MAGNETOHYDRODYNAMIC COUETTE FLOW OF VISCOPLASTIC MEDIA

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The effect of plastic fluid properties on MHD Couette flow in an annular gap with a radial magnetic field is investigated.

As the technological applications of magnetohydrodynamics increase, so also does the number of conducting media employed as the working substance. The properties of real fluid media can no longer be described satisfactorily by the approximation of a Newtonian fluid. The motion of dispersive media consisting of several phases in the channels of MHD apparatus, the MHD flow of liquid metals in electromagnetic fields for temperatures not greatly in excess of the metling point, the plastic flow of metals in electromagnetic fields when there are large loads acting, etc., can no longer be described by the equations of magnetohydrodynamics for Newtonian fluids. Moreover a series of investigations have showed [1, 2] that in strong transverse magnetic fields small departures from Newtonian properties can exert a more pronounced influence than in hydrodynamic flow. Thus the study of the MHD flow properties of non-Newtonian fluids is of serious interest.

In particular the investigation of the MHD flow of conducting viscoplastic fluids in channels with a transverse magnetic field has revealed certain peculiarities connected with the presence of plastic properties in the medium. Characteristic for this type of flow is the appearance of a zone moving like a solid body, i.e., a zone with a constant velocity across the channel. Here the dimensions of such a zone depend not only on the plastic properties of the fluid but also on the strengths of the external magnetic and electric fields.

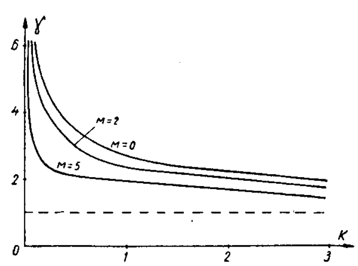

Fig. 1. The dimensions of the flow zone of a viscoplastic fluid as a function of Oldroyd's plasticity parameter.

We consider the pecularities of MHD Couette flow of a conducting viscoplastic fluid in an annular channel. Let the fluid be situated in the space between two coaxial nonconducting cylinders of radii r_1 and r_2. The external cylinder is fixed, and the inner cylinder has a force applied to it which causes it to move parallel to the axis with a velocity v_z. The external magnetic field of induction $B_r = B_0 r_1/r$ is radial. When motion occurs an azimuthal current of density $j_\varphi = \sigma v_z B_r$ will flow in the fluid.

In this case the equation of steady state motion may be written in the form

$$\frac{d\tau_{zr}}{dr}+\frac{\tau_{zr}}{r}-j_\varphi B_r=0, \qquad (1)$$

where τ_{zr} is the tangential shear stress in the fluid. In the case of viscoplastic fluids [3]

$$\tau_{zr}=\tau_0\left(\operatorname{sign}\frac{dv_z}{dr}\right)+\eta\frac{dv_z}{dr}. \qquad (2)$$

The limiting shear stress τ_0 and the plastic viscosity η are rheological characteristics of the medium. For the present formulation of the problem $dv_z/dr <$ < 0, i.e. $\operatorname{sign}(dv_z/dr) = -1$.

Setting Eq. (2) in Eq. (1) and introducing the dimensionless quantities $v = v_z/v_0$ and $x = r/r_1$, we obtain the equation for determining the axial velocity profile:

$$\frac{d^2v}{dx^2}+\frac{1}{x}\frac{dv}{dx}-M^2\frac{v}{x^2}=\frac{K}{x}, \qquad (3)$$

where $M = B_0 r_1(\sigma/\eta)^{1/2}$ is the Hartmann number, and Oldroyd's dimensionless parameter [3] $K = \tau_0 r_1/\eta v_0$ characterizes the plastic properties of the medium. The presence of a limiting shear stress τ_0 leads to the existence of a zone $r_0 \le r \le r_2$ close to the inner cylinder where the viscoplastic fluid is at rest while at the boundary $r = r_0$, $|\tau_{zr}| = \tau_0$.

If the force applied to the inner cylinder is small so that the stress on it is less than τ_0, then all the fluid in the annular gap will remain at rest. In this case the viscoplastic fluid will behave like an elastic solid body. Thus the boundary conditions for v are

$$v=1 \quad \text{for} \quad x=1,$$
$$v=0 \quad \text{for} \quad x=\gamma=r_0/r_1. \qquad (4)$$

Taking (4) into account the solution of equation (3) is for ($M \ne 1$)

$$v=-\frac{K\gamma+(1-M^2-K)\gamma^{-M}}{(1-M^2)[\gamma^M-\gamma^{-M}]}x^M+$$
$$+\frac{K\gamma+(1-M^2-K)\gamma^M}{(1-M^2)[\gamma^M-\gamma^{-M}]}x^{-M}+\frac{K}{(1-M^2)}x. \qquad (5)$$

In order to find γ, i.e., the radius r_0 where the flow zone separates from the fluid which is at rest, we make use of the equation $\left.\frac{dv}{dx}\right|_{x=\gamma}=0$, which follows from

Eq. (2), since $|\tau_{zr}| = \tau_0$ at the boundary. Taking (5) into account we have

$$K = \frac{2(M^2-1)}{\gamma[\gamma^M - \gamma^{-M} - 2\gamma^{-1} - M^{-1}(\gamma^M - \gamma^{-M})]} = F(M, \gamma). \qquad (6)$$

Fig. 1 gives the function $\gamma = f(K)$ for various values of the parameter M. It is clear that an increase in the magnetic field leads to a decrease in the flow zone, i.e. in strong magnetic fields only a thin layer of fluid in contact with the inner cylinder moves in the axial direction. Thus the plastic properties are more pronounced in the presence of a transverse magnetic field. The results of [1, 2] confirm this conclusion.

We shall consider the gradual variation of velocity profile in a channel as the velocity of motion of the inner cylinder v_0 increases for fixed values of M, τ_0 and η. For small $v_0 (K \to \infty)$ $\gamma \to 1$, i.e., the flow zone is concentrated close to the inner cylinder (Fig. 2 gives the curve corresponding to $v_0 = v_0^{(1)}$). Here the velocity profile is determined by expression (5) and the value of γ is taken from Eq. (6). As v_0 increases $\gamma \to \beta = r_2/r_1$, and for $v_0 = v_0^* = (\tau_0 r_1/\eta) F(M, \beta)$ all the fluid in the annular channel will move. For values of v_0 larger then v_0^*, the velocity profile in the channel will also be described by Eq. (5), but we must set $\gamma = \beta$. As v_0 is further increased the dimensionless profile v_z/v_0 approaches the profile of a Newtonian fluid (K = 0).

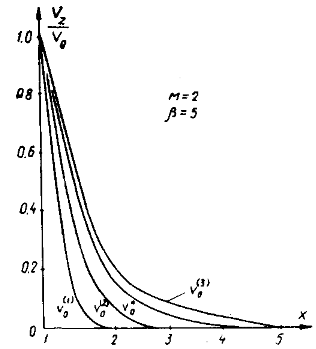

Fig. 2. The axial velocity profile of a conducting viscoplastic fluid for various values of velocity of the inner cylinder ($v_0^{(1)} < v_0^{(2)} < v_0^* < v_0^{(3)}$).

It may be shown that there is a transition to a nonconducting viscoplastic fluid ($M \to 0$).

REFERENCES

1. L. K. Martinson and K. B. Pavlov, Magnitnaya Gidrodinamika [Magnetohydrodynamics], **4**, 61, 1965.

2. L. K. Martinson and K. B. Pavlov, Magnitnaya Gidrodinamika [Magnetohydrodynamics], **3**, 69, 1966.

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