COMPARISON OF MAGNETIC SYSTEMS FOR OBTAINING A STRONG MAGNETIC FIELD IN A LARGE VOLUME

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Magnitnaya Gidrodinamika, Vol. 3, No. 4, pp. 124–134, 1967

UDC 621.3.044.5:538.4

Magnetic systems composed of ordinary materials (copper, steel) are compared in relation to their ability to create a strong magnetic field in a large volume. Formulas are obtained for estimating the power losses and weights of magnetic systems of four types. It is shown that any of these systems can create a field of 6 Wb/m^2 and for an MHD generator with a gas conductivity of 2.0 mho/m and a gas velocity of 10^3 m/sec the excitation losses are only 10% of the generated power.

Introduction. The creation of a strong uniform magnetic field in a volume on the order of hundreds of cubic meters is one of the central problems involved in the realization of a power-station MHD generator. Together with a search for means of creating such a field employing superconducting coils, it is desirable to evaluate the possibility of using materials with ordinary conductivity. From preliminary calculations and experience with the design of relatively small experimental MHD devices it is known that in magnetic systems made of ordinary materials the excitation losses are excessively large. However, transition to larger magnetic systems for full-scale MHD generators is accompanied by a scale effect and the relative excitation losses decrease. In this connection it is desirable to examine existing magnetic systems, select the most practical of them and estimate the relative excitation losses.

In this paper we discuss four types of magnetic systems: 1) an iron-free coaxial magnetic system with tangential magnetic field; 2) an iron-free magnetic system of the elliptic type, with a coil whose cross section is formed by two intersecting ellipses or circles; 3) an O-shaped magnetic system with a magnetic circuit; 4) a rhombiform magnetic system with a magnetic circuit.

At the beginning of each section we give an approximate analysis of the power and weight characteristics without allowance for the effect of the end sections of the coils. Then these estimates are refined by taking into account the effect of the end sections, the magnetic systems are optimized with respect to the power and weight characteristics, and a quantitative estimate is obtained for the relative exciting power.

1. Coaxial magnetic system. The cross section of the coaxial system is shown in Fig. 1. The current density vector j is directed normal to the plane of the drawing. The cross section of the generator duct has the shape of a ring and is bounded by the radii R_1 and R_2. A shortcoming of this system is the variability over the duct cross section of the magnetic induction, which is inversely proportional to the radius. Although of no importance for MHD generators, it is simple in form and therefore it is desirable to begin our study with precisely this type of magnetic system. In our calculations we will employ the mean induction B_m at the mean radius $R_m = 0.5(R_1 + R_2)$.

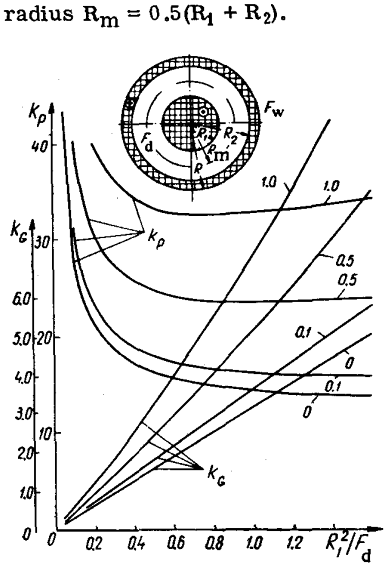

Fig. 1

In accordance with the total current law

$$2\pi R_m B_m/\mu_0 = AW, \tag{1.1}$$

where AW is the number of complete ampere-turns of the field coil,

$$AW = jF_c = jF_w k_f. \tag{1.2}$$

Here j, A/m^2, is the current density in the coil; F_c, m^2, is the cross section of the coil copper; $k_f = F_c/F_w$ is the fill factor. The cross section of the coil $F_w = \pi R_1^2$. The cross-sectional area of the duct $F_d = \pi(R_2^2 - R_1^2)$. Equating expressions (1.1) and (1.2) with account for the expressions for F_w and F_d, we obtain

