THE SYNTHESIS OF MULTIPHASE SELF-OSCILLATION SYSTEMS IN MAGNETOHYDRODYNAMICS

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The development of alternating-current MHD equipment involving the use of an external variable magnetic field runs up against difficulties that are generated by the manner in which energy is used, where the latter is supplied from an outside source. The presence of a variable magnetic field, particularly in the case of large-scale MHD installations, makes it necessary to offset the substantial reactive power taken from the source. In the case of liquid-metal MHD installations, where the value of the electrical conductivity σ is sufficiently large, it is sometimes possible to offset a low cos φ by means of capacitors in LC oscillating circuits, whereas in the case of plasma generators such a procedure is unrealistic. Thus, in the majority of cases the alternating-current MHD installations must be self-oscillating. Both in this case and in the pump regime we may find the complete absence of reactive power utilization from the external source, and when the power grid is operating at a frequency lower than the eigenfrequency of the self-oscillation system, the reactive power may be supplied to the grid with a leading cos φ.

A considerable number of papers [1-6] has been devoted to the problem of studying self-oscillation MHD installations. We know of studies in which the processes of self-excitation have been examined in traditional electrical alternating-current machinery [7]. Of particular interest here is the development of multiphase balance systems, i.e., systems requiring a constant active power from an external source. Self-oscillation inductive-excitation systems were examined in [6], and the basic advantage of these lies in the fact that the system of reactive-energy accumulators is comprised of a combination of inductances. Since the energy reserve per unit volume of magnetic field is greater by two-three orders of magnitude than the energy stored in a unit volume of the electric field, in high-capacity power installations such oscillating circuits may prove to be effective. It is therefore the goal of the present study to investigate the fundamental possibilities of synthesizing multiphase self-oscillation MHD induction-excitation systems.

Let us present some qualitative concepts which lie at the base of the synthesis of these systems. From the Maxwell equations and from Ohm's law we can derive the relationship

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$$-\frac{\partial}{\partial t}\int\limits_V \left(\frac{\varepsilon \mathbf{E}^2}{2}+\frac{\mu \mathbf{H}^2}{2}\right) dV=\int\limits_V \frac{\mathbf{J}^2}{\sigma} dV-\int\limits_V \mathbf{E}_{st}\mathbf{J} dV, \qquad (1$$

which characterizes the possible transformation of energy in the electromagnetic field wit out any consideration given to the power of radiation. The integration is carried out ove some volume V.

In the notation of (1) we make the assumption that an infinite power is utilized in the liquid-metal circuit of the acceleration device, thus making it possible in the left-h side to eliminate the term characterizing the change in the kinetic energy of the liquid metal. Here, $\mathbf{E}_{st}\mathbf{J}$ contains the term $(\mathbf{v}\times\mathbf{B})\mathbf{J}=-\mathbf{v}(\mathbf{J}\times\mathbf{B})$, characterizing the contribution of en from the moving liquid metal, i.e., the mechanical power generated by the pressure differe For the sake of simplicity, the useful work performed in the active loads of external elec tric circuits is taken into consideration in the integral of the Joule losses.

The solution of this synthesis problem is equivalent to the development of MHD equip- ment in whose working section $\mathbf{J}$, $\mathbf{E}$ and $\mathbf{H}$ are harmonic vector functions. If the vectors E and H are orthogonal, and the ratio of their amplitudes is equal to $\sqrt{\mu/\varepsilon}$, the right-hand side in (1) vanishes. Here the second harmonic in the integral of the Joule losses is off by E_{st}. A similar situation obtains, for example, in single-phase MHD installations with LC oscillation circuit. In this case, the condition of constancy for the energy required from the source is not satisfied. However, if in various portions of the volume V we have symmetrically positioned current $\mathbf{J}_p = \mathbf{J}_0 \sin(\omega t + 2\pi p/N)$, p = 1, ..., N, and if in these same portions of the volume H changes in accordance with the same laws in the absence of E then it is also possible to achieve a situation in which the left-hand side of (1) is equa to zero. Consequently, under symmetrical conditions, given identical shifts in current ph it is possible to achieve constancy in the accumulated magnetic energy in the absence of electrical-energy accumulators. This circumstance serves as the basis for the synthesis of symmetrical self-oscillating multiphase MHD inductive-excitation equipment.

