A CONSTANT TEMPERATURE MHD GENERATOR WITH SERIES CONNECTED ELECTRODES

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The isothermal flow of a gas in an MHD generator with series connected electrodes is considered in a quasi-one-dimensional approximation. Numerical calculations pertinent to the optimization of isothermal MHD generators with constant electrical efficiency are performed.

1. In MHD generators with series connected electrodes the direction of the electric field vector is given by the direction of the coupling between the oppositely laying segmented electrodes. Hence the angle of inclination of the equipotential surfaces is a design parameter of these generators. The short circuiting of the electrodes is shown schematically in Fig. 1.

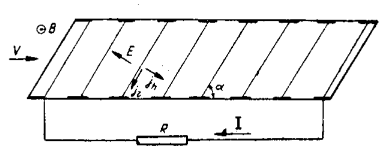

Fig. 1. Montardy-type MHD generator.

Expressions for the specific power and the local electrical efficiency of this generator type were first given by A. Montardy [1]. In the present work, the quasi-one-dimensional flow in a Montardy type MHD generator is considered. Let us write the system of quasi-one-dimensional equations defining the flow in an ideally segmented Montardy type MHD generator operating with a single load,

$$\rho v \frac{dv}{dx} = -\frac{dp}{dx} + j_y B, \qquad (1)$$

$$\rho v \frac{d}{dx}\left(c_p T + \frac{v^2}{2}\right) = j_x E_x + j_y E_y, \qquad (2)$$

$$j_x = \frac{\sigma[E_x - \beta(E_y - vB)]}{1+\beta^2}, \qquad (3)$$

$$j_y = \frac{\sigma(E_y - vB + \beta E_x)}{1+\beta^2}, \qquad (4)$$

$$p = \rho g R T, \quad \rho v F = G, \quad F\left(j_x + \frac{j_y E_y}{E_x}\right) = I. \qquad (5), (6), (7)$$

where I denotes the current flowing through the load resistance. In the above expressions, the standard notation has been used.

Note that (7) is valid only if the angle α of inclination of the segmentation (see Fig. 1) is equal to the angle of inclination of the equipotential planes in the plasma. Hence the potential drops at the electrodes should not be signficant.

Let us consider some of the cases where Eqs. (1)–(7) can be integrated. In the following, the magnetic field and the gas temperature are assumed to be constant. The following symbols are introduced:

$$k_y = \frac{E_y}{vB}, \quad \gamma = \frac{E_x}{E_y}, \quad k_x = \frac{E_x}{E_{x0}},$$

$$\eta_h = \frac{v_1^2}{2c_p T_1^*}, \quad \eta_e = \frac{\mathbf{j}\cdot\mathbf{E}}{j_y v B}, \quad \eta = \frac{T_1^* - T^*}{T_1^*},$$

where E_{x0} is the longitudinal component of the electric field vector under open circuit conditions, the quantities at the generator entrance and those corresponding to stagnation conditions are denoted by index 1 and asterisks, respectively.

a) Let us assume that $\gamma = -\beta$. In this case the specific power reaches its maximum corresponding to the given k_x value [1]. The Hall parameter is assumed to be a function of the gas parameter,

$$\beta = c/\rho. \qquad (8)$$

Such an approximation is justified by the definition of β,

$$\beta = \frac{Be}{m_e \nu}, \quad \nu = \theta N \sqrt{\frac{3kT}{m_e}}$$

where ν is the average collision frequently between the electrons and atoms, θ is the mean collision cross section, $N = \rho/m_n$ is the number of neutral particles per unit volume, hence

$$\beta = \frac{e m_n B}{m_e \sqrt{3kT/m_e}\,\theta\rho}.$$

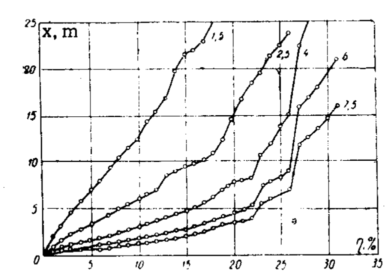

Fig. 2. The function $x = x(B, \eta)$ for $G = 1$ kg/sec under optimum operational conditions. The numbers assigned to the curves denote the respective B_1 (tesla) values.

