SELF-EXCITATION OF SOME ELECTROMECHANICAL SYSTEMS

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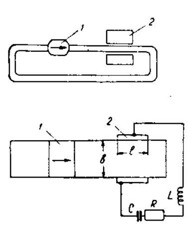
The author examines the transient process of self-excitation in an autonomous electromechanical system consisting of an acceleration device (pump) and a MHD generator. It is shown that given a sufficient liquid velocity the system is self-exciting, and if the pump head falls with increase in velocity, then a stationary and stable regime can be obtained in the linear electric circuit as a consequence of this factor alone.

To be specific, by an electromechanical system we shall understand a closed circuit (see figure) filled with a liquid-metal medium of mass M and containing an acceleration device (pump, injector)—1 and a MHD generator—2 with an external circuit including lumped parameters—inductance L and capacitance C. The total resistance of generator and load is denoted by R.

It is assumed that the cross section F of the channel is constant, so that the velocity of the liquid is the same over its entire length and the acceleration of the liquid dv/dt = ∂v/∂t (there is no convective term $(v \cdot \nabla)v$). In this approximation the liquid moves like a solid nondeformable conductor, so that the concept "electromechanical system" also includes ordinary electric machines on a single shaft (motor-generator).

The characteristic of the acceleration device p(v), of which the torque characteristic of the electric motor is an analog, is assumed given.

The object of this research was to determine the conditions of existence in a strictly linear electric circuit, where dL/dI = dR/dI = 0, of stationary regimes of operation of the system as a result of self-excitation in the absence of sources of emf, apart from the generator with series or parallel connection of the excitation winding.



The starting equations are as follows:

Kirchhoff's law for the electric circuit

$$L\frac{dI}{dt}+RI+\frac{1}{C}\int I dt=NI, \qquad (1)$$

where NI = E is the generator emf, and the law of conservation of momentum

$$M\frac{dv}{dt}=F[p(v)-\beta v^2-\alpha I^2], \qquad (2)$$

where βv^2 is the flow friction, and αI^2 the ponderomotive force.

In order to ascertain the significance of the constants, we write the corresponding expressions for a simple ideal MHD generator, i.e., a device without fringe effects and leakage, with series-connected winding:

$$L=\frac{\mu_0 w^2 bl}{h}; \quad N=\frac{\mu_0 wb}{h}v; \quad M=\rho l_1 bh;$$

$$\alpha=\frac{\mu_0 w}{h^2}; \quad \beta=\frac{\xi\rho}{2}l_1\frac{b+h}{2bh}. \qquad (3)$$

Here, μ_0 is the magnetic constant, w the number of turns of the exciting winding, ρ the density of the liquid, l_1 the length of the loop carrying the liquid, bh the cross section of the channel, and ξ the resistance coefficient.

We will begin by examining a simple known particular case.

1. It is assumed that the conditions $\alpha I^2 \ll p(v)$ and $p(\dot{v}) = \beta v^2$ are satisfied; then v = const. Also let the capacitance be infinitely large (1/C = 0). Then the behavior of the system is described by the equation

$$\frac{dI}{dt}=\frac{N-R}{L}I, \qquad (4)$$

whose solution is

$$I=I_0\exp\left(\frac{N-R}{L}t\right), \qquad (5)$$

where I_0 is the initial magnitude of the current at t = 0.

When N < R the solution I = 0 is stable with respect to variation of the initial condition, since I(t) → 0. If N > R, the current increases without limit no matter how small I_0, i.e., the solution I = 0 is unstable.

The problem was exhaustively investigated in [1, pp. 30, 85, 679], from which it follows that the case examined may be termed soft self-excitation, the parameter N the negative resistance, and the value N = R the bifurcation value of N.

However, when dN/dI = dR/dI = 0, it is impossible to obtain a stationary regime I ≠ 0. There has to be a real root of the equation N − R = 0, i.e., a decrease in N with decrease in I at R = const or an increase in R with increase in I at N = const.

This is achieved by introducing nonlinearities—magnetic N = N(I) due to saturation of the magnetic system or ohmic R = R(I). A similar effect is also given by the capacitive nonlinearity C = C(I) thanks to the use of special capacitors—variconds. All these possibilities have been thoroughly studied and used in practice [4]; however, they also possess disadvantages: a certain increase in losses necessary to obtain a stable regime, extra weight (magnetic systems) or limited power (variconds). In a number of cases their use is excluded by the specific design of the generator.