$$j = \frac{B_m}{\mu_0 k_f R_1}\left(1 + \frac{R_2}{R_1}\right). \tag{1.3}$$

The exciting power per unit length of the magnetic system

$$\frac{P}{L} = 2\rho j^2 \pi R_1^2 k_f = \frac{2\rho B_m^2}{\mu_0^2 k_f}\pi\left(1+\frac{R_2}{R_1}\right)^2 = \frac{2\rho B_m^2}{\mu_0^2 k_f}k_p, \tag{1.4}$$

where $k_p = \pi\left(1+\frac{R_2}{R_1}\right)^2 = \pi\left(1+\sqrt{1+\frac{F_d}{\pi R_1^2}}\right)^2$ is a coefficient taking into account the effect of the system geometry on the quantity P/L.

The least possible exciting power per unit length of a magnetic system of the coaxial type obtained for an infinite decrease of current density does not depend on the duct dimensions,

$$\lim_{R_1\to\infty}\frac{P}{L}=\frac{2\rho B_m^2}{\mu_0^2 k_f}4\pi. \quad (1.5)$$

The weight of the coil copper can be calculated from the relation

$$\frac{G_c}{LF_d}=2\gamma_c\frac{\pi R_1^2}{F_d}=2\gamma_c k_f k_G. \quad (1.6)$$

We will take into account the end sections of the coil assuming that the length of the end section is approximately equal to R_2. Then the exciting power and the weight of copper in the system increase as

$$\frac{L+R_2}{L}=1+\frac{\sqrt{F_d}}{L}\sqrt{\frac{1}{\pi}+\frac{R_1^2}{F_d}}. \quad (1.7)$$

Figure 1 presents the curves $k_p=f\left(\frac{R_1^2}{F_d}\right)$ and $k_G=f\left(\frac{R_1^2}{F_d}\right)$ with allowance for the effect of the end sections at the values of $k_L=\frac{\sqrt{F_d}}{L}=0;\ 0.1;\ 0.5; 1.0$, indicated against the corresponding curves.

With allowance for the effect of the end sections expressions (1.4) and (1.6) take the form

$$\left(\frac{P}{L}\right)_{es}=\frac{2\rho B_m^2}{\mu_0^2 k_f}k_p\times$$

$$\times\left[1+\frac{\sqrt{F_d}}{L}\sqrt{\frac{1}{\pi}+\frac{R_1^2}{F_d}}\right]=\frac{2\rho B_m^2}{\mu_0^2 k_f}(k_p)_{es}; \quad (1.8)$$

$$\left(\frac{G_c}{LF_d}\right)_{es}=2\gamma_c k_f k_G\times$$

$$\times\left[1+\frac{\sqrt{F_d}}{L}\sqrt{\frac{1}{\pi}+\frac{R_1^2}{F_d}}\right]=2\gamma_c k_f(k_G)_{es}. \quad (1.9)$$

2. **Magnetic system of elliptic type.** The cross section of this system is shown in Fig. 2. The magnetic field in the working space of such a system is strictly uniform [1, 2]. The shape considered is a particular instance of the more general case in which the conductors in the cross section are bounded by ellipses with any ratio of the axes.

In order to construct a picture of the plane-parallel magnetic field for this system of currents we start from an examination of the magnetic field of a single circular conductor with current density j uniformly distributed over the cross section [3]. The field created by two currents can be found by Maxwell's graphic method from the field distribution for one of them. In this method the field distributions of the two individual conductors are superimposed on each other so that the conductors are located relative to each other in the same way as in the actual current system. The resultant lines of magnetic induction are then drawn through those points of intersection of the lines of both fields for which the sum of the corresponding values of the vector magnetic potential A is equal to some constant. The field distribution diagram constructed by this method for two intersecting circles at $\alpha = 120°$ is shown in Fig. 3.

