DEFORMATION OF CURRENT-CARRYING JETS BY NONVISCOUS ELECTRICALLY CONDUCTING LIQUID

V. I. Morozova, E. V. Poklonskii, and A. I. El'kin†

UDC 537.84

1. Formulation of the Problem. In [1] the use of a system of liquid-metal current-carrying jets with noncircular transverse cross section as a liquid-metal electromagnet has been proposed for producing a magnetic field periodic in azimuth. The stability of such a system of jets with respect to constriction-type perturbations has been investigated in [2].

In the present work the change in the form of the transverse cross section of the jet under the action of the current flowing along it is investigated. The jets are assumed to have infinite extension in the z direction, which implies the independence of the parameters (velocity, form of the cross section) with respect to this coordinate. At the initial instant of time the velocity is uniform and has a single component along the z axis; the current density j_0 is constant in time, uniform, and parallel to this axis. The last assumption requires some explanation.

In industrial equipment [1], in which a certain transitional process occurs after switching on the electric supply source, this assumption is valid under certain conditions: firstly, that the diffusion time of the magnetic field and the current density in the liquid metal (skin period) be smaller than the time for significant deformation of the jet; and secondly, that the current density perturbations caused by the velocity perturbations due to the deformation of the jet be small. These requirements are satisfied under the condition

$$Rm = \mu_0 \sigma v^* a < 1, \qquad (1)$$

where $v^* = \sqrt{\mu_0/\rho}\, j_0 a$, a is the characteristic dimension. The quantity Rm in (1) may be regarded as a magnetic Reynolds number constructed from the characteristic rate of the deformation of the jet v^*.

†Deceased.

Translated from Magnitnaya Gidrodinamika, No. 1, pp. 17-20, January-March, 1986. Original article submitted April 16, 1985.

The analysis below is carried out in a coordinate system moving with the initial jet velocity. Then the problem becomes two-dimensional, i.e., the velocity vector lies in the xOy plane and all the quantities depend on the corresponding coordinates. Assuming the liquid to be nonviscous and incompressible, we write the momentum and continuity equations in the form

$$\frac{d\mathbf{v}}{dt}=-\nabla p-\mathbf{i}B_y+\mathbf{j}B_x,\quad \frac{d\mathbf{r}}{dt}=\mathbf{v},\quad \operatorname{div}\mathbf{v}=0. \tag{2}$$

Here and below the equations are written in nondimensional form, where the width of the jet a and current density j_0 are taken as the base quantities; the remaining base quantities are introduced through the formulas

$$p^*=\mu_0 j_0^2 a^2,\quad v^*=\sqrt{\mu_0/\rho}\,j_0 a,$$

$$B^*=\mu_0 j_0 a,\quad t^*=a/v^*=j_0^{-1}\sqrt{\rho/\mu_0}.$$

In the system of Eqs. (2), $\mathbf{r}$ is the radius vector of a moving particle; $\mathbf{i}, \mathbf{j}$ are unit vectors of the Cartesian coordinate system. The magnetic field induction in (2) can be expressed as a function of the coordinates of points if the solution for the z-component of the vector potential of the jet

$$A_z(M)=\frac{1}{2\pi}\int_S \ln\frac{1}{r_{MN}}\,ds_N \tag{3}$$

and Maxwell's equations

$$B_x=\frac{\partial A_z}{\partial y},\quad B_y=-\frac{\partial A_z}{\partial x} \tag{4}$$

are used. Here the base potential A_0 is introduced through the formula $A_0 = a^2\mu_0 j_0$ and Eq. (3) is written for a single jet.

Disregarding the effect of surface tension we take

$$p_\Gamma=0 \tag{5}$$

as the boundary condition. Thus the problem reduces to the solution of the system of Eqs. (2)-(4) with boundary condition (5).

Below we shall consider either isolated jets or systems of azimuthally periodic jets with currents alternating in direction. In the latter case, assuming that the jets are deformed identically, we can investigate the deformation of a single jet as before and use the superposition principle for determining the magnetic field.