Thus in the T = const and B = const approximation the application of (8) is plausible.

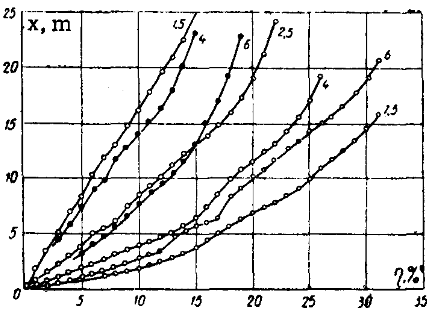

Fig. 3. The function $x = x(B, \eta)$ for $G = 2 \cdot 10^3$ kg/sec under optimum operational conditions. The numbers assigned to the curves denote the values of B_1(tesla). Circles: first case, dots: second case.

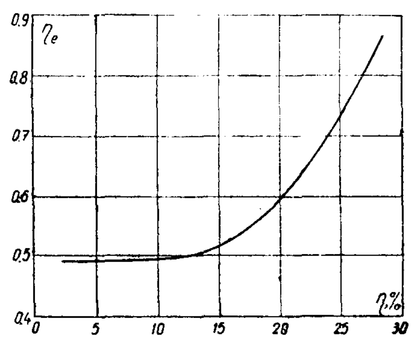

Fig. 4. The function $\eta_e(\eta)$ for B = 4, 6, and 7.5 tesla, G = 1 kg/sec.

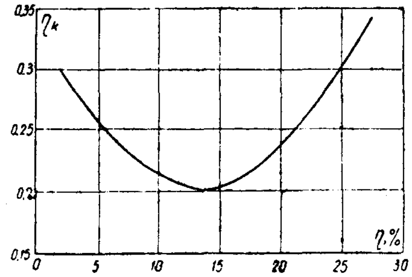

Fig. 5. The function $\eta_k(\eta)$ for B = 4, 6, and 7.5 tesla, G = 1 kg/sec.

In view of the T = const assumption we shall assume that σ = const.

Using (3), (4), (7), (6), the condition $\gamma = -\beta$, and the assumptions $\beta = c/\rho$, σ = const we obtain k_y = const.

In particular, if $k_y = 1/2$, each generator section operates in the regime of maximum specific power: $P = (\mathbf{j}\cdot\mathbf{E})/(\sigma(vB)^2)$ for a Faraday type generator with infinitely segmented electrodes ($j_x = 0$). For this case the solution can be written as

$$\frac{v^2 - v_1^2}{2} = gRT \ln\left\{\left(\frac{p}{p_1}\right)^{\frac{1-k_y}{k_y}} \times \right.$$

$$\left. \times \left[\frac{p_1^2(k_y-1)^2 + (k_y gRTc)^2}{p^2(k_y-1)^2 + (k_y gRTc)^2}\right]^{\frac{1-2k_y}{2k_y(1-k_y)}}\right\},$$

$$x = \int_p^{p_1} \frac{p^2 + (cgRT)^2}{[p^2(k_y-1)^2 + (k_y gRTc)^2]v}\, dp.$$

b) Let us consider a flow with constant electrical efficiency.

The quantities σ and β are arbitrary functions of the temperature and pressure.

The solution can be written in the following form:

$$v = \sqrt{\frac{2\eta_e}{1-\eta_e} gRT \ln\frac{p}{p_1} + v_1^2},$$

$$x = \int_p^{p_1} \frac{(1+\beta)^2\left[1 + \dfrac{\eta_e(1+\beta\gamma)\sigma BGgRT}{I(1+\beta^2)p\gamma}\right]}{(1-\eta_e)\sigma B^2 v}\, dp,$$

$$\gamma = \frac{I(1-\eta_e)}{\dfrac{GgRT\sigma B\eta_e}{p} + \beta I + \dfrac{I^2(1+\beta^2)p}{\sigma BGgRT} + I\beta\eta_e}.$$

2. **Optimization of a generator of constant electrical efficiency.** The length of a constant efficiency generator is a function of the gas parameter values at the generator entrance and the magnitudes of I and η_e.

