THE ELECTRICAL RESISTANCE OF A LIQUID-METAL CURRENT-CARRYING JET

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UDC 537.84:532.522

Liquid-metal current-carrying jets are used in a number of technological processes [1, 2] and devices [3]. For the stable functioning of such devices it frequently becomes necessary to maintain a constant current magnitude within the jet. However, this is hindered by the development of MHD instabilities which periodically cause the jet to alter its shape and, as a consequence, the magnitude of its electrical resistance. This may lead to significant fluctuations in the current flowing through the jet. However, when the current reaches some critical value, the length of the jet segment that has not been disrupted [4, 5] may become smaller than the specified jet length, which results in the complete breakdown of the jet and to the disappearance of the current flowing through the jet, or to a discontinuity in the jet (on ignition of an electric arc at the point of separation).

As is well known [6], two types of MHD instabilities form on a jet: constricted and helical. However, as our estimates show, the increments in one or another type of instability are almost identical. This indicates that at the instant at which the constricted instability leads to the separation of the jet, the helical instability results only in the jet's deviation from the unperturbed position by a magnitude on the order of the jet radius. This is clearly illustrated in the photograph shown in Fig. 1. These concepts make it possible for us, in the mathematical simulation of the evolution of electrical resistance in a liquid-metal current-carrying jet, to limit ourselves to an examination exclusively of the constricted MHD instability. A detailed investigation was undertaken in [7] into the effect exerted by the constriction-type MHD instability on the electric resistance of a liquid-metal cylinder formed in the electrical explosion of a thin wire, and the authors of this study will adhere to this methodology in the following. However, the development of constriction-type MHD instabilities on a liquid-metal cylinder such as that represented by a melted wire, and on a liquid-metal jet, despite its many common characteristics, exhibits unique differences. All constrictions developed on a liquid-metal cylinder "last" for the same length of time. Therefore, the liquid-metal cylinder will break apart into several liquid-metal bridges after a certain period of time, and these bridges will be connected to each other by arcs. However, in the liquid-metal jet the perturbations that originate in the nozzle forming the jet begin to develop at the instant that the jet leaves the nozzle and they are then carried in the direction of jet motion. With a finite jet length it is the constriction farthest removed from the nozzle that is most developed (see Fig. 1); it is precisely in the region of this constriction that we find the maximum probability of arc ignition. In the last stage of constriction development, it is here that we find a particularly intensive increase in the electrical resistance of the jet, since in addition to the pronounced reduction in the cross-sectional area at the point of the constriction we have strong heating of the liquid metal, thus increasing its resistivity.

The situation described here is confirmed qualitatively by oscillograms showing the voltage drop across the jet as a function of time (Fig. 2). The measurements were conducted on a jet formed by the eutectic alloy Ga—In—Sn 10 cm in length, flowing out of a vertically situated nozzle 5 mm in diameter, with an industrial current of 357 A. The parameters of the electrical circuit were chosen so as to produce a current governed by the resistance of that segment of the circuit external to the jet. The experiment was conducted on an installation described in detail in [6].

Using the linear model from [4] let us examine the change over time in the electrical resistance of the liquid-metal current-carrying jet. We will look at the evolution of jet resistance brought about by the development of a single perturbation harmonic with the wave number k, corresponding to the maximum increment (a detailed examination can also be undertaken for perturbations of arbitrary shape, expanded into Fourier series).

We will neglect the natural submersion of the free-falling vertical jet that comes about as a consequence of the force of gravity, i.e., we will limit our examination to the horizontal jets, and to jets exhibiting high discharge velocities out of the nozzle, and to jets that are short in the sense that the increments in velocity during the time of fluid motion outside of the nozzle are small in comparison to the initial discharge velocity of the jet. Moreover, we will assume that

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Translated from Magnitnaya Gidrodinamika, Vol. 26, No. 1, pp. 93—97, January—March, 1990. Original article submitted January 5, 1989.

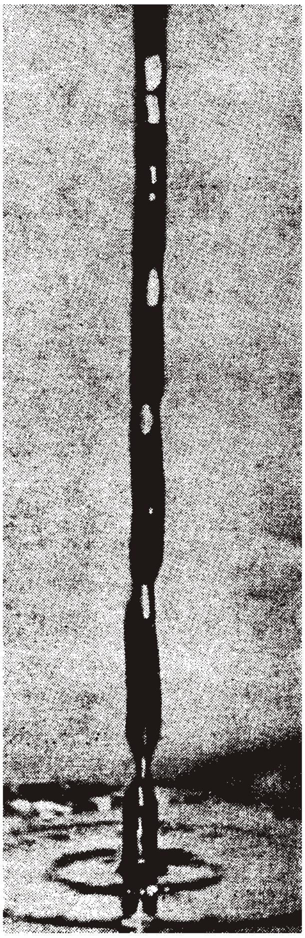

Fig. 1. An illustration of developed constricted and helical instabilities in a liquid-metal jet.