The magnetic field obtained in the working space is strictly homogeneous. We will determine the value of the magnetic induction at the center of the system

$$B_y=\frac{\mu_0 j}{2\pi}\iint\frac{x\,dx\,dy}{x^2+y^2}=\frac{\mu_0 j}{\pi}\int_{-a_1}^{a_2}x\,dx\int_0^{a_3}\frac{dy}{x^2+y^2}=$$
$$=\mu_0 j_c k_f R\cos(\alpha_p/2), \quad (2.1)$$

where

$$a_3=\sqrt{R^2-(x-R\cos(\alpha_p/2))^2}.$$

We estimate the excitating power losses per unit length of the elliptic magnetic system

$$\frac{P}{L}=2\rho j_c^2 F_w k_f, \quad (2.2)$$

where (Fig. 2)

$$F_w=R^2(\pi-\alpha_p+\sin\alpha_p). \quad (2.3)$$

It follows from (2.1) that

$$j_c=B_y[\mu_0 R k_f\cos(\alpha_p/2)]^{-1}. \quad (2.4)$$

Substituting (2.3) and (2.4) into Eq. (2.2), we obtain

$$\frac{P}{L}=\frac{2\rho B_y^2}{\mu_0^2 k_f}\frac{\pi-\alpha_p+\sin\alpha_p}{\cos^2(\alpha_p/2)}=\frac{2\rho B_y^2}{\mu_0^2 k_f}k_p. \quad (2.5)$$

We determine the weight of copper in the coil per unit volume of the working space,

$$\frac{G_c}{LF_d}=2\gamma_c k_f\frac{F_w}{F_d}=$$
$$=2\gamma_c k_f\left(\frac{\pi}{\alpha_p-\sin\alpha_p}-1\right)=2\gamma_c k_f k_G. \quad (2.6)$$

For an infinite decrease in current density ($\alpha_p\to 0$) we obtain the minimum value of the exciting power per unit length

$$\left(\frac{P}{L}\right)_{min}=\lim_{\alpha_p\to 0}\frac{P}{L}=\frac{2\rho B_y^2}{\mu_0^2 k_f}\pi. \quad (2.7)$$

We will estimate the effect of the end sections on the power and weight characteristics. We assume that the length of the end section is approximately equal to 2R. Just as for the coaxial system, we find that the power and the weight of the system increase with the ratio $k_L=\frac{\sqrt{F_d}}{L}$. Figure 2 presents the curves $k_p = f(\alpha_p)$ and $k_G = f(\alpha_p)$ for the values of k_L = 0; 0.1; 0.5; 1.0, indicated on the graph.

With allowance for the effect of the end sections expressions (2.5) and (2.6) take the form (here $k_R = (\alpha_p - \sin\alpha_p)^{-1/2}$)

$$\left(\frac{P}{L}\right)_{es}=\frac{2\rho B_y^2}{\mu_0^2 k_f}k_p\left[1+\frac{2\sqrt{F_d}k_R}{L}\right]=\frac{2\rho B_y^2}{\mu_0^2 k_f}(k_p)_{es}; \quad (2.8)$$

$$\left(\frac{G_c}{LF_d}\right)_{es}=2\gamma_c k_f k_G\left[1+\frac{2\sqrt{F_d}k_R}{L}\right]=2\gamma_c k_f(k_G)_{es}. \quad (2.9)$$

3. O-shaped magnetic system with steel magnetic circuit. The cross section of such a system is shown in Fig. 4. One of its disadvantages is a certain nonuniformity of the magnetic field in regions adjacent to the upper and lower parts of the magnetic circuit at large inductions in the working space. If the induction in the gap exceeds the saturation induction (B_i = 2.0 Wb/m^2), the regions of the magnetic circuit bounding the gap from above and below are saturated and the reluctance of these paths sharply increases. This must be taken into account by means of a corresponding increase in the effective length of the air gap.

We calculate the value of the radius r (Fig. 4) from the condition that on the circle defined by that radius the induction is equal to the steel saturation induction. Accordingly, the magnetic flux per unit length of the system is equal to

$$\frac{\Phi}{L}=B_\delta(2d+b)=2B_i h, \quad (3.1)$$

where B_δ is the induction in the air gap, 2d is the width of the air gap, b is the width of the field coil, and $h = h' + \pi r/2$ is the thickness of the yoke of the magnetic circuit.

