

AN AC CONDUCTION PUMP WITH INDUCTIVE REACTIVE-POWER COMPENSATION

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1. Alternating-current conduction pumps are fully competitive for low hydraulic power [1, 2] but have a major shortcoming: the low $\cos \varphi$ due to the unfavorable relation between the resistive and reactive power levels. If one considers such a pump without allowance for the end effects with the armature reaction compensated, one can neglect the eddy-current losses by comparison with the input power to get

$$\operatorname{tg} \varphi = \frac{P_r}{P_a} = \frac{1}{q} \frac{V'}{Rm} \frac{(1+k)^2}{k}; \quad \eta_e = \frac{P_h}{P_a} = \frac{1+k}{k}.$$

Here P_r is the reactive power in producing the pulsating magnetic field, P_a the active power supplied to the channel, P_h the hydraulic power developed by the pump, $q = v/\omega l$ the quasistationarity parameter (v velocity of liquid metal in channel, l channel length), $Rm = \mu_0 \sigma v l$ the magnetic Reynolds number, V' and V_c the volumes of the nonmagnetic gap in which the magnetic field is produced and of the channel, $k = U_c/(E-U_c)$ the load coefficient ($k < -1$), U_c the voltage applied to the channel, $E = vBh$, and h channel width. The other symbols are those generally used.

The tasks of optimizing the efficiency and $\cos \varphi$ with respect to the load parameter conflict one with the other. One can balance the low $\cos \varphi$ only with devices external to the channel. In a conduction or induction pump, one balances out the reactive power usually by means of additional capacitors, but inductive reactive power compensation is also possible [3]. Oscillatory modes have been considered [4, 5] for a liquid-metal conduction generator with inductive or capacitive compensation of the reactive power.

Figure 1a shows the theoretical circuit for a two-phase pump with inductive compensation, which is described by

$$\begin{aligned} Ri_1 + \omega_2 lh \frac{dB_1}{dt} + \omega_1 lh \frac{dB_2}{dt} + B_1 vh &= U \sin \omega t; \\ Ri_2 + \omega_2 lh \frac{dB_2}{dt} - \omega_1 lh \frac{dB_1}{dt} + B_2 vh &= U \cos \omega t \end{aligned} \quad (1)$$

and by the feedback equations, which are analogous to those given in [3],

$$B_1 = \frac{L_2}{\omega_2 hl} i_1 - \frac{L_1}{\omega_1 hl} i_2; \quad B_2 = \frac{L_2}{\omega_2 hl} i_2 + \frac{L_1}{\omega_1 hl} i_1. \quad (2)$$

Here L_1 and L_2 are the inductances of the additional and main windings correspondingly, w_1 and w_2 are the numbers of turns on those windings, R the resistance, which includes the internal resistance in the channel and the resistance in the exciting windings, and B_1 and B_2 the magnetic inductions in the channels.

It follows from (1) and (2) that

$$\begin{aligned} (L_1 + L_2) \frac{di_1}{dt} + \left(R + \frac{L_2 v}{\omega_2 l} \right) i_1 - \frac{L_1 v}{\omega_1 l} i_2 &= U \sin \omega t; \\ (L_1 + L_2) \frac{di_2}{dt} + \left(R + \frac{L_2 v}{\omega_2 l} \right) i_2 + \frac{L_1 v}{\omega_1 l} i_1 &= U \cos \omega t. \end{aligned} \quad (3)$$

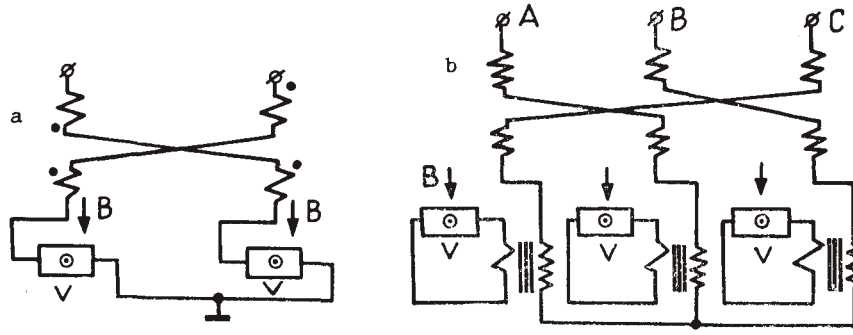


Fig. 1. Schemes for two-phase pump (a) and three-phase pump (b) with inductive reactive-power compensation.

