MAGNETIC FIELD DUE TO THERMOELECTRIC CURRENTS IN A FAST REACTOR

A. Gailitis and Ya. Freiberg

UDC 537.84

Equivalent-circuit calculations have been used [1] to estimate the thermoelectric currents arising in the liquid metal coolant in a fast reactor. The reactor tank with the first loop and core can be taken to resemble a continuous short-circuited conductor with inhomogeneous conductivity, and the bulk thermoelectric currents can be estimated from the description elements for continuous media.

The reactor core is considered approximately a volume uniformly penetrated by long thin vertical stainless-steel rods immersed in liquid sodium. Part of the volume is occupied by nonconducting fuel rods, and the structure is characterized by the filling coefficients Ω_{St}, Ω_{Na}, Ω_f correspondingly (the proportions of the core volume occupied by steel, sodium, and fuel). A constant temperature gradient along the rids is maintained throughout the core. The core as a whole is immersed in an larger tank for the first loop, which in the main is filled with sodium.

A thermo-emf is produced at the vertical contact surfaces between the sodium and steel, and the vertical gradient in the temperature T results in a current with density

$$\mathbf{j}=\sigma(-\nabla\varphi+S\nabla T),$$

in which φ is potential σ conductivity, and S the thermal potential.

These currents are identified [1] by measuring the magnetic field outside the core, but in the core, the currents in the steel and sodium parts are mainly in opposite directions, and the magnetic field they produce does not extend very far. Part of the current circulates in the external tank, which gives a nonzero mean current in the core and results in a large-scale magnetic field outside it.

We consider a characteristic core element (Fig. 1) constituted by the longitudinal contacting volumes of steel and sodium. The transverse dimension is much less than the length h, which is equal to the height of the core. Along the element there is a constant temperature gradient $\tau \equiv dT/dz = \Delta T/h$, in which z is the coordinate along the element and ΔT the temperature difference between the ends. The ends of the sodium part of the element are joined to the external passive circuit.

We have a one-dimensional treatment for the longitudinal current within the element because of its highly elongated form. The following equations describe the current within the sodium (subscript Na) and the steel (St) $\mathbf{j}_{Na} = -\sigma_{Na}\nabla\varphi_{Na}$, $\mathbf{j}_{St} = \sigma_{St}(-\nabla\varphi_{St} + E)$, where $E = (S_{Na}-S_{St})\tau$, together with the boundary conditions at the contact surface between the sodium and steel:

$$j_{Na}/\sigma_{Na}-j_{St}/\sigma_{St}=E \quad \text{and} \quad \varphi_{Na}=\varphi_{St}$$

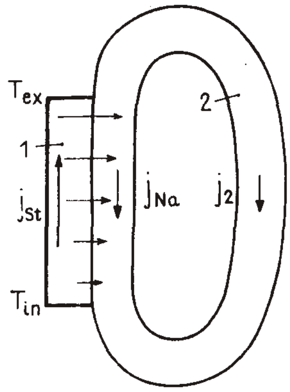
with the solution $\varphi_{Na} = \varphi_{St} = \varphi \equiv \text{const } z$. The mean current density in the element $j = \Omega_{Na}j_{Na} + \Omega_{St}j_{St}$ differs from zero if the potential difference produces a current in the external circuit.

The core consists of a set of such elements, and the linear solution applies for the potential and current averaged over the elements if a linear vertical potential distribution applies at the outer surface with bulk current flow outside the core. Here one can use the familiar electrostatic solution for polarization of an ellipsoidal body [2] when the shape of the body provides a uniform electric field within the volume and a continuous transition to the potential field outside the body.

The solution is particularly simple if one assumes that the core is an ellipsoid of rotation, while the surrounding tank is a confocal ellipsoid (Fig. 2).

Translated from Magnitnaya Gidrodinamika, No. 4, pp. 96-100, October-December, 1992. Original article submitted March 17, 1992.

 0024-998X/92/2804-0404$12.50



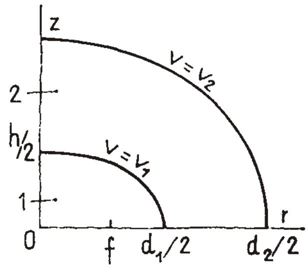


Fig. 1

Fig. 2

Fig. 1. Formulation.

Fig. 2. Ellipsoidal section of more model.

We use a cylindrical coordinate system z, ρ, φ and elliptical one v, u, φ; for a flattened ellipsoid we have

$$z = f\,\mathrm{ch}\,v\cos u,\quad \rho = f\,\mathrm{sh}\,v\sin u,$$

and for an elongated one

$$z = f\,\mathrm{sh}\,v\cos u,\quad \rho = f\,\mathrm{ch}\,v\sin u,$$

in which the focal distance f is determined by the height ha and diameter d_1 of the core ellipsoid (see Fig. 2).