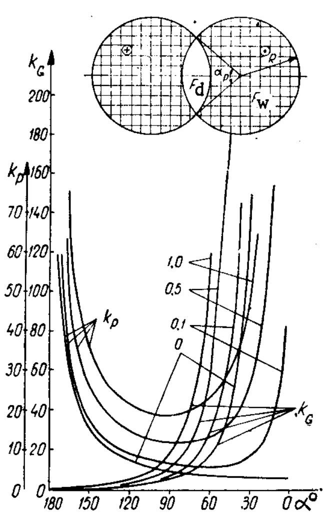

Fig. 2

With allowance for the saturated regions of the magnetic circuit we assume that the length of the magnetic line of force in air is approximately equal to 2a + h. Then the magnetizing force needed to carry the magnetic flux across the air gap is equal to [using (3.1)]

$$AW_\delta=\frac{B_\delta}{\mu_0}(2a+h)=\frac{B_\delta}{\mu_0}\left[2a+\frac{B_\delta}{2B_i}(2d+b)\right]= jF_c=jF_w k_f. \quad (3.2)$$

Here we neglect the magnetizing force needed to carry the flux through the magnetic circuit.

We have $F_w = 2ab$; $2d = F_d/2a$, whence we find the current density in the field coil,

$$j=\frac{AW_\delta}{F_w k_f}=\frac{B_\delta}{2abk_f\mu_0}\left[2a+\frac{B_\delta}{2B_i}(2d+b)\right]. \quad (3.3)$$

The exciting power per unit length of the system

$$\frac{P}{L}=2\rho j^2F_wk_f=2\rho\frac{(AW_\delta)^2}{F_wk_f}=\frac{2\rho B_\delta^2}{2abk_f\mu_0^2}\left[2a+\frac{B_\delta}{2B_i}\left(\frac{F_d}{2a}+b\right)\right]^2=\frac{2\rho B_\delta^2}{\mu_0^2k_f}k_p. \quad (3.4)$$

As before, the coefficient k_p reflects the dependence of the exciting power on the geometry of the system.

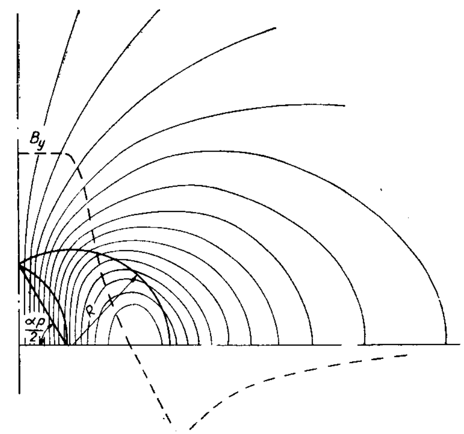

Fig. 3

Since this coefficient is simultaneously a function of the duct height and the coil width at a certain duct cross section F_d, we first find the value of one of these parameters, e.g., 2a, at which k_p reaches a minimum, and then introduce the value $2a = f(F_d, b)$ into the calculation formulas. Solving the equation $\partial k_p/\partial a = 0$ for 2a, we obtain

$$(2a)_{opt}=\frac{B_\delta b}{4B_i}\left[1+\sqrt{1+24\frac{B_iF_d}{B_\delta b^2}}\right]. \quad (3.5)$$

We introduce the notation $\frac{B_\delta}{B_i}=k_B;\ \frac{b}{\sqrt{F_d}}=k_F;\ \frac{\sqrt{F_d}}{L}=k_L;$ $\left[1+\sqrt{1+24\frac{1}{k_B}\frac{1}{k_F^2}}\right]=k_a$. Then $(2a)_{opt}=\frac{b}{4}k_Bk_a$. Substituting the value of $(2a)_{opt}$ into the expression for k_p, we get

$$k_{p(\min\ "a")}=\frac{1}{k_a}\left[k_a+2\left(1+\frac{4}{k_ak_Bk_F^2}\right)\right]^2. \quad (3.6)$$

Passing to the limit in (3.6), we obtain

$$k_{p(\min\min)}=\lim_{k_F\to\infty}k_{p(\min\ "a")}=\frac{1}{2k_B}(2k_B)^2=2k_B=2\frac{B_\delta}{B_i}. \quad (3.7)$$