We shall seek such values of these quantities which, in combination with the stagnation gas parameters given at the generator entrance and the given dimensionless effective power η, lead to a minimum generator length.

The quantities γ_1, η_e, and η_k were chosen as independent variables. The optimization has been carried out numerically with the help of a "Ural-4" computer. The program compared the generator lengths computed for various combinations of the γ_1, η_e, and η_k parameters at given values of η, B, and the mass flow G, and stored the smallest of the computed lengths. As a result of this procedure, we obtained a family of two-parameter optimum curves $x = x(\eta, B, G)$.

To assure the validity of the one-dimensional approximation, we shall restrict our consideration to a class of generators whose apex angle does not exceed a certain limiting value (20° has been assumed here). Since for increasing the mass flow the tangent of the apex angle increases proportionally to the square root of the mass flow, an increase of the latter reduces the region of applicability of the one-dimensional approximation.

Computations were performed for the following cases:

$T^* = 3200°$ K, $p^* = 50$ atm $k = 1.11$, $gR = 264$

$T^* = 2530°$ K, $p^* = 11$ atm $k = 1.155$, $gR = 294$.

The first case corresponds to combustion products resulting from coal burning in air with 50% oxygen content and preheated to 750° C (oxygen cycle).

The following values were assumed for the independent variables:

η_e 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95;
$-\gamma_1$ 0.1, 0.3; 0.5, 0.8, 1, 1.3, 1.5, 1.8, 2, 3, 4, 5;
η_k 0.055, 0.105, 0.15, 0.2, 0.25, 0.3, 0.35, 0.38.

The parameters B and G were given as

B (tesla): 1.5, 2.5, 4, 6, 7.5; G (kg/sec): 1, $2 \cdot 10^3$.

The second case computed corresponds to the combustion products of coal burned in air preheated to 900° C with G = $2 \cdot 10^3$ kg/sec, and B = 4 and 6 tesla.

The following values were assigned to the independent variables:

η_e 0.4, 0.5, 0.6, 0.65, 0.7, 0.73, 0.75, 0.8, 0.83, 0.9;
$-\gamma_1$ 0.1, 0.3, 0.5, 0.8, 1, 1.3, 1.5, 1.8, 2; 3, 4, 5;
η_k 0.15, 0.17, 0.19, 0.20, 0.21, 0.22, 0.23, 0.24.

In both cases the conductivity and the Hall parameter were computed after Frost [2].

The function $x = x(\eta, B)$ corresponding to the first case with G = 1 kg/sec (laboratory size generator) is shown in Fig. 2. The distributions corresponding to the first and second cases with G = $2 \cdot 10^3$ kg/sec (large scale power station) are shown in Fig. 3.

As can be seen, the given generator type makes it possible to extract a significant fraction of the enthalphy (η_1 = 31%, η_2 = 19%) at moderate values of the generator length. A comparison of Figs. 2 and 3 shows that the limitations imposed on the computations by the one-dimensionality of the approximation reduce significantly the quantity of the generators being compared [sic] (for equal B values, the curves in Fig. 2 lie somewhat lower than those in Fig. 3).

The function $\eta_e = \eta_e(\eta)$ is shown in Fig. 4 for the first case with the following parameter values: B = 6, 4, and 7.5 tesla; G = 1 kg/sec (in reality, the function changes stepwise for the parameter values used in these computations). At low power levels η_e = 0.5. This is caused by the fact that at small generator lengths the condition of local optimization ($\gamma = -\beta$, $k_y = \eta_e = 1/2$) must be fulfilled.

At larger power outputs, η_e and the internal efficiency η_0 become high (for B = 6 tesla and η = 31%, $\eta_e = \eta_0 = 0.9$).

The function $\eta_k(\eta)$ is shown in Fig. 5 for several B values (4, 6, and 7.5 tesla) with G = 1 kg/sec.

At smaller power outputs the dimensionless kinetic energy satisfies the condition of local optimum. Since at larger outputs $\eta_k > \eta$, in this region the dimensionless kinetic energy should be sufficiently large.

Hence a constant temperature MHD generator with a single load may prove to be an effective energy converter.

REFERENCES

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