When the energy reserve in the system is constant, then the energy from the external source is also kept constant. In the applied magnetic fields the Faraday and Hall emf or thermo-emf can be used as E_{st}. Having introduced the drift velocities $\mathbf{V}^*$ and $\mathbf{V}_t^*$, produce by the Hall and Nernst effects, we can write that $E_H=(\mathbf{V}+\mathbf{V}^*+\mathbf{V}^*_{st})\times\mathbf{B}_t$ It should be noted that constancy of energy consumption occurs only when developed self-oscillations exist. In the periodic processes the energy stored within the system undergoes change [6]. Thus the multiphase MHD inductive-excitation devices being examined here exhibit the property of equilibrium only at the limit cycles.

Let us examine the synthesis problem on the example of MHD generators (MHDG) exhibiti Hall nonlinearity. Here we will assume that the conditions formulated in [6] have been sa tisfied; under these conditions each MHDG is presented equivalently in the form of a four- amplification element with concentrated parameters. Such an approximation is valid when using an incompressible working fluid, or in the case of a separate pair of sectioned MHDG electrodes with a compressible working fluid. Within the scope of this last case we can examine plasma MHDG, for which the mobility of the charge carriers is rather high. We wi then examine a system consisting of N elementary MHDG, each of which is characterized by Faraday and Hall emf, with corresponding internal resistances. In this case, in the expre sion for E_{st}, the terms with $\mathbf{V}$ and $\mathbf{V}^*$ are retained. In liquid-metal MHDG, where the Hall effect is absent, we have $\mathbf{V}^* = 0$.

In order to achieve the above-cited quantitative relationships governing the distribu tion of currents and magnetic fields, the elementary MHDG can be combined in the following manner. The active resistance R_p and the inductance L_{pn} are connected in series into the Faraday circuit of the elementary MHDG with the number p, and each of these serves to gene ate the magnetic field in the corresponding MHDG with the number n (n, p = 1, ..., N). If the p-th circuit does not affect the magnetic field in the n-th MHDG, then $L_{pn} = 0$. Thus the linkage between the individual circuits is accomplished through the magnetic field and moreover, may be achieved by the connection in series of the individual MHDG in the direct of the Hall current in a circuit with a common direct-current source. The general schema of the multiphase system for the case N = 3 is shown in the figure. Squares denote the e mentary MHDG.

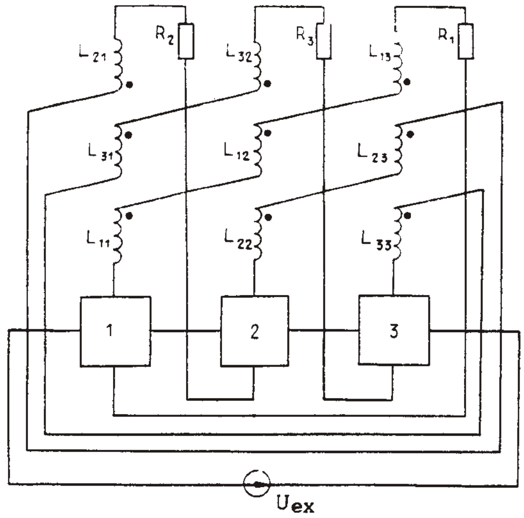

Fig. 1

Let us compile a system of equations to describe the operation of a multiphase system with inductive-excitation MHDG. We will write the Kirchhoff laws for the Hall and Faraday circuits with provision for the expressions of the corresponding emf, derived from Ohm's law for a moving medium:

$$
\begin{gathered}
U_{ex} = -R_H i_H N - k \sum_{n=1}^{N} B_n i_n; \\
R_H = l/(\sigma\Delta h); \quad R_y = h/(\sigma\Delta l); \\
k = \eta/(\sigma\Delta); \\
\left(\sum_{n=1}^{N} L_{pn}\right) \mathrm{d}i_p/\mathrm{d}t + R_p i_p = \\
= -vB_p h + kB_p i_H - R_y i_p; \\
p = 1, \ldots, N,
\end{gathered}
\tag{2}
$$

where U_{ex} is the voltage generated by the external source; i_H is the current in the Hall circuit; i_p ($p = 1, \ldots, N$) are the currents in the Faraday circuits; η is the mobility of the charge carrier; ℓ, h, and Δ denote the length, width, and height of the elementary MHDG. For the sake of simplicity, here we examine the case in which the elementary MHDG are identical. The magnetic induction B_p in the working volume of the p-th MHDG is determined, in the light of the inductive couplings, by the currents in the windings of the corresponding electromagnets. In the case of iron-free magnetic systems we have the following linear relationship:

$$
B_p = -\sum_{n=1}^{N} \chi_{np} i_n; \quad |\chi_{np}| = L_{np}/(w_{np} l h); \quad n, p = 1, \ldots, N,
\tag{3}
$$

where w_{np} is the number of terms in the corresponding winding.