With account for (3.7) the minimum exciting power for an infinite decrease in current density tends to the value

$$\left(\frac{P}{L}\right)_{\min}=\frac{2\rho B_\delta^2}{\mu_0^2k_f}2\frac{B_\delta}{B_i}. \quad (3.8)$$

The weight of coil copper per unit working volume can be calculated from the relation

$$\frac{G_c}{LF_d}=2\gamma_c\frac{F_w}{F_d}k_f=2\gamma_c k_f k_G\,, \tag{3.9}$$

where

$$k_G=\frac{(2a)_{opt}\,b}{F_d}=\frac{b^2}{4F_d}k_Bk_a=0.25k_ak_Bk_F^2\,. \tag{3.10}$$

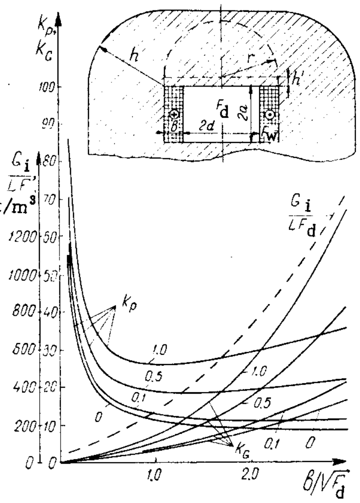

Fig. 4

The weight of steel in the magnetic circuit per unit working volume

$$\frac{G_i}{LF_d}=\frac{\gamma_i h}{F_d}2\left[2d+2a+2b+\pi\frac{h}{2}\right]=$$

$$=\gamma_i k_Bk_F^2\left[1+\frac{4}{k_ak_Bk_F^2}\right]\times$$

$$\times\left[2+\frac{k_ak_B}{4}+\frac{4}{k_ak_Bk_F^2}+\frac{\pi}{4}k_B\left(\frac{4}{k_ak_Bk_F^2}+1\right)\right]. \tag{3.11}$$

Figure 4 shows graphs of $k_p=f(k_F)$ and $k_G=f(k_F)$ for the values of k_L = 0; 0.1; 0.5; 1.0, indicated against the curves. In the calculations it was assumed that $k_B=3$, i.e., $B_\delta=6.0$ Wb/m^2.

With allowance for the effect of the end sections the exciting power and the weight of coil copper increase as

$$1+\frac{l_{es}}{L}=1+\frac{2d+b}{L}=1+k_L\left(k_F+\frac{4}{k_ak_Bk_F}\right). \tag{3.12}$$

With account for the effect of the end sections the expressions for the exciting power and the weight of coil copper take the form

$$\left(\frac{P}{L}\right)_{es}=\frac{2\rho B^2}{\mu_0^2k_f}k_p\times$$

$$\times\left[1+k_L\left(k_F+\frac{4}{k_ak_Bk_F}\right)\right]=\frac{2\rho B_\delta^2}{\mu_0^2k_f}(k_p)_{es}; \tag{3.13}$$

$$\left(\frac{G_c}{LF_d}\right)_{es}=2\gamma_ck_fk_G\times$$

$$\times\left[1+k_L\left(k_F+\frac{4}{k_ak_Bk_F}\right)\right]=2\gamma_ck_f(k_G)_{es}. \tag{3.14}$$

4. Magnetic system with rhombiform duct and steel magnetic circuit. The cross section of the system is shown in Fig. 5. Intensification of the magnetic field with the magnetic circuit unsaturated is achieved by reducing the angle β at the apex of the rhomb. A rigorous analytic solution has been obtained for the particular case of a square [4]. It has been shown that for any thickness of the current sheet the magnetic field in the working space remains homogeneous and equal in magnitude to $\sqrt{2}B_n$, where B_n is the normal component of the induction at the wall of the magnetic circuit, which we will assume equal to $B_i=2.0$ Wb/m^2.

