THE FLOW OF AN ELECTRICALLY CONDUCTING FLUID IN CROSSED CONSTANT AND VARIABLE MAGNETIC FIELDS

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Let us examine the problem involved in the development of flow for an electrically con- ducting fluid in a rectangular container with nonconducting walls and a lateral cross secti of $2a \times 2d$, with this container situated within crossed constant and variable magnetic fiel $B(B_1(t), 0, B_0)$, with the variable magnetic field directed along the axis of the container. Sections of this type were dealt with in [1].

If the longitudinal dimension of the container is considerably larger than the transve dimension, the motion of the fluid may be regarded as rectilinear and the terminal effects need not be taken into consideration. Thus, we have only a single velocity component $u(u(z, y), 0, 0)$, the flow is assumed to be gradient-free, and the resulting magnetic field within the channel has the form $B(B_1(t) + b(t, y, z), 0, B_0)$. Turning to dimensionless qua tities $\tilde{d} = d/a$, $\tilde{z} = z/a$, $\tilde{y} = y/a$, $\tilde{u} = u/v_0$, $\tilde{t} = t/T_0$, $\tilde{B}_1 = B_1/B_0$, $\tilde{b} = b/(Rm\,B_0)$, $v_0 = a/T_0$, $\tilde{B}_{10} = B_{10}/B_0$, where T_0 is the characteristic time for the change in magnetic field, we deri a system of equations (the tilde is dropped in the following), describing the flow:

$$\frac{\partial u}{\partial t}=\frac{1}{Re}\Delta u+St\frac{\partial b}{\partial z};\quad Rm\frac{\partial b}{\partial t}=\Delta b+\frac{\partial u}{\partial z}-\frac{\partial B_1}{\partial t};\tag{1}$$

$$j_y=\frac{\partial b}{\partial z};\quad j_z=-\frac{\partial b}{\partial y}.\tag{2}$$

The complexes St, Re, and Rm are constructed on the basis of v_0. The conditions of symmetr applicable to this problem allows us to deal exclusively with the region $0 \le y \le 1$, $0 \le z \le d$, specifying the following boundary conditions at the axes of symmetry:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}=0;\quad \left.\frac{\partial b}{\partial y}\right|_{y=0}=0;\quad u|_{z=0}=0;\quad \left.\frac{\partial b}{\partial z}\right|_{z=0}=0.\tag{3}$$

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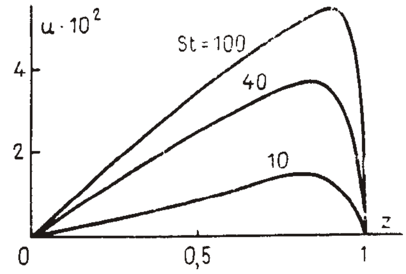

Fig. 1

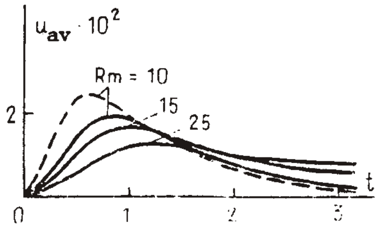

Fig. 2

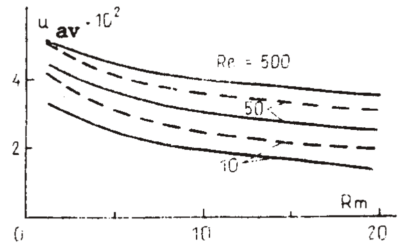

Fig. 3

Fig. 1. Distribution of velocity at the axis of the container with t = 0.6, Re = 10, Rm = 10.

Fig. 2. $u_{av}(t)$ for Re = 10 and St = 100 (dashed line) and 40 (solid line).

Fig. 3. $u_{av\,m}(Rm)$ for St = 100 (dashed line) and 40 (solid line).

