUCTING LIQUID THROUGH DISTRIBUTING AND CONVERGING COLLECTORS IN A TRANSVERSE MAGNETIC FIELD

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1. References 1 and 2 have examined the motion of an electrically conducting liquid in the magnetic field through a collector system which included distributing and converging collectors and a flow section (active zone), wherein the motion through the collectors proceeded through a transverse field, while in the active zone the field was oriented longitudinally.

In the computational model [1, 2] the flow parameters were averaged across the collector height (in the field direction).

In the present work the parameters have not been averaged over the height; this made it possible to examine the MHD effects which are dependent on the expansion of the collector in the field direction and to estimate the accuracy of the averaging across the height and of certain other assumptions in [2].

2. Within the noninductive approximation, we examine the plane-parallel laminar flow of a viscous electrically conducting fluid through a collector in a transverse magnetic field (Fig. 1). After introduction of dimensionless parameters, the system of equations for the stream function ψ and the vorticity ζ can be written in the following form:

$$\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}=\frac{1}{\mathrm{Re}}\Delta\zeta-\mathrm{N}\frac{\partial^2\psi}{\partial x^2}, \quad \Delta\psi=\zeta; \tag{1a}$$

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}, \quad \zeta=\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}. \tag{1b}$$

where $\mathrm{Re}=v_0 a\rho/\mu$, and $\mathrm{N}=\sigma aB^2/\rho v_0$. The collector width a and the standard flow velocity v_0 at its entrance were selected as the basic reference values.

The no-slip condition was imposed on the impermeable walls, while on the penetrable walls (entry or exit from the active zone) it was assumed that the normal velocity had the given magnitude and that the tangential component was zero. The given velocity profile may be imposed by the placement of regulating devices, such as for example, throttle plates at the entry (and exit) of the active zone. Calculations were carried out both for the case of a uniform and a parabolic velocity profile at the entry (exit) of the distributing (converging) collectors and for uniform, or linear exit (entry). Boundary conditions of the Dirichlet type were determined for the stream function ψ for the normal velocity component. For the vorticity ζ Tomma [3] conditions accurate to the first order were set. For the inpenetrable wall

$$\zeta=\frac{2(\psi_{\omega+1}-\psi_\omega)}{\Delta n^2}, \tag{2}$$

where Δn is the distance on the normal from the nodal point next to the wall $\omega + 1$ to its projection on the wall ω.

Analogous conditions were imposed at the entry and exit of the distributing collector (Fig. 1a):

$$\zeta_\omega=\frac{2(\psi_{\omega+1}-\psi_\omega)}{\Delta y^2}-\frac{\partial v_\omega}{\partial x}, \quad \zeta_\omega=\frac{2(\psi_{\omega+1}-\psi_\omega)}{\Delta x^2}+\frac{\partial u_\omega}{\partial y}. \tag{3}$$

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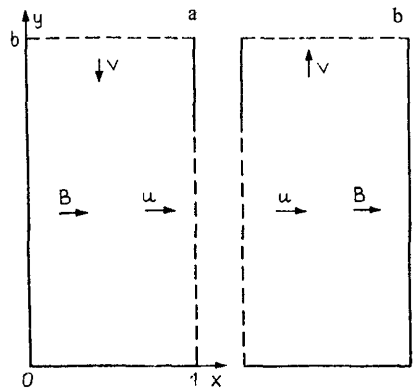

Fig. 1. Flow schematic for the electrically conducting liquid through collectors in a transverse magnetic field: a) distributing collector; b) converging collector.

3. The problem has solved by a numerical method using a rectangular uniform finite-difference grid. The sought stationary solution was obtained as the limit for $t \to \infty$ solution of the nonstationary equations [3]. In order to obtain reliable results in the solution of the vortex transport equations (1a) several computational methods were used [3]: 1) an implicit scheme of the alternating direction method; 2) a scheme with differences opposite to the flow; and 3) the "leap-frog" scheme.

3.1. The implicit scheme of the alternating direction method is based on the splitting up of the time step; it requires only the inversion of the three-diagonal matrix. It is absolutely stable and with the applied modification [3] is accurate to the order $O(\Delta t^2, \Delta x^2, \Delta y^2)$. The finite-difference analog for the vortex transport at the point (i, j) has the form:

$$\frac{\zeta^{n+1/2}-\zeta^n}{\Delta t/2} = -u\frac{\delta\zeta^{n+1/2}}{\delta x} - v\frac{\delta\zeta^n}{\delta y} + \frac{1}{\mathrm{Re}}\frac{\delta^2\zeta^{n+1/2}}{\delta x^2} + \frac{1}{\mathrm{Re}}\frac{\delta^2\zeta^n}{\delta y^2} - \mathrm{N}\frac{\delta^2\psi^{n+1/2}}{\delta x^2};$$

