LETTERS TO THE EDITORS

MAGNETOHYDRODYNAMIC FLOWS OF VISCOPLASTIC FLUIDS

L. K. Martinson and K. B. Pavlov

Magnitnaya Gidrodinamika, Vol. 2, No. 3, p. 152, 1966

UDC 538.4

Certain hydrodynamic and magnetohydrodynamic flows of viscoplastic media are characterized by the formation of zones of quasi-solid motion in which the structure of the viscoplastic medium is preserved [1,2]. In this case the flows separate into several independent regions: zones of viscous motions and zones of quasi-solid motions. The boundaries of these zones are determined from the condition of equality of the tangential shear stress τ and the limiting shear stress τ_0 characterizing the viscoplastic medium.

The flows in the viscous zones are described by differential equations that sometimes coincide in form with the differential equations of motion of a viscous Newtonian fluid. In considering the zones of quasi-solid motions it is necessary to use integral conditions, as in [1, 2]. Attempts to describe the flows in the quasi-solid zones without using integral conditions lead to incorrect results, as happened in [3]. Thus, for example, the dimension of the zone of quasi-solid motion of a viscoplastic medium z_0 in a plane MHD channel is given [2] by

$$(P+\sigma E_0 B_0) z_0/\tau_0 = \operatorname{ch} M(1-z_0/L),$$

whereas in [3] the corresponding erroneous relation [Eq.(19)] is

$$(P+\sigma E_0 B_0)\frac{\operatorname{sh} M z_0/L}{\tau_0} = \frac{M}{L}\operatorname{ch} M(1-z_0/L).$$

It is easy to confirm that in this case the conditions of stationary motion of the quasi-solid zone are not satisfied, which, in the last analysis, renders certain conclusions of [3] invalid.

REFERENCES

1. M. P. Volarovich and A. M. Gutkin, ZhTF, **16**, 3, 321, 1946.

2. L. K. Martinson and K. B. Pavlov, Magnitnaya gidrodinamika [Magnetohydrodynamics], 4, 61, 1965.

3. T. Carpkaya, A. I. Ch. Eng. Journ., **7**, 2, 324, 1961.

23 March 1966

VARIOUS DEFINITIONS OF THE MAGNETIC REYNOLDS NUMBER

A. I. Vol'dek

Magnitnaya Gidrodinamika, Vol. 2, No. 3, pp. 152–154, 1966

UDC 538.4

E. I. Yantovskii [1] correctly draws attention to the desirability of eliminating differences in the definition of the magnetic Reynolds number

$$R_m = \mu\sigma v l \quad (1)$$

and suggests that v be regarded as the relative velocity of the magnetic field and the conducting medium. However, Yantovskii's proposal does not exhaust the matter and fails to eliminate certain more profound internal contradictions.

According to [1], for induction MHD machines (IMHDM) with a travelling magnetic field in (1) one should set

$$v = v_c - v_M, \quad (2)$$

where v_c is the velocity of the field, or so-called synchronous velocity, and v_M is the velocity of the medium.

The slip

$$s = \frac{v_c - v_M}{v_c}, \quad (3)$$

whence

$$v_M = (1-s) v_c, \quad (4)$$

and therefore instead of (2) we can also write

$$v = s v_c \quad (5)$$

or

$$v = \frac{s v_M}{1-s}. \quad (6)$$

Generally speaking, Yantovskii's proposal cannot be objected to. Moreover, R_m enters into the expressions characterizing the electromagnetic relationships in IMHDM in precisely this form when v is determined from (2), (5), or (6) and

$$l = \tau/\pi, \quad (7)$$

where τ is the pole pitch (length of half-wave) of the travelling magnetic field.

Incidentally, for IMHDM τ/π is a very characteristic geometric dimension. In particular, from a comparison of the expressions for the magnetic induction of flat induction machines with double and single inductors it follows [2] that for the latter τ/π plays the part of an equivalent air gap.

Yantovskii reproves the authors of [3, 4] for their supposedly unjustified assumptions. However, this reproof relates more to the form than to the essence of the matter, since these assumptions are basically equivalent to the substitution in (1) of a value of v in accordance with (6), which coincides with Yantovskii's own proposal.

More serious contradictions are revealed when we compare induction (IMHDM) and conduction (CMHDM) machines.

Let flat linear IMHDM and CMHDM have the same dimensions of the active part of the duct and the same working media moving at the same absolute velocities v_M. Moreover, 1) let the armature reaction of the CMHDM be compensated, and the IMHDM have lateral strips with infinitely high conductivity, and 2) let the operating regimes of the two machines be such that the magnetic field inductions B and the current densities j in the working media of both machines are the same (here for the IMHDM we have in mind the effective values of j and B for the resultant field). Neglecting differences in the manifestation of fringe effects in the two machines, it is easy to conclude that the MHD processes in them proceed with the same intensity and that they develop the same electromagnetic heads and powers. It would likewise be natural to assume that under these conditions R_m is also the same in both machines. However, if we agree with Yantovskii's reasonable proposal, then for the CMHDM we must substitute $v = v_M$ in (2), and for the IMHDM the value of v from (6). These two values of v will coincide only at s = 0.5, while at $s \gtrless 0.5$ the relations between these velocities, and hence the values of R_m, may be quite different.