For a rhomb with an arbitrary angle β we may assume that the magnetic field outside the current sheet will also be homogeneous. Moreover, from the solution of the problem for a square it follows that the magnetizing force needed to create a field of given magnitude in the working space is the same as if the entire current were concentrated in an infinitely thin sheet over the inner perimeter of the magnetic circuit. We will assume that this also holds true for a rhomb with any angle at the apex.

For an infinitely thin current sheet with linear current density i_0, A/m, the tangential component of the magnetic field strength $H_t=i_0$. The normal component $H_n=H_t\,\mathrm{tg}\,(\beta/2)$. The resultant strength $H=H_n/\sin(\beta/2)$ and the resultant value of the magnetic induction

$$B=B_i/\sin(\beta/2). \tag{4.1}$$

With allowance for the above assumption, the ampere-turns of the gap

$$AW=i_0\Pi_i/2, \tag{4.2}$$

where Π_i is the inner perimeter of the magnetic circuit.

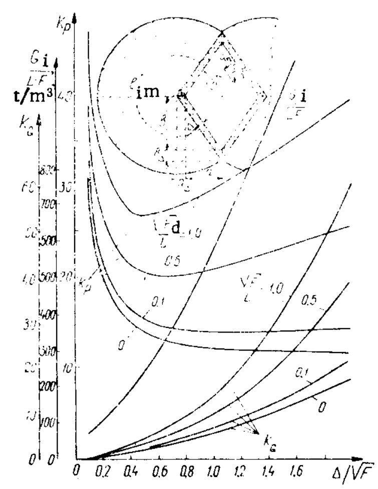

Fig. 5

From Fig. 5 we obtain the following relations for the geometric dimensions of the cross section:

$$F_d=2ab,\quad a=\sqrt{\frac{F_d}{2}\,\mathrm{tg}\,(\beta/2)}\,,$$

$$b=\frac{a}{\operatorname{tg}(\beta/2)},\quad a_\Delta=\frac{\Delta}{\cos(\beta/2)},\quad b_\Delta=\frac{a_\Delta}{\operatorname{tg}(\beta/2)}. \tag{4.3}$$

Assigning the necessary value of the induction in the duct, we find

$$i_0=\frac{B}{\mu_0}\cos(\beta/2). \tag{4.4}$$

The exciting ampere-turns can be expressed in the form

$$AW=jF_c=jF_w k_f. \tag{4.5}$$

It follows from the drawing that

$$F_w=(a+a_\Delta)(b+b_\Delta)-ab=$$
$$=\frac{(a+a_\Delta)^2-a^2}{\operatorname{tg}(\beta/2)}=\frac{2\Delta}{\sin\beta}(\sqrt{F_d}\sin\beta+\Delta). \tag{4.6}$$

The inner semiperimeter of the magnetic circuit

$$\frac{\Pi_i}{2}=\frac{2}{\sin(\beta/2)}(a+a_\Delta)=\frac{2\sqrt{F_d\sin\beta}+4\Delta}{\sin\beta}. \tag{4.7}$$

Equating the right sides of Eqs. (4.2) and (4.5), we determine the current density in the field coil,

$$j=\frac{i_0}{F_w k_f}\frac{\Pi_i}{2}=\frac{B}{2\Delta\mu_0 k_f}\frac{(\sqrt{F_d\sin\beta}+2\Delta)}{(\sqrt{F_d\sin\beta}+\Delta)}. \tag{4.8}$$

We find the exciting power per unit length of the magnetic system

$$\frac{P}{L}=2\rho j^2F_w k_f=$$
$$=\frac{2\rho B^2}{\mu_0^2k_f}\operatorname{ctg}(\beta/2)\cdot\frac{(\sqrt{\sin\beta}+2\Delta')^2}{\Delta'(\sqrt{\sin\beta}+\Delta')}=\frac{2\rho B^2}{\mu_0^2k_f}k_p. \tag{4.9}$$

Here, to simplify the notation, we have taken $\Delta'=\Delta/\sqrt{F_d}$. The weight of copper per unit working volume of the magnetic system

$$\frac{G_c}{LF_d}=2\gamma_c\frac{F_w}{F_d}k_f=2\gamma_c k_f\frac{2}{\sin\beta}\Delta'(\sqrt{\sin\beta}+\Delta')=$$
$$=2\gamma_c k_f k_G. \tag{4.10}$$