$$\frac{\zeta^{n+1}-\zeta^{n+1/2}}{\Delta t/2} = -u\frac{\delta\zeta^{n+1/2}}{\delta x} - v\frac{\delta\zeta^{n+1}}{\delta y} + \frac{1}{\mathrm{Re}}\frac{\delta^2\zeta^{n+1/2}}{\delta x^2} + \frac{1}{\mathrm{Re}}\frac{\delta^2\zeta^{n+1}}{\delta y^2} - \mathrm{N}\frac{\delta^2\psi^{n+1/2}}{\delta x^2}. \quad (4)$$

In this equation the values for u, v, and ψ are taken from the $(n + {}^1/_2)$ step and are determined by the linear extrapolation from the known values u, v, ψ at the (n − 1) and n time steps. The boundary values for the (n + 1) step which are required for the evaluation of ζ^{n+1} in the interior of the region are also determined by linear extrapolation over time from known boundary values ζ in the preceding two steps.

In Eqs. (4) and further on $\delta\zeta/\delta x$ and $\delta^2\zeta/\delta x^2$ are specified by approximations with central differences for $\partial\zeta/\partial x$ and $\partial^2\zeta/\partial x^2$ at the point (i, j).

3.2. The second scheme with differences opposite to the flow is a single-step explicit two-layer time scheme which provides statistical stability for convective terms with a first order in accuracy; it is conservative and transportable.

The finite-difference analog for the vortex transport equations at the (i, j) point in the n-th time step has the following form:

$$\frac{\zeta_{i,j}^{n+1}-\zeta_{i,j}^{n}}{\Delta t} = -\frac{u_r\zeta_{jr}-u_l\zeta_{jl}}{\Delta x} - \frac{v_r\zeta_{ir}-v_l\zeta_{il}}{\Delta y} + \frac{1}{\mathrm{Re}}\left(\frac{\delta^2\zeta^n}{\delta x^2}+\frac{\delta^2\zeta^n}{\delta y^2}\right) - \mathrm{N}\frac{\delta^2\psi^n}{\delta x^2}, \quad (5)$$

where

$$u_r = {}^1/_2(u_{i+1,j}^n + u_{i,j}^n), \quad u_l = {}^1/_2(u_{i,j}^n + u_{i-1,j}^n),$$

$$v_r = {}^1/_2(v_{i,j+1}^n + v_{i,j}^n), \quad v_l = {}^1/_2(v_{i,j}^n + v_{i,j-1}^n),$$

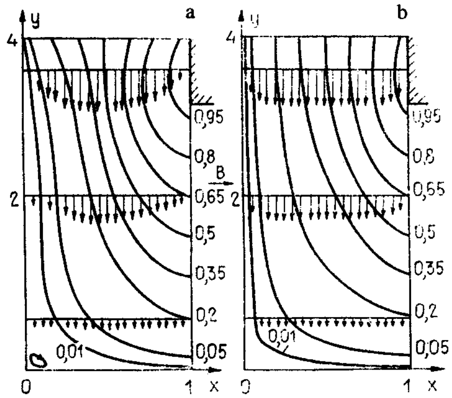

Fig. 2

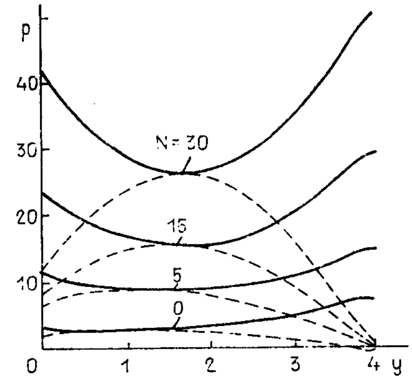

Fig. 3

Fig. 2. Streamlines for the liquid and distributions of the y-component of velocity for the flow with uniform velocity profiles at the entry and the exit of the distributing collector: a) Re = 10, N = 0; b) Re = 10, N = 50.