The current distribution in this axisymmetric case is put in terms of the current function $\Psi(z, \rho)$, which is related to the azimuthal magnetic field by $B = \mu_0\Psi/\rho$. The solution is written as closed expressions [3] and amounts to satisfying the continuity of the potential and current function (normal component of the current density) at the surface of the inner ellipsoid,

$$\varphi_1 = \varphi_2 \text{ and } \Psi_1 = \Psi_2 \quad \text{for} \quad v = v_1,$$

and the conditions for the current not to penetrate the outer boundary:

$$\Psi_2 = 0 \quad \text{for} \quad v = v_2.$$

The subscript 1 denotes the interior region and subscript 2 the outer one. The limiting values of the coordinates v_1 and v_2 are defined by h and d_1 together with the diameter d_2 of the tank in accordance with Fig. 2.

The distribution in the core is independent of h/d_1:

$$\varphi_1 = C_1 z,\qquad \Psi_1 = (-\sigma_1 C_1 + \alpha)\rho^2/2,$$

in which $\sigma_1 = \Omega_{Na}\sigma_{Na} + \Omega_{St}\sigma_{St}$, $\alpha = \Omega_{St}\sigma_{St}(S_{Na} - S_{St})\tau$. The solution in the external medium is

$$\varphi_2 = [C_2 + D_2 U(v)]z,\qquad \Psi_2 = -\sigma_2[C_2 + D_2 Q(v)]\frac{\rho^2}{2}.$$

The boundary conditions give

$$C_1 = \frac{\alpha}{\sigma_1 + \varkappa\sigma_2},\qquad D_2 = \frac{C_1}{U(v_1) - Q(v_2)},\qquad C_2 = D_2 Q(v_2),$$

$$\varkappa = -\frac{Q(v_1) - Q(v_2)}{U(v_1) - Q(v_1)}.$$

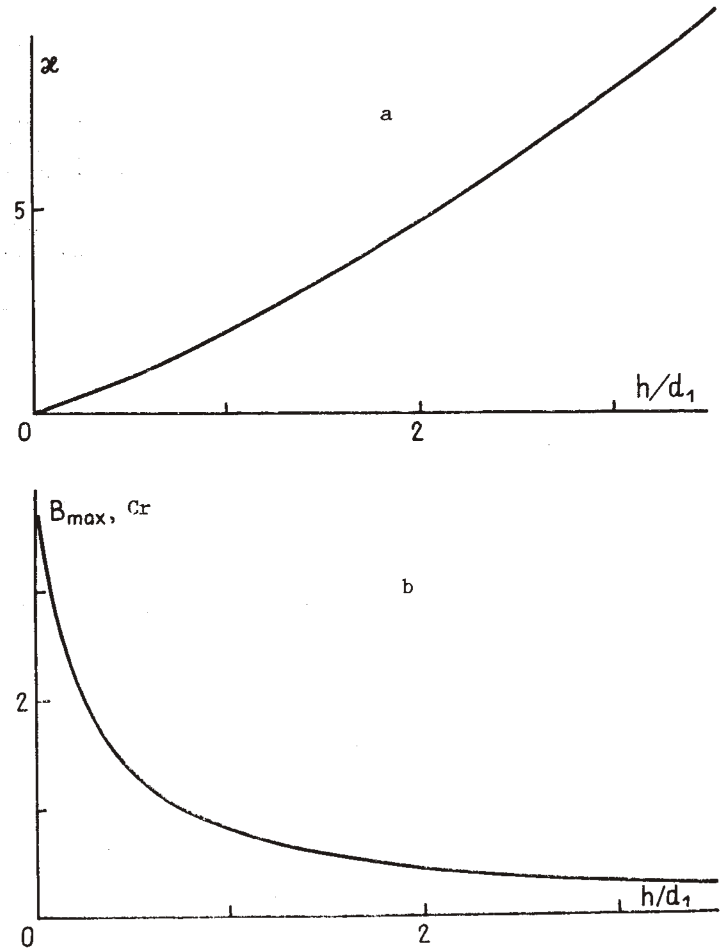


Fig. 3. External resistance coefficient $\varkappa$ (a) and maximum field B_{max} (b) as functions of relative core height in the BN-600 reactor.