2. With all the above assumptions a certain difference in the behavior of the system is observed when the generator employs a Hall emf (as distinct from the Faraday emf in §1). If the nonuniformity of the parameters in the channel is neglected, the open-circuit voltage of the Hall current generator has the form

$$U=\omega\tau vBl=\eta\left(\frac{\omega\mu_0}{h}\right)^2 lvI^2=QI^2,$$

where $\omega\tau$ is the Hall parameter, η the electron mobility, and $Q = \eta(\omega\mu_0/h)^2 lv$; substituting QI^2 for NI in (1), we obtain the equation of the transient

$$L\frac{dI}{dt}=QI^2-RI, \tag{6}$$

whose solution has the form

$$I=RI_0\exp\left(-\frac{R}{L}t\right)\left\{R-QI_0\left[1-\exp\left(-\frac{R}{L}t\right)\right]\right\}^{-1}. \tag{7}$$

In connection with the quadratic dependence of voltage on magnetic field in the Hall current generator the current buildup, i. e., the beginning of self-excitation, depends on the magnitude of the initial current, and, in particular, it is necessary that $I_0 > R/Q$, which provides an example of "hard" self-excitation.

The Hall emf increases quadratically with increase in current and magnetic field. However, since the internal resistance of the generator increases according to the same law, when $\omega\tau \gg 1$ the load voltage ceases to increase and approaches a limit, which in principle makes it possible to achieve a stationary and stable regime.

In what follows the influence of the Hall effect is disregarded. The present section serves only to illustrate the complication introduced by employing a Hall emf.

3. It is interesting to consider the possibility of obtaining stationary currents in the case of strict linearity of the electric circuit thanks to the use of the autonomy of the system and the dependence of the velocity on the current in the circuit. In this case it is necessary to keep in mind the commensurability of the pressure developed by the acceleration device and the ponderomotive force retarding the liquid in the generator. In order to simplify the solution, we make assumptions that enable us to arrive directly at a clear result. Thus, let the mass of the liquid $M = 0$ and let there be no friction, i. e., $\beta = 0$. Moreover, let the characteristic p(v) have the very simple form

$$p=p_0-kv, \tag{8}$$

i. e., the pump head falls with increase in velocity. Then from Eq. (2)

$$v=(p_0-\alpha I^2)/k. \tag{9}$$

As before, we will assume that $C = \infty$ in Eqs. (1) and write N in the form: $N = \gamma v$ (γ = const). Then

$$E=NI=\gamma I(p_0-\alpha I^2)/k. \tag{10}$$

Here we already have a nonlinear relation E(I).

Substituting Eqs. (9) and (10) in Eq. (1) gives

$$\frac{dI}{dt}=I\left[\left(\gamma\frac{p_0}{kL}-\frac{R}{L}\right)-\gamma\frac{\alpha}{kL}I^2\right] \tag{11}$$

or

$$\frac{dI}{dt}=I(a-bI^2), \tag{12}$$

where

$$a=\gamma\frac{p_0}{kL}-\frac{R}{L};\quad b=\gamma\frac{\alpha}{kL}. \tag{13}$$

From Eq. (12) it is immediately clear that the steady-state current

$$I_k=\sqrt{\frac{a}{b}}=\sqrt{\frac{1}{\alpha}(p_0-kv_k)}. \tag{14}$$

Integration of Eq. (12) gives the form of the transient

$$I^2=a\{b+[(a/I_0^2)-b]\exp(-2at)\}^{-1}. \tag{15}$$

When $a > 0$ we obtain an increase or decrease in current with time (depending on the sign of the difference $a - bI_0^2$) which tends to the steady-state value (14).

The dependence of the course of the process on the initial current I_0 is connected with the fact that it is related with the initial velocity of the liquid; therefore the condition of initiation of self-excitation is actually the same as the simple case of §1, i. e., $\gamma v > R$. When $a < 0$ the current decreases to zero at any I_0.

One more important detail of the system should be noted: at $k < 0$, i. e., when the pressure of the acceleration device increases with increase in velocity, a is always less than zero and $I(t) \to 0$.

We obtain the same result if we take into account the finiteness of the capacitance, i. e., consider an ac MHD generator with mechanical nonlinearity, neglecting the inertia of the liquid.

From Eq. (1) we obtain

$$L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{1}{C}I=\gamma\frac{dv}{dI}\frac{dI}{dt}I+\gamma v\frac{dI}{dt}. \tag{16}$$

Substituting v from (9) and grouping the terms, we have

$$\frac{d^2I}{dt^2}+\omega^2I=(a_1-b_1I^2)\frac{dI}{dt};$$

$$\omega^2=\frac{1}{LC};\quad a_1=\gamma\frac{p_0}{kL}-\frac{R}{L}=a;\quad b_1=3\frac{\alpha}{kL}\gamma=3b. \tag{17}$$

Equation (17) accurately coincides with the equation of a vacuum-tube generator (48.16) from [2]. It gives a stable self-oscillating regime with a steady-state current amplitude

$$A_k = 2\sqrt{\frac{a_1}{b_1}},$$

where in our notation the equation for the amplitude has the form

$$\frac{dA}{dt} = \frac{1}{2}A\left(a_1 - \frac{1}{4}b_1A^2\right),$$

analogous to Eq. (12).

The difference between the electromechanical and radio-engineering systems consists in the fact that in our case the nature of the nonlinearity is purely mechanical (variation of velocity to obtain equality of motive force and resistance), whereas in the vacuum-tube generator the nonlinearity is created by the special form of the tube characteristic.