Using expression (3), we can rewrite Eqs. (2) as follows:

$$
\begin{gathered}
\mathrm{d}\mathbf{i}/\mathrm{d}t = \mathbf{A}_1(\mathbf{i})\,\mathbf{i}; \quad v^* = kU_{ex}/(hR_H N); \\
\mathbf{A}_1(\mathbf{i}) = -\mathbf{L}^{-1}\mathbf{R} + h(v + v^*)\,\mathbf{L}^{-1}\mathbf{A} - (k^2/R_H N)\,Q\,\mathbf{L}^{-1}\mathbf{A}.
\end{gathered}
\tag{4}
$$

Here $\mathbf{A}$, $\mathbf{L}$, and $\mathbf{R}$ are the quadratic matrices of order N, in which we find the elements $\delta_{pm} \cdot \sum_{n=1}^{N} L_{pn}$, $\delta_{pm}(R_p + R_y)$ and χ_{mp} (δ_{pm} is the Kronecker delta) at the intersection of the p-th row and the m-th column; $\mathbf{i}$ is the vector column with the components i_n ($n = 1, \ldots, N$); $Q = \mathbf{A}\mathbf{i}\cdot\mathbf{i}$ is quadratic relative to the $\mathbf{i}$ form. The dot identifies a scalar product; $\mathbf{L}^{-1}$ is the matrix that is the reciprocal of $\mathbf{L}$.

The synthesis problem for alternating-current self-oscillators now reduces to an anal sis of system (4). To satisfy the self-excitation conditions it is necessary that the ro of the characteristic equations for the matrix $A_1(0)$ contain multiple roots with a zero r part or roots with a positive real part. The development of the self-excitation process, the possibility for the existence of a stable oscillating regime, and its characteristics these are all governed by the behavior of the quadratic form of Q. Since we are dealing with questions of synthesis in this study, which generates the symmetric currents in all phases, we will subsequently assume that Q is of an elliptical quadratic form. Here the matrix $A_Q=(A+A^t)/2$ of the quadratic form of Q is positively determinate. The superscr t denotes transposition.

We will now examine the positively determinate quadratic form of $Q_1 = Li\cdot i$, which ha the physical sense of the total magnetic energy stored in the system. The change in this reserve of magnetic energy during the operation of the MHD unit is determined from the fo lowing equation:

$$dQ_1/dt=[LA_1(i)+A_1^t(i)L]i\cdot i=F(i)i\cdot i;$$

$$F(i)=2[h(v+v^*)-k^2Q/(R_H N)]A_Q-2R.$$

Since both matrices R and A_Q generate positively determinate quadratic forms, the system be self-exciting with sufficiently large values of $(v + v^*)h$. In this case, the property of el lipticity for Q ensures the self-excitation of the system under the action of perturbatio no matter how small. Satisfaction of the self-excitation conditions is equivalent to the situation in which the right-hand side of Eq. (5) can be evaluated from below in some vic nity about the coordinate origin of the current phase space by some positively determinat quadratic form, i.e., $dQ_1/dt > 0$ when $i \neq 0$, which gives evidence of the accumulation of magnetic energy in the system. Outside of a rather large vicinity about the coordinate o gin the right-hand side of (5) is bounded from above by some negatively determinate quadr form, i.e., in the given region of the phase space we have extinction of the self-oscilla and the return of the accumulated energy to the external sources. Thus, as a result of s excitation trajectory (4) enters some elliptical layer in the phase space. For purposes of synthesizing MHD installations with purely harmonic currents in the phases and with a constant reserve of magnetic energy, additional requirements must be imposed at the limit cycle on the matrices A, R, and L.