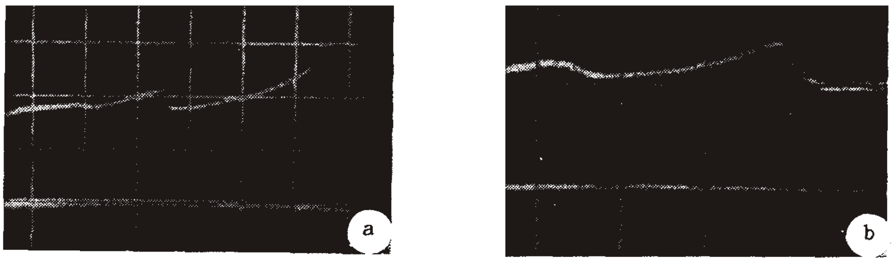

Fig. 2. Oscillograms of voltages applied to a liquid-metal jet with a current leading to (a) and not leading to (b) the destruction of the jet and the ignition of an electric arc (in case a). The time divisions represent 5 msec.

in each cross section of the jet the current is distributed uniformly through that cross section, i.e., we will neglect the skin effect.

We will further assume that the heating of the liquid metal proceeds adiabatically. This assumption can be validated by the fact that in this phase of the heating, where the temperature of the metal in the constriction zone becomes sufficiently high for significant exchange of heat to occur between the metal in the jet and the ambient medium, the rate of subsequent heating and destruction of the jet is quite high.

Fig. 3

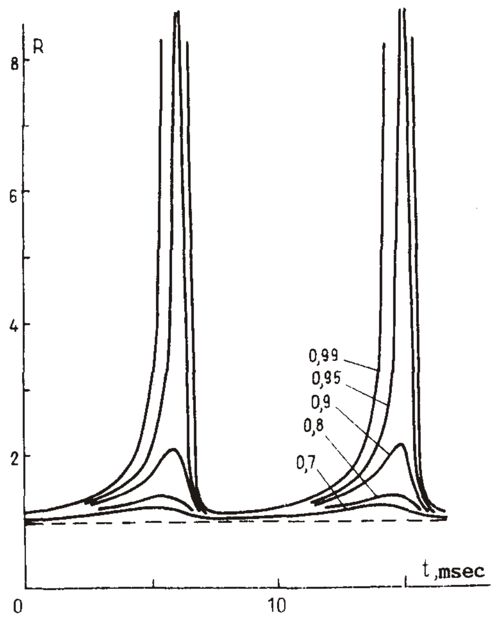

Fig. 4

Fig. 3. Formulation of the problem.

Fig. 4. Relative jet resistance as a function of time for values of ξ_{max} indicated in the figure.

Let us now examine a liquid-metal jet within a coordinate system whose z axis is directed along the axis of the jet in the direction of flow, with the coordinate origin (z = 0) coinciding with the outlet of the nozzle (Fig. 3). The jet length is equal to L. We will assume that in this coordinate system a sinusoidal axisymmetric perturbation ξ is formed at the surface of the jet, with an increment δ, a wave number k, an initial amplitude ξ_0, and an initial phase φ_0:

$$\xi = \xi_0 \sin\left(k(z+v_0 t)+\varphi_0\right) \exp\left(\delta z/v_0\right),$$

where v_0 is the jet discharge velocity. In this expression the magnitudes of the perturbations have been made dimensionless with respect to the unperturbed radius of the jet. The form of the expression for ξ reflects the fact that the perturbations are not propagated in the direction of the jet, i.e., the fixed point of the jet with coordinate $z = z_0 - v_0 t$ exhibits the same phase at any instant of time t.

In calculating the resistance of the liquid-metal jet it is necessary to take into consideration the fact that due to Joule heating the specific electrical resistance of the liquid metal is increased. The heating of the metal proceeds with particular intensity at the points of constriction. In order to account for the relationship between the specific resistance and the temperature, we will employ the method proposed in [7].

With consideration of the adopted assumptions we can write the quantity of heat dQ evolved by a unit volume of liquid metal within time dt:

$$dQ = j^2 \rho\, dt, \tag{1}$$

where j is the current density in the given cross section of the jet; ρ is the specific electrical resistance of the liquid metal in this cross section. The expression for the density of the electric current in the jet has the form

$$j = j_0/(1-\xi)^2,$$

where ξ is the dimensionless perturbation of the surface; j_0 is the current density in the unperturbed jet.

