POSSIBILITY OF INCREASING THE FLOW STABILITY IN A BOUNDARY LAYER BY MEANS OF CROSSED ELECTRIC AND MAGNETIC FIELDS

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Reference [1] examined the problem of reducing the drag of a flat plate by stopping the increase in boundary layer thickness along it and creating a velocity profile more stable than the Blasius profile. The possibility of controlling boundary layer flow by means of forces exponentially distributed along the normal was considered as an example. Since that paper was mainly concerned with a general outline of the basic idea, it is worthwhile examining the hydrodynamic aspects of the problem in greater detail.

As shown in [2], an exponential distribution of forces along the normal to a surface can be realized sufficiently accurately by means of a system of alternating electrodes and magnetic poles in the form of strips arranged along the flow surface. In this case the expression for the body force acting on the fluid has the form

$$F=\frac{\pi^2 j_0 B_0}{K(\sin\varphi)K(\cos\varphi)}\,e^{\frac{-\pi}{a+b}y}, \tag{1}$$

where j_0 is the current density at the electrode surface, B_0 is the induction of the magnetic field at the pole surface, $K(\sin\varphi)$ and $K(\cos\varphi)$ are complete elliptic integrals of the first kind and $\varphi=\frac{\pi a}{a+b}$, a and b are the widths of the magnetic pole and electrode, respectively, and y is a coordinate reckoned along the normal to the plate surface.

Since the coefficient of the exponential reaches a maximum at $a = b$, we will confine ourselves to a consideration of that case as the most favorable from the standpoint of minimum expenditure of energy to create a force of given magnitude. In this case the force may be expressed as

$$F=2.87 j_0 B_0 e^{\frac{-\pi}{2a}y}. \tag{2}$$

We will write the equation of the boundary layer in the usual notation for the case of body force (2) in the absence of a pressure gradient

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{2.87 j_0 B_0}{\rho}e^{\frac{-\pi}{2a}y},$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

with boundary conditions

$$u|_{x=0}=f(y);\quad v|_{x=0}=0;\quad u|_{y=0}=0;$$

$$v|_{y=0}=0;\quad u|_{y\to\infty}\to u_0. \tag{4}$$

We introduce the new variables

$$\bar u=\frac{u}{u_0};\quad \bar v=\frac{2a}{\pi\nu}v;\quad \xi=\frac{\nu\pi^2}{4a^2u_0}x+C;\quad \bar y=\frac{\pi}{2a}y.$$

Then the equations take the form

$$\bar u\frac{\partial \bar u}{\partial \xi}+\bar v\frac{\partial \bar u}{\partial \bar y}=\frac{\partial^2 \bar u}{\partial \bar y^2}+ze^{-\bar y};\quad \frac{\partial \bar u}{\partial \xi}+\frac{\partial \bar v}{\partial \bar y}=0, \tag{5}$$

and

$$\bar u|_{\xi=C}=f(\bar y);\quad \bar v|_{\xi=C}=0;\quad \bar u|_{\bar y=0}=0;$$

$$\bar v|_{\bar y=0}=0;\quad \bar u|_{\bar y\to\infty}\to 1, \tag{6}$$

where $f(\bar y)$ is the given inlet profile.

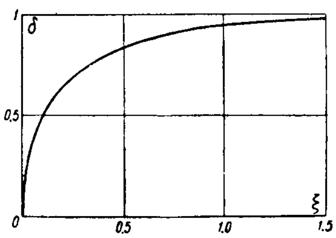

Fig. 1

The coefficient of the exponential in the first of Eqs. (5)

$$z=1.16\frac{a^2 j_0 B_0}{u_0\rho\nu}. \tag{7}$$

We turn to the case z = 1. This case is interesting in that system (5) has an exact solution

$$\bar u=1-e^{-\bar y};\quad \bar v\equiv 0,$$

satisfying the conditions $\bar u|_{\bar y\to\infty}=1$ and $\bar u|_{\bar y=0}=0$. This solution is the asymptotic profile of the boundary layer as $\xi\to\infty$.