First of all, we have to make specific the concept of symmetry. We will examine onl that case in which harmonic currents with equal amplitudes and frequency exist in all pha Here the currents in the adjacent MHDG differ in phase by $2\pi/N$ ($N \geq 3$). When N = 2 we wi regard the regime in which the phase currents are shifted by $\pi/2$ as symmetrical. This de nition of symmetry corresponds to the concept of a symmetrical star-shaped spider of curr in electrical engineering. The limit cycle in the current phase space corresponds to the operating regime of the MHD installation. It follows from (5) that in order to keep cons the reserve of magnetic energy at the limit cycle it is necessary to maintain a constant value for Q, since conversely the right-hand side of (5) would contain fourth-order harmo and would not be equal to zero in the limit cycle. Moreover, in the limit cycle we must satisfy the identity $Ri\cdot i \equiv$ const, and since R is a single-diagonal matrix, then for the existence of a symmetrical spider of phase currents we have the necessary condition $R_1 = R_2 = \ldots = R_N = R$, i.e., $R = (R + R_y)E$, where E is a unit matrix. Analogously, the const of Q in the limit cycle is equivalent to the condition L = LE, where L denotes the total inductance in the circuit of each phase of the symmetrical installation.

It follows from the derived conditions for R and L that on the limit cycle the matri of system (4) is constant and representable in the form $A_2=aA-bE$ where a and b are cer positive numbers. Since the vector function of the harmonic currents of the limit cycle is a solution for the system of linear differential equations with matrix A_2, the narrowi of the matrix A_2 to a minimum linear space containing the limit cycle is representable by an obliquely symmetrical matrix. Further, in accordance with the requirement of stabilit for the limit cycle we have to define the matrix A so that the spectrum of the constricti of A_2 to the orthogonal addition to the subspace with the limit cycle contains no eigenva with a positive real portion. As a result we can formulate the general requirement (condi tions) with respect to the parameters of system (4) under which synthesis of the symmetri ly multiphase self-oscillation installation is possible on the basis of elementary MHDG:

I) the active resistance $R_p + R_y$ and the total induction $\sum_{n=1}^{N} L_{pn} (p=1, \ldots, N)$ in the circuits of all phases are identical;

II) the matrix A with accuracy to the nondegenerate transformation is representable in the form of the sum of the unit matrix with a positive coefficient and some block-diagonal matrix; at least one block of the last matrix is obliquely symmetrical and corresponds to the minimum linear subspace containing the limit cycle; the other blocks contain a spectrum that has no eigenvalues with a positive real part;

III) the positive coefficient for the unit matrix in the expression for A must exceed the value of $R + R_y$, which is necessary to ensure the self-excitation of the MHD device.

In the presence of mutual inductances in the excitation windings within the matrix L nonzero elements outside of the principal diagonal appear, and these have the sense of the total mutual inductance between the corresponding Faraday circuits. The matrix L retains its nondegenerate symmetry, while the quadratic form of Q_1 has the sense of the total magnetic energy of the system. Equation (5) is also valid for the modified quadratic form of Q_1. Consequently, the conditions of equality for the active resistances in the phases and the constancy of Q on the limit cycle are retained. The conditions imposed on the matrix A_2 are also maintained. Condition III also remains without change. The condition of equal inductances in I must be replaced with the analogous condition of compensation for the second harmonics in Q_1 on the limit cycle.

As an example of the application of the derived conditions, let us examine the problem of synthesizing balanced self-oscillating three-phase MHDG. Under the condition of equality of the excitation windings we have studied three particular cases in which the matrix A has the form

$$A_3=\begin{pmatrix} \chi & -\chi & \chi \\ \chi & \chi & -\chi \\ -\chi & \chi & \chi \end{pmatrix}; \quad A_4=\begin{pmatrix} \chi & 0 & -\chi \\ -\chi & \chi & 0 \\ 0 & -\chi & \chi \end{pmatrix}; \quad A_5=\begin{pmatrix} \chi & 0 & \chi \\ \chi & \chi & 0 \\ 0 & \chi & \chi \end{pmatrix}.$$

The polarity of the winding connections, indicated by the dots in the figure, corresponds to the case $A=A_3$. Here conditions I and II are satisfied and in the case of self-excitation (condition III is satisfied) in the phases of the MHDG stable harmonic currents of frequency $\omega = \sqrt{3}(R + R_y)/L$ arise, and these form a symmetrical three-phase spider. When $A=A_4$, the qualitative behavior of the system in the limit cycle and its stability are conserved. However, the self-oscillation frequency diminishes and amounts to $\omega = (R + R_y)/\sqrt{3}L$. When $A=A_5$ system (4) also exhibits a limit cycle to which the symmetrical harmonic self-oscillations of the phase currents correspond. However, in this case condition II is not satisfied: in the block-diagonal portion of the matrix, similar to A, we find a block with a positive eigenvalue. The self-oscillations with a frequency of $\omega = \sqrt{3}(R + R_y)/L$ are therefore unstable.