4. It is a shortcoming of the above approach that ignoring the mass (inertia) of the liquid prevents one from drawing a perfectly definite conclusion concerning the possibility of a stationary regime. Therefore it is desirable to consider the equations with allowance for the term Mdv/dt, while retaining the assumptions of the previous section regarding the characteristic of the acceleration device:

$$L\frac{dI}{dt} = I(\gamma v - R), \quad M\frac{dv}{dt} = F(p_0 - kv - \alpha I^2). \quad (18)$$

Differentiating the first of these equations with respect to time and substituting dv/dt from the second, we obtain

$$\frac{d}{dt}\left(\frac{1}{I}\frac{dI}{dt}\right) = m - n\frac{1}{I}\frac{dI}{dt} - qI^2, \quad (19)$$

where

$$m = \frac{Fp_0}{LM}\gamma - \frac{RkF}{LM}; \quad n = \frac{kF}{M}; \quad (20)$$

$$q = \frac{\alpha F}{LM}\gamma. \quad (21)$$

Making the substitution y = ln I, we obtain the second-order equation

$$y'' + ny' + q\exp 2y = m. \quad (22)$$

Equations of this type have been investigated for a long time; however, their solutions have still not been found [3].

This Eq. (22) has a simple first integral when n = = 0, i.e., when the pump pressure does not depend on velocity (k = 0)

$$I' = \frac{dI}{dt} = \pm I\sqrt{m\ln\frac{I^2}{I_0^2} - q(I^2 - I_0^2) + \frac{1}{L^2}(\gamma v_0 - R)^2}. \quad (23)$$

The basic Eq. (19) can be analyzed by going over to the phase plane (I', I)

$$\frac{dI'}{dI} = \frac{I'}{I} - n + \frac{I(m - qI^2)}{I'}. \quad (24)$$

Equation (24) has two singular points: A (I' = 0; I = = 0) and B (I' = 0; $I = \sqrt{m/q}$), corresponding to two states of equilibrium of the system. In the neighborhood of the point A, i.e., at low current, the solution of Eq. (19) has the form

$$I = I_0 \exp\frac{m}{n}t, \quad (25)$$

which coincides with Eq. (5), i.e., when m/n = = (N − R)/L > 0 the state of equilibrium at point A is unstable and self-excitation begins at that point. In the neighborhood of point B we assume $I \sim I_k = \sqrt{m/q}$, and equation (24) takes the form:

$$\frac{dI'}{dI} = \frac{-nI' - 2m(I - I_k)}{I'}. \quad (26)$$

It coincides with the equation of a linear oscillator with friction [1, p. 56] and when n > 0 characterizes absolutely stable states of equilibrium of the "focus" type. Depending on the sign of the difference $n^2/4 - 2m$ as equilibrium is approached at point B we may get either damped oscillations or a damped aperiodic process (critical motion).

The phase trajectory as a whole has the form of a line starting from point A and spiraling onto point B.

The motion near point B, where $I \sim I_k$, $v \sim v_k = R/\gamma$, is described by the linearized equation

$$(I - I_k)'' + n(I - I_k)' + 2m(I - I_k) = 0, \quad (27)$$

the roots of whose characteristic equation are

$$P_{1,2} = -\frac{n}{2} \pm \sqrt{\frac{n^2}{4} - 2m} = \frac{kF}{M} \times$$

$$\times\left[-\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{2M}{kFL}\left(R - \frac{\gamma p_0}{k}\right)}\right]. \quad (28)$$

Hence there follows the stability condition: k > 0, i.e., a fall in head with increase in velocity. When k = 0 we get undamped oscillations about an equilibrium state of the "center" type with a frequency $\sqrt{2m} = \sqrt{(2F/LM)p_0\gamma}$. For any I, and not only close to I_k, the phase trajectory as a whole, a closed curve symmetrical about the I axis, is constructed in accordance with expression (22).

The relations obtained make it possible to trace the behavior of the system with increase in pump head p_0. Initially, at $p_0 = 0$ there is no current in the system. As the pressure increases, the liquid velocity increases in correspondence with the growth of head, but the current still remains equal to zero, since absence of current is still a stable state of equilibrium. Any small initial currents damp with time. As soon as the head and hence the velocity reach the critical value $v_k = R/\gamma$, the increase in velocity ceases, the system becomes self-exciting, and subsequently, if the increase in head continues, there is an increase not in liquid velocity but in the current in accordance with the relation $I_k = \sqrt{(p_0 - kv_k)/\alpha}$, i.e., we get "blocking" of the channel. If the electric circuit is cut in at a point when the velocity has already exceeded the critical value v_k, a transient process of current buildup and decrease in velocity is initiated, both current and velocity tending to the steady-state values at the point of stable equilibrium.

SUMMARY

1. The unique condition of onset of self-excitation is a sufficiently large initial velocity ensuring N > R.

2. The mechanical nonlinearity ensures a stationary and stable operating regime of the electromechanical system considered if the head in the acceleration device falls with increase in velocity.

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