2. Method of Solution. We used LING numerical method [3] for the computation of nonsteady-state flows of an incompressible liquid with free boundaries. Quadrangular cells of the computational grid move together with the liquid. The coordinates, velocities, and accelerations are specified at the nodes of the cells; the pressure is over some small area and the double integrals were transformed to integrals over the boundary of the region of integration.

After averaging, the momentum equation has the form

$$\mathbf{v}_{i,j}^{n+1}=\mathbf{v}_{i,j}^{n}+\tau(\mathbf{D}_{i,j}^{n+1}+\mathbf{f}_{i,j}^{n}). \tag{6}$$

Here $\mathbf{D}_{ij}$ is the acceleration at point (i, j) created by the pressure gradient; $\mathbf{f}_{i,j}$, acceleration caused by mass forces (electrical volume force); τ, time step; n, step number. All the quantities on the right-hand side of Eq. (6) are known except $\mathbf{D}_{i,j}$, which depends on the pressure at (n + 1)-th time step.

The new coordinates of the nodes are found from the equation

$$\mathbf{r}_{i,j}^{n+1}=\mathbf{r}_{i,j}^{n}+\tau\mathbf{v}_{i,j}^{n+1}. \tag{7}$$

In the method used the continuity equation reduces to the requirement of conservation of the area of the cells:

$$s_{i,j}^{n+1}=s_{i,j}^{0}. \tag{8}$$

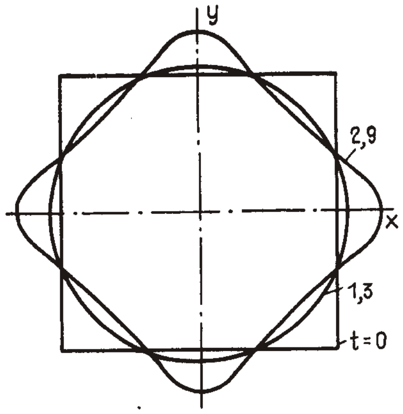

Fig. 1. Deformation of a solitary jet of square cross section.

Using (6)-(8) we can obtain a Poisson-type equation for the pressure. After finding this equation the new values of velocities and coordinates of the cell nodes are found from Eqs. (6) and (7). The process is then repeated.

In the computation of the electrical body force in (6), the volume of computations was significantly reduced by transforming from surface to contour integrals in Eqs. (3), (4) using Green's formulas. As a result, we have

$$B_x(M)=\frac{1}{2\pi}\oint_L \ln\frac{1}{r_{MN}}\,dx_N,\quad B_y(M)=\frac{1}{2\pi}\oint_L \ln\frac{1}{r_{MN}}\,dy_N. \tag{9}$$

Replacing the contour of integration in (9) by a broken line and integrating over the rectilinear segments, we finally get

$$B_x(M)=-\frac{1}{4\pi}\sum_{i=1}^{N-1}\Delta x_i K_i(M),$$

$$B_y(M)=-\frac{1}{4\pi}\sum_{i=1}^{N-1}\Delta y_i K_i(M).$$

Here

$$K_i(M)=\frac{1}{r_i^2}\left\{(\mathbf{R}_{i+1},\mathbf{r}_i)\ln R_{i+1}^2-(\mathbf{R}_i,\mathbf{r}_i)\ln R_i^2+2C_i\left(\operatorname{arctg}\frac{(\mathbf{R}_{i+1},\mathbf{r}_i)}{C_i}-\operatorname{arctg}\frac{(\mathbf{R}_i,\mathbf{r}_i)}{C_i}\right)\right\},$$

$$\Delta x_i=x_{i+1}-x_i,\quad C_i=\sqrt{R_i^2r_i^2-(\mathbf{R}_i,\mathbf{r}_i)^2},$$

$$R_i^2=(x_i-x)^2+(y_i-y)^2,\quad r_i^2=(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2,$$

$$R_{i+1}^2=(x_{i+1}-x)^2+(y_{i+1}-y)^2,$$

$$(\mathbf{R}_{i+1},\mathbf{r}_i)=(x_{i+1}-x)(x_{i+1}-x_i)+(y_{i+1}-y_i)(y_{i+1}-y),$$

$$(\mathbf{R}_i,\mathbf{r}_i)=(x_i-x)(x_{i+1}-x_i)+(y_i-y)(y_{i+1}-y_i),$$

$\{x_i, y_i\}_{i=1}^{N}$ are the points on the boundary of the cross section of the jet, numbered counterclockwise.