We eliminate i_2 in (3) and get in the linear approximation

$$\frac{d^2 i_1}{d\tau^2} + \frac{R}{\rho} (1 + \kappa) \frac{di_1}{d\tau} + i_1 = \bar{U} \sin(\nu\tau + \theta), \quad (4)$$

in which

$$\begin{aligned} \tau &= \omega_0 t; \quad \nu = \omega / \omega_0; \quad \omega_0 = [(L_1 v / \omega_1 l)^2 + R^2 (1 + \kappa)^2]^{1/2} / (L_1 + L_2); \\ \rho &= \frac{1}{2} (L_1 + L_2) \omega_0; \quad \bar{U} = \frac{U_V}{\omega_0 (L_1 + L_2)} [(1 + \kappa_2)^2 + \kappa_1^2]^{1/2}; \\ \theta &= \text{arctg} \frac{1 + \kappa_2}{\kappa_1}; \quad \kappa = L_2 v / (\omega_2 l R); \\ \kappa_1 &= R (1 + \kappa) / \omega (L_1 + L_2); \quad \kappa_2 = \frac{L_1 v}{\omega (L_1 + L_2) l \omega_1}. \end{aligned}$$

The current equation takes a form analogous to (4) in a pump with capacitive compensation, but $\omega_0 = 1/\sqrt{LC}$; $\rho = \sqrt{L/C}$; $\bar{U} = \omega CU$. Consequently, in the case of inductive compensation, there is a tuned circuit, only with parameters somewhat different from an ordinary LC circuit. Then (4) gives the frequency response in the linear approximation without allowance for the eddy currents and three-dimensional effects.

The phase shift is as follows between current and voltage in a pump with inductive compensation: $\varphi = \theta + \theta_1 - \pi/2$, in which $\theta_1 = \text{arctg}[(\kappa_1^2 + \kappa_2^2 - 1)/(2\kappa_1)]$. In the case of practical interest $\kappa_1 \ll 1$, the phase shift is zero for $\kappa_2^2 - 1 = 0$, which corresponds to the frequency of the voltage in phase $\omega = \omega_1 = L_1 v / [\omega_1 l (L_1 + L_2)]$; here $\omega_0 \approx \omega_1$. From (3) we get the complex current amplitudes as

$$I_1 = \frac{\bar{U}}{L_2 v / \omega_2 l + R} \frac{1 - iN(1 - \omega_1/\omega)}{1 + [N(1 - \omega_1/\omega)]^2}; \quad I_2 = iI_1, \quad (5)$$

in which $N = 1/\kappa_1$.

When the frequency of the external supply is $\omega = \omega_1$, there is resonance and the current coincides in phase with the voltage, and no reactive power is drawn from the source. For $\omega > \omega_1$, the pump is an inductive load for the line and requires power to produce the magnetic field; and for $\omega < \omega_1$, the pump supplies reactive power to the line, i.e., acts as a reactive power compensator.

The complex current amplitude is as follows for a pump with capacitive compensation:

$$I = \frac{\bar{U}}{L v / \omega l + R} \frac{1 - iN_1(1 - \omega_0^2/\omega^2)}{1 + [N_1(1 - \omega_0^2/\omega^2)]^2}, \quad (6)$$

in which $N_1 = \omega L_2 / (L_2 v / \omega_2 l + R)$.