The weight of steel per unit working volume of the magnetic system

$$\frac{G_i}{LF_d}=\gamma_i\frac{F_i}{F_d}=\gamma_i\frac{2}{F_d}\frac{\Pi_i}{4}l_{im}=$$
$$=\frac{\pi\gamma_i}{\sin^2\beta}\frac{90+(\beta/2)}{90}(\sqrt{\sin\beta}+2\Delta'). \tag{4.11}$$

Passing to the limit in expression (4.9), we obtain the minimum possible exciting power per unit length of the magnetic system

$$\left(\frac{P}{L}\right)_{min}=\lim_{\Delta'\to\infty}\left(\frac{P}{L}\right)=\frac{2\rho B^2}{\mu_0^2k_f}4\operatorname{ctg}(\beta/2). \tag{4.12}$$

The power requirements and the weight of copper in the field coil increase when the end sections are taken into account as

$$1+\frac{l_{es}}{L}=1+\frac{\Delta+\sqrt{\tfrac{1}{2}F_d}\operatorname{tg}(\beta/2)}{L}=$$
$$=1+\frac{\sqrt{F_d}}{L}\left(\Delta'+\sqrt{\tfrac{1}{2}\operatorname{tg}(\beta/2)}\right), \tag{4.13}$$

and expressions (4.9) and (4.10) are written in the form

$$\left(\frac{P}{L}\right)_{es}=\frac{2\rho B^2}{\mu_0^2k_f}k_p\left[1+\frac{\sqrt{F_d}}{L}(\Delta'+\sqrt{\tfrac{1}{2}\operatorname{tg}(\beta/2)})\right]=$$
$$=\frac{2\rho B^2}{\mu_0^2k_f}(k_p)_{es}; \tag{4.14}$$

$$\left(\frac{G_c}{LF_d}\right)_{es}=2\gamma_c k_f k_G\left[1+\frac{\sqrt{F_d}}{L}(\Delta'+\sqrt{\tfrac{1}{2}\operatorname{tg}(\beta/2)})\right]=$$
$$=2\gamma_c k_f(k_G)_{es}. \tag{4.15}$$

Figure 5 shows the curves $k_p=f(\Delta')$ and $k_G=f(\Delta')$ for $k_L=0$; 0.1; 0.5; 1.0. The angle $\beta=39°$, which corresponds to B = 6.0.

5. Estimation of excitation losses as a fraction of the total power of the MHD generator. The power of a MHD generator per unit length of duct can be expressed in the form

$$\frac{P_{gen}}{L}=k_y(k_y-1)\sigma_{pl}V^2B^2F_d, \tag{5.1}$$

where k_y is the load factor, σ_{pl} the plasma conductivity, and V the plasma velocity.

The ratio of exciting power to generated power

$$\psi=\frac{P}{P_{gen}}=\frac{2\rho B^2}{\mu_0^2k_f}\frac{k_p}{k_y(k_y-1)\sigma_{pl}V^2B^2F_d}=\frac{2}{k_fk_y(k_y-1)}\times$$
$$\times k_p\frac{1}{\mu_0^2\sigma_{pl}\sigma_cV^2F_d}=\frac{C}{R_{m1}R_{m2}} \tag{5.2}$$

where $C=\frac{2k_p}{k_fk_y(k_y-1)}$ is a quantity depending on the geometry of the duct and the load factor; $R_{m1}=\mu_0\sigma_cV\sqrt{F_d}$ and $R_{m2}=\mu_0\sigma_{pl}V\sqrt{F_d}$ are the magnetic Reynolds numbers for the copper and the plasma, respectively.