3. Results of Computations. The results of computations for solitary current carrying jets of square and rectangular cross sections, and for systems of six and twelve azimuthally periodic jets with currents alternating in direction, are shown in Figs. 1-4.

A solitary jet with square transverse cross section at the initial instant executes a periodic motion involving transition from square cross section, through circular, back to square rotated by 45° with respect to the initial section, and later going through circular

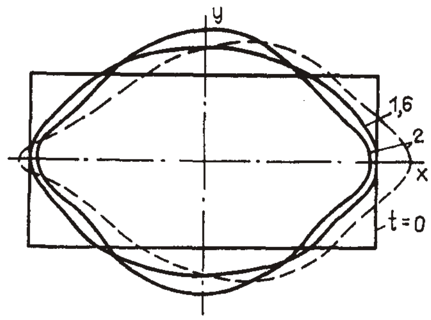

Fig. 2. Deformation of a solitary jet of rectangular cross section and comparison with the deformation of a similar jet from a system of six jets (dashed curve).

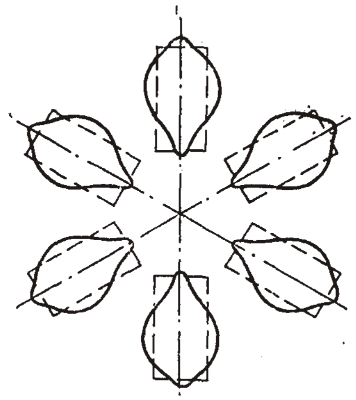

Fig. 3

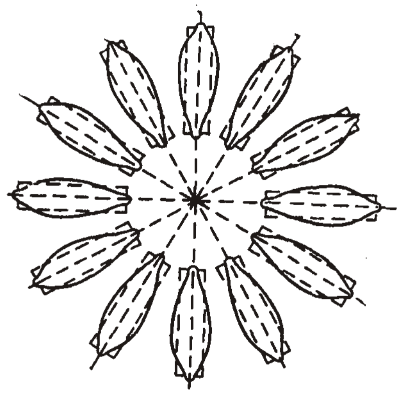

Fig. 4

Fig. 3. Deformation of the cross section of a system of six current-carrying jets during time t = 2.

Fig. 4. Deformation of the cross section of a system of 12 current-carrying jets during time t = 2.4.

cross section and returning to the initial position (Fig. 1). Similar deformations occur with a solitary rectangular cross section (Fig. 2). Deformations of the systems of jets are accompanied by their radial separation (Figs. 3 and 4).

We note that the present formulation uses only geometric criteria of similarity; therefore the dependences of the deformations on time, presented in Figs. 2-4, are universal.

LITERATURE CITED

1. Yu. I. Arkhangel'skii, V. P. Volkov, E. V. Murav'ev, S. L. Nedoseev, P. V. Romanov, L. N. Rudakov, V. D. Ryutov, E. A. Tsygankov, and R. E. Shatalov, "Conditions of operation of construction materials in a pulsed thermonuclear reactor at relativistic electron beams," in: Problems of Atomic Science and Engineering. Thermonuclear Synthesis [in Russian], Inst. Atomic Energy, Moscow, No. 1(3), 39-51 (1979).
2. E. V. Poklonskii and A. I. Él'kin, "Stability of a system of current carrying jets with currents alternating in direction," Magn. Gidrodin., No. 4, 39-45 (1982).
3. C. W. Hirt, I. L. Cook, and T. D. Butler, "Lagrangian method for calculating the dynamics of an incompressible fluid with free surface," J. Comp. Phys., No. 5, 103-124 (1970).