We know that

$$\rho = \rho_0(1+\beta Q), \tag{2}$$

where $\beta = \alpha/(cd)$; α, c, and d represent the temperature coefficient for the change in resistance, the specific heat capacity, and the density of the liquid metal. From (1) and (2) we obtain the equation

$$dQ/(1+\beta Q) = \rho_0 j^2 dt.$$

Having carried out the integration, we find

$$\rho = \rho_0(1+\beta Q) = \rho_0 \exp\left(\rho_0\beta \int_0^{t_1} j^2 dt'\right). \tag{3}$$

The time t_1 during which the heating takes place will be reckoned from the instant at which the liquid metal is discharged from the nozzle. Thus,

$$t_1 = z/v_0. \tag{4}$$

Now, in expression (3) we will make the transition from integration over time to integration over the magnitude of the perturbation ξ. For this we will turn to the coordinate system y, t, connected to the jet and moving together with the jet: $y - y_0 = z - v_0 t$, where $ky_0 = \varphi_0$. Then the expression for the dimensionless perturbation assumes the form

$$\xi = \xi_0 \sin ky \exp\{[y-(y_0-v_0 t)]\delta/v_0\},$$

which allows us to write

$$t = \frac{1}{\delta}\ln\frac{\xi}{\xi_0 \sin ky} + \frac{y_0-y}{v_0}.$$

With consideration of (4) we will turn in expression (3) to integration over ξ:

$$\rho(y,t) = \rho_0 \exp\left[\frac{\rho_0\beta j_0^2}{\sigma}\int_{\xi_0 \sin ky}^{\xi}\frac{d\xi}{\xi(1-\xi)^4}\right]. \tag{5}$$

The integral in the right-hand side of (5) is taken in quadratures:

$$\int_{\xi_0 \sin ky}^{\xi}\frac{d\xi}{\xi(1-\xi)^4} = \left[\ln\frac{\xi}{1-\xi} + \frac{3\xi}{1-\xi} + \frac{3\xi^2}{2(1-\xi)^2} + \frac{\xi^3}{3(1-\xi)^3}\right]_{\xi_0 \sin ky}^{\xi} \tag{6}$$

Knowing the cross-sectional area $S(y, t) = \pi r_0^2(1-\xi)^2$ of the jet, we can calculate the magnitude of the resistance for the entire jet:

$$R(t) = \int_{y_0-v_0 t}^{y_0-v_0 t+L}\frac{\rho(y,t)}{S(y,t)}dy. \tag{7}$$

Having substituted expression (5) into (7), and with consideration of (6), we obtain the relationship from which, by means of numerical integration, we find the relationship between the electrical resistance of the jet and time.

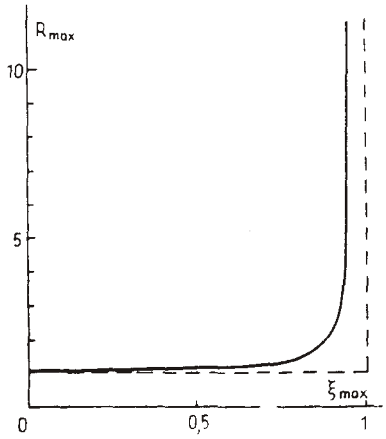

Fig. 5. Relative maximum jet resistance as a function of ξ_{max}.

Figure 4 shows the resistance of the jet as a function of time for various initial perturbation amplitudes. The amplitudes have been selected so that the maximum perturbation ξ_{max} lies within the limits $0 \leq \xi_{max} < 1$. Comparison of experimental (see Fig. 2) and theoretical (see Fig. 4) relationships between voltage and time yields their qualitative coincidence. Figure 5 shows the maximum electrical resistance of the jet (referred to the resistance of the unperturbed cylinder) as a function of the maximum magnitude of the perturbation that can develop on the jet (referred to the radius of the unperturbed jet).

In comparing the theoretical results with the experimental data, it is necessary to take into consideration that under real conditions the perturbations are random in nature; the jet of liquid metal in the experimental installation moves downward, and this leads to its submersion, for which no provision was made in the model. These factors must be taken into consideration in utilizing the described results, as well as the traditional errors of the linear model. Nevertheless, the constructed mathematical model can be used for qualitative description of the electrical resistance of a current-carrying jet of an electricity-conducting fluid.

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