Comparison of (5) and (6) shows that the LC tuned circuit has a higher quality factor, since there is a quadratic dependence of the current amplitude on frequency, in contrast to a system with inductive compensation. Consequently, an inductive-compensation pump allows more extensive control than a capacitive one while maintaining the requirement $\cos\varphi \approx 1$. Similar results can be derived for three-phase and multiphase pumps. The inductive tuned circuit is better than a capacitive one as regards mass and size at low frequencies (around 50 Hz) [4, 5].

2. These formulas do not incorporate the parameter nonlinearity in the excitation system. To incorporate the nonlinearity, we use the [6] linearization method in the form employed in [4]. Here the equation describing the nonlinear system containing currents close to harmonic is replaced by an equivalent linear one with coefficients determined as functions of the current amplitude.

Then (4) becomes

$$\frac{d^2 i_1}{dt^2} + f(i_1) \frac{di_1}{dt} + i_1 = I \sin(\nu\tau + \theta). \quad (7)$$

In states close to harmonic, the difference between the dissipative term and the right-hand side in (7) is small, so in accordance with [6] we seek the solution to (7) as

$$i_1 = a \cos(\nu t + \theta + \psi),$$

in which a and ψ are slowly varying functions of time, which are defined by

$$da/d\tau = Q(a, \psi); \quad d\psi/d\tau = p(a, \psi). \quad (8)$$

Here $Q(a, \psi) = -\delta(a)a - [I/(1 + \nu)]\cos\psi$; $p(a, \psi) = 1 - \nu + [I/a(1 + \nu)]\sin\psi$; $\delta(a) = R(1 + \kappa)/2\rho$.

We equate the right-hand sides in (8) to zero for the stationary states to get the frequency dependence of the amplitude and phase:

$$(1 - \nu^2)a^2 + \delta^2(a)a^2(1 + \nu)^2 = I^2; \quad \text{tg } \psi = (1 - \nu)/\delta. \quad (9)$$

Resonance sets in for $\nu = 1$, and the amplitude is

$$2a^2\delta^2(a) = I^2. \quad (10)$$

In mild excitation [4], the linearized inductances can be represented as $L_1 = L_{10}(1 - \gamma a^2)$; $L_2 = L_{20}(1 - \gamma a^2)$; $\gamma a^2 < 1$, and with capacity compensation

$$\delta = \frac{1}{2} \frac{\nu}{\omega l} \sqrt{L_0 C} \frac{1 - \alpha a^2}{1 - \gamma a^2}, \quad (11)$$

in which $\alpha = 1 + L_0 \nu / (\omega l R)$. With inductive compensation

$$\delta = \frac{1 + \kappa(1 - \gamma a^2)}{\{\kappa_{10}^2(1 - \gamma a^2)^2 + [1 + \kappa_{20}(1 - \gamma a^2)]^2\}^{1/2}}, \quad (12)$$

in which $\kappa_{20} = L_{20} \nu / (\omega_2 l R)$; $\kappa_{10} = L_{10} \nu / (\omega_1 l R)$.

We substitute (11) and (12) into (9) to determine the amplitude and phase as functions of I and ν . For $\nu = 0$, these formulas give the above particular solutions to the linear equation.

The stability of the stationary states is [6] defined by

$$\begin{aligned} \alpha Q'_a(a, \varphi) + P'_\varphi(a, \varphi) &< 0, \\ Q'_a(a, \varphi) \cdot P'_\varphi(a, \varphi) - Q'_\varphi(a, \varphi) P'_a(a, \varphi) &> 0. \end{aligned}$$

3. These results were used for a three-phase conduction pump with intermediate transformer, for which Fig. 1b shows the theoretical circuit. The pump has three subchannels, which can work independently, in parallel, or in series. The characteristics were calculated on the assumption of load symmetry, with a constant speed in the channel, and with allowance for the electrically conducting shell and the inhomogeneity in the current leakage in the channel in accordance with [7].