Fig. 3. Distributions of the dimensionless pressure $p = (\bar{p} - \bar{p}_{ex})/\rho v_0^2$, along the collector on the line x = 0.5, where $\bar{p}_{ex}$ is the pressure at the exit from the collector system. Continuous lines refer to the distributing collector, dashed lines to the converging. Re = 10.

$$\zeta_{jr} = \begin{cases} \zeta^n_{i,j}, & u_r > 0 \\ \zeta^n_{i+1,j}, & u_r < 0 \end{cases}, \quad \zeta_{jl} = \begin{cases} \zeta^n_{i-1,j}, & u_l > 0 \\ \zeta^n_{i,j}, & u_l < 0 \end{cases}$$

$$\zeta_{ir} = \begin{cases} \zeta^n_{i,j}, & v_r > 0 \\ \zeta_{i,j+1}, & v_r < 0 \end{cases}, \quad \zeta_{il} = \begin{cases} \zeta^n_{i,j-1}, & v_l > 0 \\ \zeta^n_{i,j}, & v_l < 0 \end{cases}.$$

3.3. The "leap-frogging" scheme is explicit and absolutely stable, to the second order of accuracy [3]. The finite-difference analog of the vorticity transport, solved relative to $\zeta^{n+1}_{i,j}$, has the form:

$$\zeta^{n+1}_{i,j} = \left[1 + \frac{2\Delta t}{\mathrm{Re}}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2}\right)\right]^{-1}\left[\zeta^{n-1}_{ij} - \frac{\Delta t}{\Delta x}(u^n_{i+1,j}\zeta^n_{i+1,j} - u^n_{i-1,j}\zeta^n_{i-1,j}) - \right. \tag{6}$$

$$\left. - \frac{\Delta t}{\Delta y}(v^n_{i,j+1}\zeta^n_{i,j+1} - v^n_{i,j-1}\zeta^n_{i,j-1}) + \frac{2\Delta t}{\mathrm{Re}\,\Delta x^2}(\zeta^n_{i+1,j} - \zeta^{n-1}_{i,j} + \zeta^n_{i-1,j}) + \frac{2\Delta t}{\mathrm{Re}\,\Delta y^2}(\zeta^n_{i,j+1} + \zeta^n_{i,j-1} - \zeta^{n-1}_{i,j}) - 2\Delta t \mathrm{N}\frac{\delta^2\psi^n}{\delta x^2}\right].$$

In the solution of the Poisson equation for the stream function ψ use was made of the method of the successive upper relaxation [3].

4. The implementation of the numerical methods was accomplished on an electronic computer ES-1033 with finite-difference grids 21 × 41 and 41 × 81, using time steps of Δt = 0.01-0.001 over a range of parameters $10 \leqslant \mathrm{Re} \leqslant 100$ and $0 \leqslant \mathrm{N} \leqslant 50$. The Re and N values actually used were: Re = 10, N = 0, 5, 15, 30, 50; Re = 100, N = 0, 4. The following relationships were used as the criteria for stability with time:

$$\max_{\Omega}\left|\frac{\psi^{n+1}_{i,j} - \psi^n_{i,j}}{\Delta t}\right| < \varepsilon, \quad \max_{\Omega}\left|\frac{\zeta^{n+1}_{i,j} - \zeta^n_{i,j}}{\Delta t}\right| < \varepsilon,$$

where $\varepsilon = 10^{-2}$-10^{-3}, and Ω is the grid region.

The condition for iteration convergence in the solution of the Poisson equation was formulated as $\max_{\Omega}|\psi^{k+1}_{i,j} - \psi^k_{i,j}| < 10^{-5}$. The initial conditions satisfied the boundary conditions for the normal velocity component and the continuity equation. The results of the computations are shown in Figs. 2-5.

Figure 2 shows a comparison of streamlines and distributions of the y-component of velocity for N = 0 and N = 50 for uniform velocity distributions at the entrance and the exit of the distributive collector having the same geometry, as that used in [4]. Using a quad-

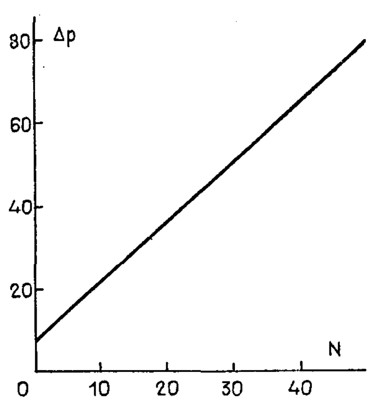

Fig. 4

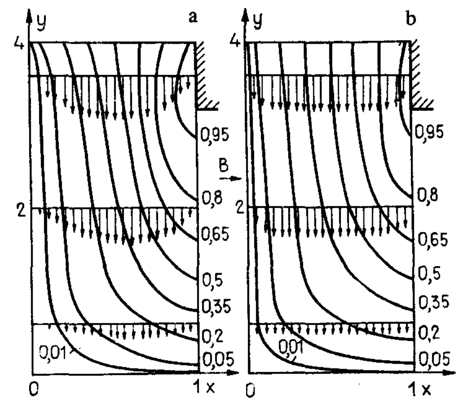

Fig. 5

Fig. 4. Pressure drop in distributing collector as a function of the Stewart number N for Re = 10.