The initial and boundary conditions at the walls of the container have the form

$$u|_{t=0}=b|_{t=0}=0;\quad b|_{z=d}=u|_{z=d}=0;\quad b|_{z=1}=u|_{z=1}=0. \tag{4}$$

System (1) with boundary conditions (3) and (4), written in vector form

$$A\frac{\partial \mathbf{w}}{\partial t}=B\frac{\partial^2 \mathbf{w}}{\partial y^2}+B\frac{\partial^2 \mathbf{w}}{\partial z^2}+C\frac{\partial \mathbf{w}}{\partial z}-\mathbf{F}, \tag{5}$$

where

$$\mathbf{w}=\begin{pmatrix}u\\ b\end{pmatrix};\quad A=\begin{pmatrix}1&0\\0&\mathrm{Rm}\end{pmatrix};\quad B=\begin{pmatrix}1/\mathrm{Re}&0\\0&1\end{pmatrix};\quad C=\begin{pmatrix}0&\mathrm{St}\\1&0\end{pmatrix};\quad \mathbf{F}=\begin{pmatrix}0\\ \partial B_1/\partial t\end{pmatrix};$$

$$\mathbf{w}|_{y=1}=0,\ \left.\frac{\partial \mathbf{w}}{\partial y}\right|_{y=0}=0;\ \mathbf{w}|_{z=d}=0;\ A_1\mathbf{w}|_{z=0}+A_2\left.\frac{\partial \mathbf{w}}{\partial z}\right|_{z=0}=0; \tag{6}$$

$$A_1=\begin{pmatrix}1&0\\0&0\end{pmatrix};\quad A_2=\begin{pmatrix}0&0\\0&1\end{pmatrix};\quad \mathbf{w}|_{t=0}=0,$$

were solved by the finite-difference method.

The time derivative in (5) was approximated by unilateral differences in advance

$$A\frac{\mathbf{w}^{n+1}-\mathbf{w}^{n}}{\Delta t}=B\frac{\partial^2 \mathbf{w}^{n+1}}{\partial y^2}+B\frac{\partial^2 \mathbf{w}^{n+1}}{\partial z^2}+C\frac{\partial \mathbf{w}^{n+1}}{\partial z}-\mathbf{F}^{n+1},$$

where $\mathbf{w}^n$, $\mathbf{w}^{n+1}$ are the values of the vector functions at the n-th and (n + 1)-th time intervals, respectively, with $\mathbf{w}^n$ held to be known from the previous interval. In order to determine $\mathbf{w}^{n+1}$, an iteration process is carried out at each time interval and this is an establishment process that is based on some fictitious time τ. The approximation of Eq. (5) with the additional fictitious term $\partial \mathbf{w}^{n+1}/\partial\tau_{\tau\to\infty} \to 0$ is achieved in accordance with the implicit scheme of variable directions [2]. The solution of this system of finite-difference equations, with consideration of boundary conditions (6), was achieved by the matrix sweeping method.

The results of the calculation for the case in which d = 6 were compared with the analytical solution of the one-dimensional problem relating to the flow of an electrically conducting fluid in the case in which $d \gg 1$ and with the induced fields for the case $B_1 = B_{10}e^{-t}$ held to be negligibly small. With values of Rm = 0.1, the difference amounted to no more than 1%, which testifies as to the high accuracy of the method.

Additional calculations were carried out for the case d = 1, $B_1 = B_{10}e^{-t}$, $B_{10} = 0.1$. The velocity distributions at the axis y = 0(u(y, z) = −u(y, z)) are shown in Fig. 1. Figure 2 shows the time-dependent quantities $u_{av} = d^{-1}\int_0^1\int_0^d u(y, z, t)dydz$ characterizing the development of the flow. The quantity u_{av} initially increases to $u_{av\,m}$, and then it diminishes, since the forces of viscosity become greater than the electromagnetic forces. Figure 3 shows $u_{av\,m}$ as a function of Rm for various values of St and Re. The increase in Rm leads to a reduction in the skin layer and, consequently, to a reduction in $u_{av\,m}$.

The model described here makes it possible to evaluate the pressure at the walls of the container, said pressure associated with the motion of the fluid.

LITERATURE CITED

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2. P. Rouch, Computational Hydrodynamics [Russian translation], Moscow (1980).