Thus, in this case there is a clear contradiction in the manner of defining R_m. This is because in IMHDM the emf induced in the working medium is expended only on the voltage drop in the working medium itself, whereas in CMHDM this emf, in the generator case, is also a source of voltage at the terminals of the machine, and in the motor (pump) case balances part of the applied voltage. This difference in the role and magnitude of the emf in the two machines also conditions the different, other things being equal, values of the velocities of the working medium relative to the magnetic field.

This contradiction can be eliminated, at least formally, if in the CMHDM we introduce instead of the effective velocity of the working medium the velocity required solely to obtain the emf needed to cover the voltage drop in the working medium. To this end, we can introduce for CMHDM the concept of slip proposed by Yu. A. Birzvalk. Professor Yu. S. Chechet [5] made a similar proposal even earlier for dc motors.

For CMHDM let v_c be that velocity of the working medium at which at a given terminal voltage U the emf E = U and therefore the current in the working medium I = 0 (ideal open circuit). Then in the generator regime $v_M > v_c$, and in the motor (pump) regime $v_M < v_c$. Hence, according to Chechet and Birzvalk, the slip of the working medium of the CMHDM is given by

$$s = \frac{v_c - v_M}{v_c},$$

which coincides with (3) for IMHDM and electric induction machines.

Then for the CMHDM we have

$$U = Bbv_c, \qquad (8)$$

where b is the duct width, and

$$E = Bbv_M, \qquad (9)$$

and the voltage drop in the working medium

$$\Delta U = rI = U - E = Bb(v_c - v_M) = Bbsv_c. \qquad (10)$$

Other things being equal, the emf in the working medium of the IMHDM is the same.

Thus, if for the CMHDM in (1) we substitute v from (2), (5), or (6), then, in this respect, the lack of coordination in the determination of v will be eliminated, but it would still be necessary to reach an understanding on the identical definition of the characteristic dimension l. However, as already noted, for IMHDM the value $l = \tau/\pi$ is quite natural, whereas for CMHDM such a value of l makes no sense.

To determine R_m in IMHDM and CMHDM we can also use the actual velocity of the medium v_M. The R_m thus determined are suitable for certain estimates, but they do not characterize the intensity of the MHD processes in various operating regimes. For example, when the operating conditions change so that $v_M = v_c$, the MHD processes cease, but R_m retains a finite value. From this standpoint it is quite expedient to introduce the concept of the slip s of the working medium in CMHDM and define the R_m of CMHDM and IMHDM in terms of the slip velocity.

Some foreign authors define the R_m of IMHDM in terms of the synchronous velocity v_c. This definition of R_m can also be extended to CMHDM.

However, the characteristic lengths still remain indeterminate or noncomparable. Moreover, in IMHDM v_c and s are functions of f_1, while in CMHDM they are functions of U and B, i.e., quantities of a different type. Therefore we should perhaps reconcile ourselves to the differences in the definition of Rm for CMHDM and IMHDM. However, in comparing these machines it is necessary to keep this difference in mind and use the appropriate reduction formulas. If, for example, for IMHDM R_m is determined in the form

$$R_{mi} = \frac{\mu_0 \sigma s v_c \tau}{\pi} = \frac{\mu_0 \sigma s \omega_1 \tau^2}{\pi^2}, \qquad (11)$$

where $\omega_1 = 2\pi f_1$ is the angular frequency of the primary current, then the equivalent value for CMHDM is

$$R_{mc} = \mu_0 \sigma v_M l = R_{mi} \frac{v_M}{s v_c} \frac{\pi l}{\tau} = R_{mi} \frac{1-s}{s} \frac{\pi l}{\tau}. \qquad (12)$$

In this case the quantity l for CMHDM may be selected in different ways.

REFERENCES

1. E. I. Yantovskii, Magnitnaya gidrodinamika [Magnetohydrodynamics], 4, 153, 1965.
2. A. I. Vol'dek, Izv. VUZ Elektromekhanika, 12, 14, 1958.
3. G. A. Popov and V. B. Tikhonov, Problems of Magnetohydrodynamics [in Russian], 3, Riga, Izd. AN LatvSSR, 5, 1963.
4. N. M. Okhremenko, Problems of Magnetohydrodynamics [in Russian], 3, Riga, Izd. AN LatvSSR, 119, 1963.
5. Yu. S. Chechet, Elektrichestvo, 9, 28, 1951.

4 March 1966