Thus, at z = 1 the thickness of the boundary layer is limited and tends to a constant value of the same order as $2a/\pi$. Consequently, by selecting a (width of electrode and magnetic pole) so that the Reynolds number calculated from the thickness of the boundary layer does not exceed $4\cdot 10^4$ [3] and j_0B_0 so that z = 1, we can prevent the transition from a laminar to a turbulent boundary layer. In other words, the system parameters must be selected so as to satisfy the two conditions

$$1.16\frac{a^2 j_0 B_0}{u_0\rho\nu}=1 \quad\text{and}\quad \frac{2u_0 a}{\pi\nu}\leqslant 4\cdot 10^4.$$

To estimate the dependence of the boundary layer thickness on the longitudinal coordinate at z = 1 we use the Karman-Pohlhausen method, selecting as the velocity profile

$$u=u_0(1-e^{-\bar y/\delta(\xi)}).$$

After the usual calculations we obtain the curve

$$(1-\delta)e^{\delta}=e^{-2\xi},$$

shown in Fig. 1.

In order to establish the effect of the nature of the inlet profile on the distance from the inlet at which the velocity distribution may be

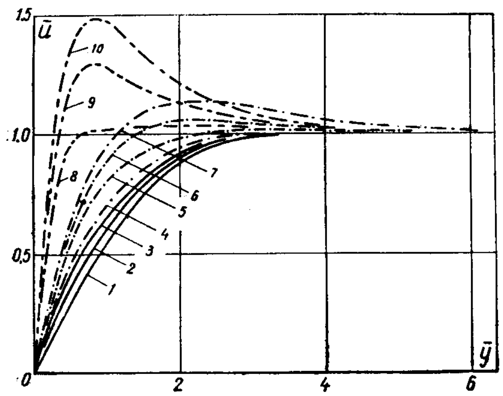

Fig. 2a. 1-ξ = 0.355; 2-ξ = 0.455; 3-ξ = = 1.355; 4-ξ = 0.455; 5-ξ = 0.655; 6-ξ = = 1.555; 7-ξ = 1.955; 8-ξ = 0.455; 9-ξ = = 0.555; 10-ξ = 0.655. Curves 2 and 3 were obtained at z = 1, curves 4, 5, 6, and 7 at z = 2, and curves 8, 9, and 10 at z = 10.

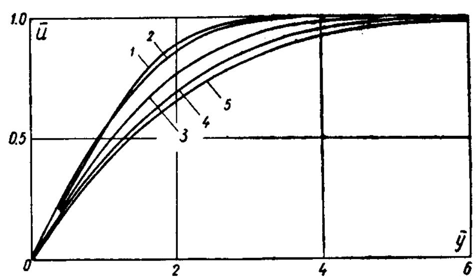

Fig. 2b. 1-ξ = 0.355; 2-ξ = 0.455; 3-ξ = = 0.855; 4-ξ = 1.355; 5-ξ = 1.675. Curves 2, 3, 4, and 5 were obtained at z = 5/16.

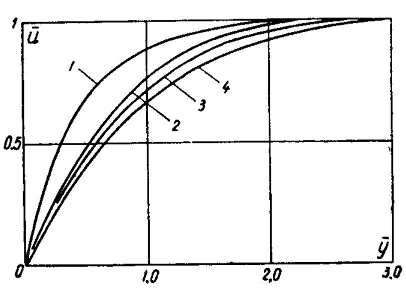

Fig. 2c. 1-ξ = 0.455; 2-ξ = 0.655; 3-ξ = 0.855; 4-ξ = 1.355. The curves were obtained at z = 1.

assumed asymptotic, and also to determine how sharply the profile is modified as z deviates from 1 and what shape it takes, we made computer calculations of system (5) with boundary conditions (6). As the inlet profile at $\xi = C = 0.355$ we took the Blasius profile and a uniform velocity profile. The calculations were made for various values of z. Some of the results obtinaed are presented in Figs. 2a, 2b, and 2c.

In all the figures the quantity $\bar{y}$ has been used as transverse coordinate for convenience of comparison.

Figures 2a and 2b correspond to the case when a Blasius profile is given at the inlet, and Fig. 2c to the case of a uniform profile. It is clear from the figures that the asymptotic profile is reached more rapidly for the Blasius than for the uniform inlet profile.

It is also interesting to note that at z greater than unity (Fig. 2a) the velocity profile becomes nonmonotonic, so that inside the boundary layer there are velocities greater than the free-stream velocity. In this case the thickness of the boundary layer increases with increase in ξ.

At $z < 1$ (Fig. 2b) the thickness of the boundary layer also increases. However, this process is slower than in the absence of electromagnetic forces.

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