EFFECT OF THE NOZZLE EXPANSION RATIO FOR AN IONIZED GAS ON THE SPECIFIC POWER OF AN MHD GENERATOR

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The object of the present note is to give a clear and simplified picture of the relationship between the rate of energy conversion in a magnetic field and the expansion ratio of a nozzle (supplying an ionized gas) with the minimum number of determining parameters.

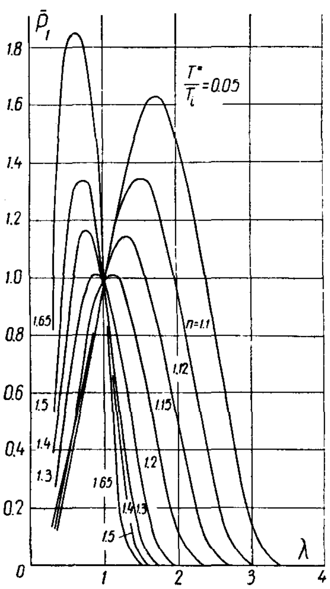

Fig. 1

For this purpose we consider the velocity dependence of the following quantities at the channel inlet:

$$P_1=\frac{1}{4}\sigma v^2B^2 \text{ (volume specific power)}; \qquad (1)$$

$$P_2=\frac{1}{4}\sigma v^2B^2F \text{ (linear specific power)}. \qquad (2)$$

Here σ is the electrical conductivity of the ionized gas; v is the gas velocity; B is the induction; and F is the cross-sectional area of the channel. The conductivity σ of an equilibrium ionized gas is a known function of temperature, pressure, and the type of gas [1]:

$$\sigma=0.532\frac{e^2}{Q\sqrt{m_ekT}}\,G\,\frac{T^{5/4}}{\sqrt{p}}\,e^{\frac{-V_i}{2kT}}. \qquad (3)$$

Here e and m_e are the electronic charge and mass, k is Boltzmann's constant, V_i is the ionization potential, p and T are the gas pressure and temperature, Q is the cross section for collisions of electrons with neutral particles, which is assumed to be constant (interaction with ions is neglected), and G is a constant depending on the statistical weights of the positive ion and the atom.

Upon expansion of the gas in the nozzle, the velocity increases but the temperature and electrical conductivity decrease; it is therefore natural to expect a maximum of P_1 and P_2 at a certain value of the velocity.

As the scale values of P_1 and P_2 at a gas velocity equal to the critical velocity

$$a_*=\sqrt{\frac{2n}{n+1}gRT_0}, \qquad (4)$$

where n is the adiabatic exponent of expansion of the gas in the nozzle, R is the gas constant, and T_0 is the stagnation temperature, we take

$$P^*_1=\frac{1}{4}\sigma(p^*,T^*)a_*^2B^2; \qquad (5)$$

$$P^*_2=\frac{1}{4}\sigma(p^*,T^*)a_*^2B^2F. \qquad (6)$$

Then, introducing the coefficient of velocity

$$\lambda=v/a_*, \qquad (7)$$

we obtain the nondimensional quantities

$$\overline{P}_1=\frac{1}{P^*_1}\cdot\frac{1}{4}\sigma[p(\lambda),T(\lambda)]v^2B^2=\frac{\sigma}{\sigma^*}\lambda^2; \qquad (8)$$

$$\overline{P}_2=\frac{1}{P^*_2}\cdot\frac{1}{4}\sigma[p(\lambda),T(\lambda)]v^2B^2F=\frac{\sigma}{\sigma^*}\lambda^2\varphi;$$

$$\sigma^*=\sigma(p^*,T^*). \qquad (9)$$

Here the expansion ratio of the nozzle is

$$\varphi=\frac{F}{F^*}=\left[\frac{n+1}{2}\,\frac{\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}}{\left(1-\frac{n-1}{n+1}\lambda^2\right)^{\frac{2}{n-1}}\frac{n-1}{n+1}\lambda^2}\right]^{1/2}; \qquad (10)$$

