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STRESSES ACTING BETWEEN THE SHAFT AND THE POLES IN A MAGNETIC FLUID SEAL

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The eccentricity dependence of the magnetic interaction forces between the shaft and the poles and also of the pressure retained by a magnetic fluid seal were considered.

We consider the traditional scheme of magnetic fluid sealing (MFS) with a magnetic shaft. Since in a real device the shaft and the poles always possess eccentricity, as a result of a nonsymmetric magnetic field distribution around the gap perimeter, a magnetic force appears which acts between the shaft and the poles. This force is in competition with the buoyancy force of the magnetic fluid filling the MFS gap. The force acting on a ferromagnetic body placed in a magnetic fluid is determined from the relation [1]

$$F = \oint_S \left(\mu_0 H^2 / 2 - \mu_0 \bar{J} H\right) \mathrm{d}S, \text{ where } \bar{J} = \int_0^H J \mathrm{d}H.$$

If in the MFS gap the magnetic field is sufficiently strong, such that $J = J_0 = \text{const}$ everywhere, then

$$F = \oint_S \left(\mu_0 H^2 / 2 - \mu_0 J_0 H\right) \mathrm{d}S. \tag{1}$$

The first term in this equation corresponds to the force acting on the body in a magnetic field, while the second stems from the buoyancy force of the magnetic fluid.

In order to determine the magnitudes of these forces it is necessary to know the magnetic field distribution in the MFS gap. This problem is solved making the following assumptions. The pole and the shaft are considered to be surfaces with an infinite magnetic permeability. The profiling of the pole is neglected and the calculation is directed toward an effective magnetic gap. The problem is assumed to be two-dimensional, since the MFS gap is much smaller than the shaft diameter and the pole thickness. Due to the introduced assumptions, the pole and the shaft are equipotential, and the magnetic field is defined by a scalar potential

$$\mathbf{H} = -\Delta U, \tag{2}$$

which allows one to employ the known solutions of the problem of electric field distribution between two infinite cylinders [2]. For the geometry and coordinates presented in Fig. 1, the solution of (2) has the form

$$U = \operatorname{arccth} \frac{x^2 + y^2 + a^2}{2ay}. \tag{3}$$

The potential on the surface of the shaft and the pole, as well as the constant a, are determined from the relations

$$R_1 = a|\operatorname{cosech} U_1|, \ R_2 = a|\operatorname{cosech} U_2|, \ D = a(|\operatorname{cth} U_1| - |\operatorname{cth} U_2|). \tag{4}$$

We perform a transformation of the coordinates into a system with an origin at the center of one of the cylinders, and then to a polar system of coordinates and, using (3), write

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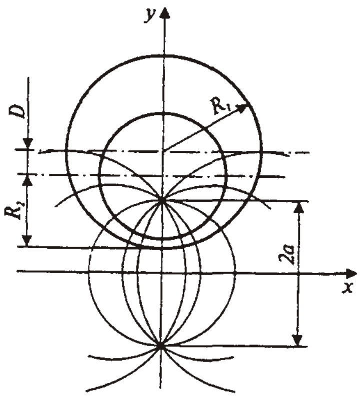

Fig. 1. Geometry of the problem.

$$H^2 = \frac{4a^2\left[r^4 + R_i^4 + 2r^2\left(R_i^2\mathrm{ch}^2U_i + a^2\right) + 4r\left(r^2 + R_i^2\right)R_i\mathrm{ch}U_i\sin\varphi + 4r^2R_i^2\sin^2\varphi\right]}{\left[4a^2R_i^2\mathrm{ch}^2U_i - \left(R_i^2\mathrm{ch}^2U_i + a^2 + r^2\right)^2 - 4r\left(r^2 + R_i^2\right)R_i\mathrm{ch}U_i\sin\varphi - 4r^2R_i^2\sin^2\varphi\right]^2}. \tag{5}$$

The distribution of the magnetic field strength on the internal cylinder (shaft) surface for the values $r = R_i$, $R_i = R_2$, $U_i = U_2$, taking into account (4), has the form

$$H = \frac{\mathrm{sh}U_2}{R_2\left(\mathrm{ch}U_2 + \sin\varphi\right)}. \tag{6}$$

The obtained relation determines the field in the gap with an accuracy within a dimensional constant. From (4) we obtain

$$a = \frac{R_2^2}{2D}\left\{\left[\left(\frac{R_1}{R_2}\right)^2 - 1\right]^2 - 2\left[\left(\frac{R_1}{R_2}\right)^2 + 1\right]\left(\frac{D}{R_2}\right)^2 + \left(\frac{D}{R_2}\right)^4\right\}^{1/2}, \quad \mathrm{sh}U_2 = \frac{a}{R_2}. \tag{7}$$

According to (6) and (7) for the coaxial cylinders for $D \to 0$ the field within the gap $H_0 = 1/R_2$ and (6) can be written in the form

$$\frac{H}{H_0} = \frac{\mathrm{sh}U_2}{\mathrm{ch}U_2 + \sin\varphi}, \tag{8}$$

where H_0 is the magnetic field strength in a uniform gap between the cylinders.