The expressions for U(v) and Q(v) are dependent on the shape of the ellipsoid. For a flattened one, we put λ = sh v and have

$$U = \operatorname{arcctg}\lambda - 1/\lambda, \qquad Q = \operatorname{arcctg}\lambda - \lambda/(1-\lambda).$$

For an elongated one (λ = ch v),

$$U = \ln\frac{\lambda+1}{\lambda-1} - \frac{2}{\lambda}, \qquad Q = \ln\frac{\lambda+1}{\lambda-1} - \frac{2\lambda}{\lambda^2-1}.$$

These expressions contain the following in the case of sphere for f → 0:

$$U = r^{-3}, \; T = -2r^{-3}, \text{ where } r = \sqrt{z^2+\rho^2} \text{ is the spherical radius.}$$

Here $\varkappa$ is the ratio of the effective conductivity in the outer tank to the conductivity in the core and is dependent on h/d_1 (Fig. 3a) and on d_1/d_2, the diameter ratio for the external and internal ellipsoids. In the highly flattened case (thin disk $h \ll d_1$), the internal resistance is small by comparison with the external ($\varkappa \to 0$), while in the highly elongated case ($h \gg d_1$), the total current in the main is restricted by the internal resistance ($\varkappa \to \infty$). For a sphere ($h = d_1$) in an unbounded tank ($d_2 \gg d_1$) we have $\varkappa = 2$. The external nonconducting boundary in a real reactor is far away by comparison with the dimensions of the core, so the reduction in $\varkappa$ due to that boundary is slight. A fairly smooth variation in $\varkappa$ over a wide range in h/d_1 indicates (Fig. 3a) that these magnetic-field estimates apply for a more accurate model for the core and tank. B_{max} is only slightly sensitive to the shape of the model (Fig. 3b), which is also an argument for the estimate.

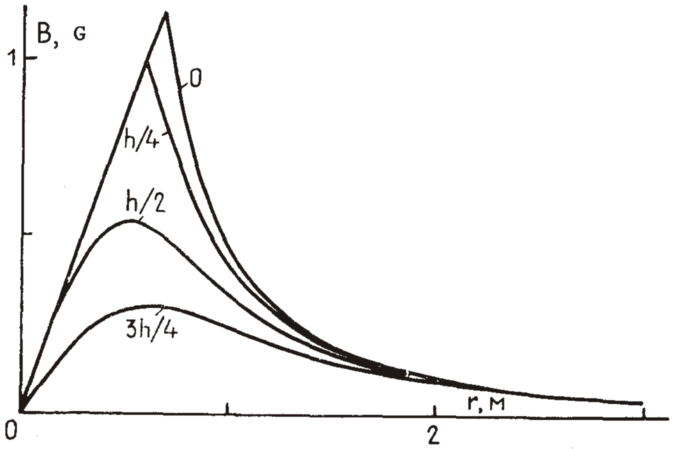


Fig. 4. Radial magnetic field distributions at the heights indicated on the curves in the BN-600 model.

The core field increases linearly with the distance from the symmetry axis. Outside the core, the field is close to that from a current dipole and decreases to zero at the outer boundary of the tank. The maximum field occurs at the outer boundary of the core ($\rho = d_1/2$, $z = 0$):

$$B_{max} = \mu_0 \frac{\alpha\sigma_2\Omega_{St}\sigma_{St}(S_{Na}-S_{St})\Delta T d_1}{4(\Omega_{St}\sigma_{Na}+\Omega_{St}\sigma_{St}+\varkappa\sigma_2)h}.$$

With a constant temperature difference between the lower and upper parts of the core (ΔT = const) and with a constant shape for the core (h/d_1 = const), the maximum field is independent of the core size. We substitute the input data corresponding to an average fast reactor such as the BN-600: $h = 0.75$ m; $d_1 = 2.0$ m; $d_2 = 16$ m; $\Omega_{Na} = 0.32$; $\Omega_{St} = 0.22$, average $T = 463°C$; $\Delta T = 170°C$; $S_{Na}-S_{St} = 9\ \mu V/°C$; $\sigma_{Na} = 4 \cdot 10^6\ \Omega^{-1}/m$; $\sigma_{St} = 10^6\ \Omega^{-1}/m$; $\sigma_2 = \sigma_{Na}$, to get the following: $B_{max} = 1.78$ G, difference in average potential between the top and bottom poles of the core $\Delta\varphi_{max} = 86\ \mu V$, total current through core 892 A, and external resistance coefficient $\varkappa = 0.651$.

Figure 4 shows the field distributions at various heights z = const above the horizontal symmetry plane z = 0 as functions of distance from the symmetry axis; the maximum field decreases as the height increases. However, the field is considerable even at levels above the top of the core ($z > h/2$).

These field estimates show that the field is sufficient for one to identify the thermoelectric currents against the background of the Earth's magnetic field, although the field is less by an order of magnitude than the [1] estimate.

LITERATURE CITED

1. I. M. Kirko, G. E. Kirko, A. G. Sheinkman, and M. T. Telichko, "Thermoelectric currents in the BN-600 reactor at Beloyarsk nuclear power station," Dokl. Akad. Nauk SSSR, **266**, No. 4, 854-856 (1982).
2. L. D. Landau and E. M. Lifshits, Electrodynamics of Continuous Media, 2nd edition [in Russian], Moscow (1982).
3. V. V. Boyarevich, Ya. Zh. Freiberg, E. I. Shilova, and E. V. Shcherbinin, Electrovortex Flows [in Russian], Riga (1985).