$$p^*/p_0 = [2/(n+1)]^{\frac{n}{n-1}}; \quad T^*/T_0 = 2/(n+1). \quad (11)$$

From the conventional theory of one-dimensional isentropic expansion of a gas in a nozzle we obtain

$$p/p^* = \left[\frac{n+1}{2}\left(1+\frac{n-1}{n+1}\lambda^2\right)\right]^{\frac{n}{n-1}}; \quad (12)$$

$$T/T^* = \frac{n+1}{2}\left(1-\frac{n-1}{n+1}\lambda^2\right). \quad (13)$$

The nondimensional expression for the conductivity is

$$\sigma/\sigma^* = (p^*/p)^{1/2}(T/T^*)^{3/4}\cdot e^{-\frac{T_i}{2T^*}\left(\frac{T^*}{T}-1\right)}. \quad (14)$$

Here $T_i = V_i/k$ is the temperature corresponding to the ionization energy. Equations (14), (13), (12), (9), and (8) show that the nondimensional specific powers depend only on the coefficient of velocity λ, the adiabatic exponent n, and the ratio of critical temperature to ionization temperature T^*/T_i.

In cases of practical importance $T^* = 2500-3500°$ K, while for a potassium seeding agent $V_i = 4.34$ eV. For potassium, $T_i = 50\,000°$ K; therefore if we adopt the characteristic value $T^* = 2500°$ K, the ratio $T^*/T_i = 0.01$.

The adiabatic exponent changes from 1.65 for monatomic perfect gases to 1.10 for the multiatomic dissociated products of combustion of hydrocarbon fuels in oxygen. In the latter case the quantity n can be assumed to be constant along the length of the nozzle only as a very rough approximation, but this is permissible in our approximate analysis. Consequently, n = 1.65–1.10, and this covers the entire range of gases likely to be used. The coefficient of velocity may vary in the range

$$\lambda = 0 \div \left(\frac{n+1}{n-1}\right)^{1/2}. \quad (15)$$

The upper limit of λ corresponds to the limiting exit velocity into a vacuum, when the gas temperature becomes zero and all the particles acquire directed motion.

Thus, the adopted limits of n and λ cover the entire range of parameters.

The results of calculations are given as graphs in Figs. 1 and 2 (for $T^*/T_i = 0.05$). These curves can be used to select the optimum velocity λ and also to ascertain the difference between the specific power and the maximum where it is not possible to select an optimum value of λ.

We also calculated similar curves for $T^*/T_i = 0.07$, which are not reproduced here. Their shape is the same.

Since in the long channels actually in use the velocity, temperature and conductivity change, the selection of λ from the curves can only be used for rough estimates.

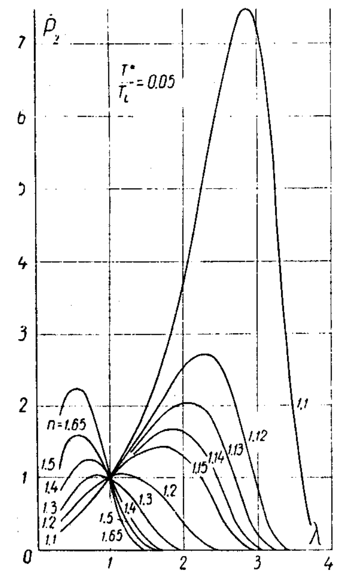

Fig. 2

The comparison of nondimensional specific powers P taken for various values of n is only possible if one takes into account the fact that the scales, i. e., the specific powers at critical pressure and temperature, depend on n.

The curves show that for expansion of multiatomic gases (e. g., products of combustion in oxygen), when n = 1.10–1.14 at supersonic velocities, i. e., at a high rate of expansion of the gas in the nozzle, the specific power is much greater than in the case of subsonic velocities. The difference is particularly great for the power per unit channel length, $\bar{P}_2$, i. e., at supersonic velocities the length of the channels is found to be much less.

At n = 1.25–1.65, the maximum specific power corresponds to subsonic flow, i. e., to a relatively low rate of expansion.

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