The effect of the scale factor on the efficiency of the system is clear from expression (5.2): the larger the generator, the smaller the exciting power in relation to the total generated power. Moreover, it is also clear from (5.2) that the relative value of the excitation losses is reduced to the same degree by increasing either the conductivity of the field coil conductors or the plasma conductivity. The small value of R_{m2} for the plasma in the denominator of (5.2) is compensated by the large value of the fictitious

Type of system	k_p	k_G	$\frac{G_i}{LF_d}$	k_p	k_G	$\frac{G_i}{LF_d}$	k_p	k_G	$\frac{G_i}{LF_d}$	$k_{p\,min}$
Coaxial	25	0.475		15	2.467		12.56	∞		12.56
Elliptic	25	0.3		15	0.51		12.56	0.712		3.14
O-shaped	25	0.672	80	15	1.453	140	12.56	2.149	180	6
Rhombiform	25	0.478	82.5	15	1.51	190	12.56	4.62	480	11.3

"magnetic Reynolds number" R_{m1} calculated from the conductivity of the copper.

In order to obtain a rough idea of the value of P/P_{gen} we will calculate it for the following values of the parameters:

$\sigma_c = 4.6 \cdot 10^7$ mho/m, the conductivity of copper at 75° C; $\sigma_p = 2.0$ mho/m, the plasma conductivity; $F_d = 20$ m^2; $V = 10^3$ m/sec; $k_p = 15$, the actual permissible value for all four magnetic systems at copper weights not too large;

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}; \quad k_y = 0.15; \quad k_f = 0.8;$$

$$C = \frac{2 \cdot 15}{0.8 \cdot 0.15 \cdot 0.85} = 294;$$

$$R_{m1} = 12.56 \cdot 10^{-7} \cdot 4.6 \cdot 10^7 \cdot 10^3 \cdot 4.48 = 259 \cdot 10^3;$$

$$R_{m2} = 12.56 \cdot 10^{-7} \cdot 2 \cdot 10^3 \cdot 4.48 = 112.4 \cdot 10^{-4};$$

$$\psi = \frac{294}{259 \cdot 10^3 \cdot 112.4 \cdot 10^{-4}} = 0.101 = 10.1\,\%.$$

6. Comparison of types of magnetic systems. The coefficients k_p and k_G introduced in §§1–4 essentially represent the exciting power and the weight of copper in the magnetic system expressed in certain conventional units identical for all types of magnetic systems. For magnetic systems with a magnetic circuit and an O-shaped or rhombiform configuration a third characteristic quantity is the weight of steel per unit working space of the magnetic system; the weight of steel is shown in Figs. 4 and 5 as a function of the relative thickness of the field coil for systems with a magnetic circuit.

It is clear from Figs. 1, 2, 4, and 5 that both the exciting power and the weight of the coil increase sharply for all types of magnetic systems with increase in the coefficient $k_L = \sqrt{F_d}/L$, i.e., with shortening of the duct. For full-scale powerful MHD generators the value of the coefficient k_L will be close to 0.25, therefore in the first approximation it is possible to consider infinitely long systems ($k_L = 0$).

We have tabulated values of the coefficients k_G and the weight of the steel at several values of k_p for all types of systems.

As follows from the table, the elliptic system possesses considerable advantages over the other types over almost the entire range of variation of the exciting power, which is a function of the relative thickness of the current sheet. It is possible that for very thin current sheets (large current densities) the rhombiform system has a certain advantage. Consequently, if there is a real possibility of reducing the Joule losses by reducing the electrical resistivity of the coil material (building superconducting coils or cooling copper or aluminum coils to the boiling point of liquid hydrogen), preference should be given to a magnetic system with an iron magnetic circuit of rhombiform cross section, although in this case the cost of the magnetic circuit must be added to the cost of the copper in the soil.

SUMMARY

1. Each of the magnetic systems considered, all consisting of ordinary materials (copper and steel), is capable of creating a stationary magnetic field with an induction of 6.0 Wb/m^2.

2. In a sufficiently large MHD generator, for example, with a duct outlet cross section of 20 m^2, a plasma conductivity of 2 mho/m, and a plasma velocity of 10^3 m/sec, the excitation losses are only 10% of the generated power.

3. The best system with respect to all the characteristics is an iron-free elliptic system with a conductor cross section in the form of intersecting circles. In certain circumstances the rhombiform system may have advantages.

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27 December 1966