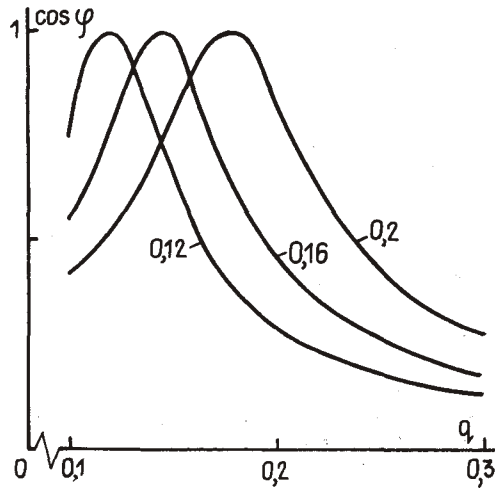


Fig. 2. Behavior of $\cos \varphi$ for a three-phase pump with inductive compensation in relation to the quasistationarity parameter q for $k_{12} = k_1/k_2$ (values given on the curves).

To consider the basic trends for such a pump with inductive compensation, we write simplified formulas for a pump in which we neglect the open-circuit current in the transformer, as well as the mutual inductances of the coils on the various rods in the exciting winding and the resistive losses in the steel. We give the equations only for the first channel, since those for the others are analogous:

$$\begin{aligned} (R_{\tau 1} + j\omega L_{\sigma 1}) I_1 + \dot{E}_0 + (R_{\Sigma} + j\omega L_{\Sigma}) I_1 + j\omega M (I_1 + I_2) &= \dot{U}_1; \\ \dot{E}_0 &= -(R'_{\tau 2} + j\omega L_{\sigma 2}) I' + U_c; \quad I' = -I_1. \end{aligned} \quad (13)$$

Here \dot{U}_i , \dot{I}_i are the complex voltages and currents in the phases ($i = 1, \dots, 3$), U_c , I' are the voltages and currents in the channels, and R_{Σ} , L_{Σ} , M are the resistance, inductance, and mutual inductance of the exciting windings on the rod correspondingly; $R_{\tau i}$, $L_{\sigma i}$ are the resistance and leakage inductance of the primary and secondary windings of the transformer correspondingly ($i = 1, 2$). A prime denotes that the parameter is referred to the transformer primary.

The channel current and electromagnetic pressure are

$$\begin{aligned} I &= (R_c)^{-1} [\kappa_{IV} (1 + \xi) \dot{U}_c - \kappa_{IQ} v \dot{B} h]; \\ \dot{p}_e &= (a R_c)^{-1} (\kappa_{pV} \dot{U}_c - \kappa_{pQ} v \dot{B} h) \dot{B}^*. \end{aligned} \quad (14)$$

Here \dot{B} , \dot{p}_e are the complex magnetic induction in a channel and the electromagnetic pressure correspondingly, ξ is the shell parameter, $R_c = h/(\sigma a l)$ the channel resistance, a channel height, and κ coefficients that correct for the current nonuniformity in the channel [7].

The resultant magnetic induction in the channel is produced by the main and additional windings, in which the currents have a phase difference of $2\pi/3$, and it for the first channel is

$$\dot{B}_1 = [\mu_0 / (2\delta k_\delta)] [k_2 - 0,5(1 + j\sqrt{3}) k_1] I, \quad (15)$$

in which $k_1 = \omega_1/2\omega_3$; $k_2 = \omega_2/2\omega_3$, with 2δ the nonmagnetic gap and k_δ the leakage coefficient. The solution to (13)-(15) defines all the characteristics.