It is evident from (1) that the magnetic pressure per unit area of the ferromagnetic surface is $p = \mu_0H^2/2$ and if one defines $p_0 = \mu_0H_0^2/2$, the relative magnetic pressure on the ferromagnetic shaft surface is

$$\bar{p} = \frac{p}{p_0} = \left(\frac{\mathrm{sh}U_2}{\mathrm{ch}U_2 + \sin\varphi}\right)^2.$$

The density of the volume forces in the magnetic fluid is given by the equation

$$f_m = \mu_0 J\nabla H,$$

by substituting into it the gradient, evaluated from (8), we determine the relative value

$$\bar{f}_m = \frac{f_m}{\mu_0J_0H_0/R_2} = -\frac{\mathrm{sh}U_2\cos\varphi}{\left(\mathrm{ch}U_2 + \sin\varphi\right)^2}.$$

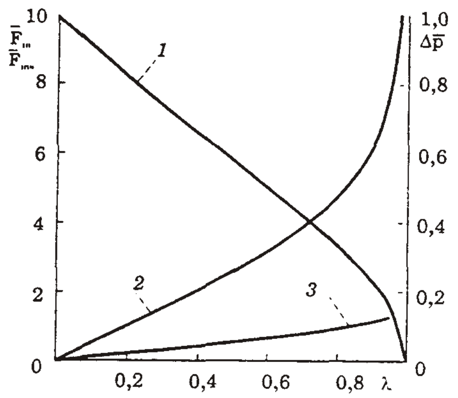

Fig. 2. Eccentricity dependence of the forces (1, 2) and of the retention pressure (3). 1) $\bar{F}_m$, 2) $\bar{F}_{ma}$, and 3) $\Delta\bar{p}$.

We perform a transformation which allows us to simplify further computations. By defining $\lambda = D/(R_1 - R_2)$, we can write

$$a = \frac{R_1 + R_2}{2}\frac{\sqrt{1-\lambda^2}}{\lambda}\left[1 - \left(\lambda\frac{R_1 - R_2}{R_1 + R_2}\right)^2\right]^{1/2},$$

since in general $(R_1 - R_2) \sim 10^{-4}$ m, and $(R_1 + R_2) > 10^{-2}$, the last term in the square brackets can be neglected. Furthermore, with a sufficient degree of accuracy it can be assumed that $(R_1 + R_2)/2 \approx R_2$.

Then $a = R_2(1 - \lambda^2)^{1/2}/\lambda$, $\sinh U_2 = (1 - \lambda^2)^{1/2}/\lambda$, $\cosh U_2 = \lambda^{-1}$, and from (7), after substitutions, we obtain

$$\frac{H}{H_0} = \frac{\sqrt{1-\lambda^2}}{1+\lambda\sin\varphi}. \tag{9}$$

By substituting (9) into Eq. (1), we obtain

$$F_m = 2\mu_0 R_2 H_0^2\left[4\lambda + \frac{\arcsin\lambda}{\sqrt{1-\lambda^2}}\right];\quad F_{ma} = 4\mu_0 R_2 J_0 H_0 \operatorname{arctg}\frac{\lambda}{1-\lambda^2}.$$

These forces are directed toward opposite sides and determine the shaft's position.

Figure 2 shows variation nature of the dimensionless force components in the λ function:

$$\bar{F}_m = \frac{F_m}{2\mu_0 R_2 H_0^2},\quad \bar{F}_{ma} = \frac{F_{ma}}{4\mu_0 R_2 J_0 H_0^2}.$$

For real parameter values, the saturation magnetization of the magnetic fluid generally is an order of magnitude lower than the magnetic field strength; therefore, the buoyancy force is practically always smaller than the magnetic attraction of the shaft in the bushing. It follows from an estimate of the orders of magnitude of the terms that the attraction force is an order, or more, greater than the magnetic buoyancy force and therefore in practical calculations, the latter can be neglected.

The pressure gradient, retaining by compression, can be determined from the relation $\Delta p = J \cdot B$.

The noncoaxial position of the shaft and the pole results in a lowering of the gradient, retaining by compression. With the assumptions we made with respect to the gradient in a uniform gap, the actual gradient is

$$\Delta\bar{p} = \sqrt{\frac{1-\lambda}{1+\lambda}}.$$

According to the variation presented in Fig. 2 for preserving the magnitude of the retaining gradient within 20% limits from the nominal, the eccentricity should not exceed 0.20-0.21.

Thus, in designing MFS and to determining the service life and the actual retaining gradient, it is necessary to take into account the noncoaxiality of the shaft and the poles.

All the results were obtained in this investigation with rather drastic approximations, in which the actual device geometry was not taken into account. However, all relations are given per unit length, and with a sufficiently large experimental statistical base, they can be used by introducing empirical coefficients.

REFERENCES

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