Fig. 5. Liquid streamlines and distributions of the y-component of the velocity for a flow with a uniform velocity profile at the entry and a linear profile at the exit of a distributing collector ($u_0 = -y/8 + 0.51$): a) Re = 10, N = 0; b) Re = 10, N = 50.

ratic parabola equation for the entry has practically no effect on the flow picture. It follows from Fig. 2 that the magnetic field leads to an equalization in the velocity distribution over the collector height and to a suppression of vorticity in its lower left corner.

It is thus evident that the averaging procedure over the collector height, as performed in [1, 2] is justified. The velocity field for the converging collector is, according to calculations, qualitatively the same as for the expanding one, with small quantitative differences.

The pressure distribution was determined by numerical integration of the Navier–Stokes equations containing $\partial p/\partial x$ and $\partial p/\partial y$:

$$\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\Delta u-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial y}; \tag{7}$$

$$\frac{\partial p}{\partial y}=\frac{1}{\mathrm{Re}}\Delta v-u\frac{\partial v}{\partial x}-v\frac{\partial v}{\partial y}+\mathrm{N}(E_z-v). \tag{8}$$

with $E_z=\frac{1}{b}\int_0^1\int_0^b v\,dy\,dx.$

The condition (8) arises from the requirement that the total current flowing into side walls equal zero, i.e., if one neglects the electrodynamic coupling of the collectors.

Pressure distributions along the collectors are shown in Fig. 3. The presence of pressure extremes are evidently related to the presence of closed currents produced by the nonuniformity of the y-component of the velocity along the collector. In this case, the initial section of the distributing collector can operate, for example, in the breaking regime, while its terminal section is working in the pump regime.

The difference in the ordinates of the curves describing the pressure distribution in the distributing and converging collectors for their assumed configuration in Fig. 3 (the presence of the point of tangency) yields a minimum value of pressure loss in the throttling grid, providing a uniform inflow of the electrically conducting liquid into the active zone.

The difference in the ordinates for the points of entry and exit to and from the collector system gives minimal values of pressure drop in that system (beyond the residual pressure loss in the active zone). It follows from Fig. 3 that the vortex currents result in a substantial increase in the pressure drop within the collector system, yet at the same time, due to their closed state, they should not exert a marked influence on the total pressure losses in each collector. The dependence of the pressure drop in the collector system on the Stewart number N, as shown in Fig. 4, is nearly linear. The pressure distribution across the collector height is nearly uniform.

The use of various computational schemes for the equations of vorticity transport (1a) yielded variations in the stream function that were less than 5-6% and slightly larger variations for the velocity. Among the methods used, the implicit scheme of the alternating direction proved to be the most economical (the machine time requirement being from 5 to 10 times less than in the explicit schemes).

Shown in Fig. 5 are streamlines and profiles of the y-component of velocity in a distributing collector for the case when the profile at the entrance to the active zone is nonuniform (the velocity is distributed according to linear law).* In this case, the increase in the field also results in an equalization of the velocity profile across the channel height. Thus, it is seen that for fixed velocity profiles on permeable collector boundaries, a strengthening of the magnetic field results in the equalization of the velocity profile over the entire collector volume.

Conclusions. 1. Computations were carried out for the velocity field and the pressure distribution for laminar flow of an incompressible, viscous electrically conducting liquid within collectors exposed to a magnetic field.

2. For a fixed velocity profile on the permeable collector boundaries an increase in the Stewart number brings about an equalization of the velocity field over the collector height.

3. The pressure distributions along the collector displays an extremum which results in an increase in pressure losses within the collector system.

LITERATURE CITED

1. A. I. Élkin, "Flow of an electrically conducting liquid through a distributing collector in a magnetic field," Magn. Gidrodin., No. 2, 117-122 (1978).
2. D. L. Broder, M. M. Nagornyi, and A. I. Élkin, "Motion of an electrically conducting liquid through a collector system having an electrodynamic coupling between the entry and exit collectors," Magn. Gidrodin., No. 1, 54-60 (1981).
3. P. J. Roache, Computational Fluid Dynamics, Hermosa (1976).
4. G. I. Zhemchuzhnikov, A. M. Benevolenskii, and A. I. Topilskii, "Model of a hybrid blanket for liquid lithium cooling for the T-20 device," in: Synthesis Division. Proceedings of the Second Soviet–American Seminar, Atomizdat, Moscow (1978), pp. 164-169.

*This statement seems to contradict the title for Fig. 5 — Translator.