When there is no Hall effect, as is the case for liquid-metal generators and pumps, the above-cited rules of synthesis are preserved in connection with the linearized portion of system (4). Let us examine the three-phase liquid-metal conduction MHDG, whose excitation windings are connected as shown in the figure, i.e., for the matrix A we have chosen a matrix of the A_3 type. Individual sections in each MHDG are connected by means of an intermediate transformer to the excitation windings and the load [3]. We will assume that the elementary MHDG are hydraulically connected in series and form a liquid-metal contour, with two elementary MHDG for each of the three phases, and to compensate the eddy currents in the primary winding of the transformer these elements are situated within the general magnetic field and are opposed to the direction of the liquid-metal flow.

Following the fomulated conditions, we will assume that $R_1 = R_2 = R_3 = R$, $L_{11} = L_{22} = L_{33} = L$, $L_{12} = L_{13} = L_{23} = L_{21} = L_{31} = L_{32} = L_1$. It is assumed that each elementary MHDG has dimensions of $\Delta \times h \times \ell = 6 \times 16 \times 150$ mm³ and is filled with liquid potassium heated to a temperature of 550°C, with a specific electrical conductivity of $2.05 \cdot 10^6$ S/m and a density of 716 kg/m³. The walls of each MHDG are 0.1 mm thick and they have a specific electrical conductivity of $0.6 \cdot 10^6$ S/m. The transformer ratio is assumed to be equal to 1. In this case, when k = 0, the condition for the development of the oscillations $R + R_y = Lv/(w\ell)$ makes it possible, with consideration of the coupling $L = \mu_0 h \ell w^2/\Delta$, to obtain the relationship for the critical number of turns $w = (\alpha + 1)/(\mu_0 \sigma v \ell)$ in the excitation windi with an inductance L. Here, for the frequency of the self-oscillations the following rep sentation is valid: $\omega = (v/\ell w)[\sqrt{3}w_0/(1 + 3w_0^2)]$. Here $\alpha_1 = R/R_y$ is the load parameter, $w_0 = w_1/w$, w_1 is the number of turns in the excitation winding, with inductance L_1. The three-phase liquid-metal MHDG is designed with allowance made for friction, the shunting action of the electrically conducting walls, the terminal effects, eddy currents within t channels, the exponential velocity profile, scattering within the transformer, and the ex citation system. With v = 50 m/sec, $w_0 = 0.577$ and $\alpha = 4$ we obtain the self-oscillation frequency ω = 50 Hz, and the hydraulic and useful power are 17.6 and 8 kW in phase, re spectively, with an efficiency of 45.5%.

If in the place of R_1, R_2, and R_3 we introduce into the installation being examined here a three-phase voltage source with an operating frequency ω, then the current in each phase corresponds in phase with the voltage, i.e., such a pump will have $\cos \varphi = 1$. With a grid frequency lower than the resonance frequency, we have a lag angle φ and in the opp site case we have a lead angle.

Thus the fundamental results of this study reduce to the formulation of principles o synthesizing self-oscillation MHD devices with inductive excitation. It is demonstrated that the compensation of the reactive power can be achieved without utilization of traditional contours. The formulated synthesis principles demonstrate the possibility of designing multiphase conduction liquid-metal MHD generators and pumps requiring no reactive power from the source at the resonance frequency. It is demonstrated that in the case of plasma MHDG with clearly defined Hall effect synthesis of purely harmonic self-oscillatio systems is possible, these retaining a constant reserve of energy in the electromagnetic field. In the case of liquid-metal MHDG the nature of establishing the self-oscillations and the extent of their deviation from the harmonic are determined by the predominant non linearity in the system [3, 4]. It should also be noted that on the basis of the above-cited principles of synthesis it is possible to design plasma and semiconductor current c verters in the absence of motion of the working fluid when the mobility of the charge car riers is high and without utilization of barrier elements.

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