If we neglect the leakage inductance of the intermediate transformer and the effects from the channel shell, the power factor is

$$\cos \varphi = \frac{1}{\{1 + r^2 [1 - \sqrt{3} k_1 q / 2 (k_1^2 + k_2^2 - k_1 k_2)]^2\}^{1/2}},$$

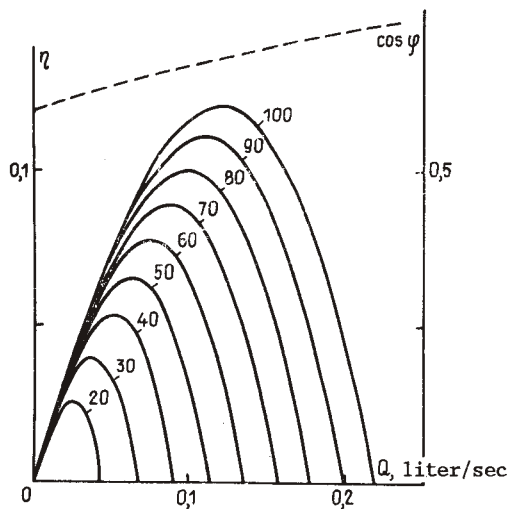


Fig. 3

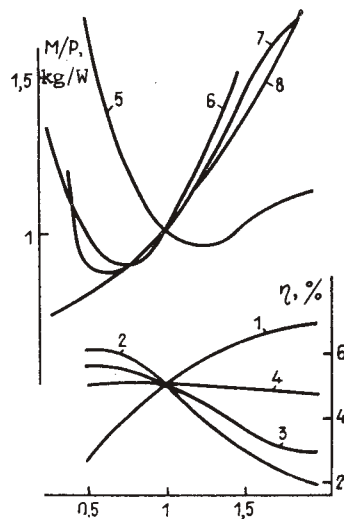


Fig. 4

Fig. 3. Dependence of $\cos \varphi$ (dashed line) and efficiency (solid line) for a three-phase pump with inductive compensation on the liquid metal flow for the U in V stated on the curves.

Fig. 4. Dependence of the efficiency (1-4) and reduced mass M/P (5-8) on l_*/l_{*0} (1, 5) v/v_0 (2, 6), a_*/a_{*0} (3, 7) and k_{12}/k_{120} (4, 8). Nominal parameters: $l_{*0} = 4$, $v_0 = 10$ m/sec, $a_{*0} = 0.3$, $k_{120} = 0.5$. The abscissa is to be understood as representing all four ratios of the numerical values of the parameters to the nominal values: $l_* = l/b$, $a_* = a/b$, $k_{12} = k_1/k_2$. When one of the parameters varies, the other three are constant.

in which

$$r = \omega L/R_0, \quad L = L_\Sigma - M, \quad R_0 = R_\Sigma + R_{T1} + \omega_3^2 \left(R_{T2} + R_c + \frac{v h \mu_0 (k_2 - 0.5 k_1)}{(2\delta) k_\delta} \right).$$

One can choose the turns on the exciting windings to compensate the reactive power for

$$q_{op} = 2(k_1^2 - k_1 k_2 + k_2^2) / (\sqrt{3} k_1).$$

Figure 2 shows that the power factor varies little near the maximum, so it is possible to vary the metal speed and supply frequency quite widely in the calculations, which provides for extensive machine control without reduction in the power factor.

Calculations were performed on a conduction pump with a hydraulic power of 10 W at a supply frequency of 50 Hz for a working body having $\sigma = 1.7 \cdot 10^6 \Omega^{-1} \cdot m^{-1}$ with allowance for all the forms of loss and for the transformer and shell parameters. In fact, it is impossible in practice to attain the maximum in the power factor and efficiency simultaneously, so a reasonable compromise must be adopted for the nominal state. Figure 3 shows the results. Correction for the transformer's leakage inductance and for the channel shell leads to a more gentle rise in the power factor by comparison with that in Fig. 2 and displaces q_{op} towards larger values. We examined how certain parameters affected the basic pump characteristics, and the results are shown in Fig. 4, which shows that deviations in certain parameters from the nominal values can affect the pump characteristics quite substantially.

This shows that one can raise the power factor substantially in a conduction pump by providing special inductive links between the winding phases.

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