DISTRIBUTION OF FLOW VELOCITIES IN A RECTANGULAR PIPE SITUATED WITH THE LONG SIDE OF ITS CROSS SECTION IN THE DIRECTION OF A MAGNETIC FIELD

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Results are presented for computer calculations of the velocity distributions in rectangular pipes with cross sections of small side ratio, situated in a transverse magnetic field, on the basis of Shercliff's exact solution for laminar flow. Experimental data are also given for the effect of a transverse magnetic field on the flow velocity of a conducting fluid (mercury) at the center of a pipe with a cross section side ratio of β = 0.133. It is established that the experimental results in both the turbulent region as well as in the region of laminar flow are in good agreement with both the semiempirical theory and the exact solution.

INTRODUCTION

It has recently been shown that the effect of a transverse magnetic field on flow in a pipe of rectangular cross section is quite different when the field is parallel to the short side of the cross section and when it is parallel to the long side [1–3]. In particular it has been shown that when the field is parallel to the long side of the cross section it exerts a very strong damping action on the turbulence caused by the sharp decrease of the resistance coefficient.

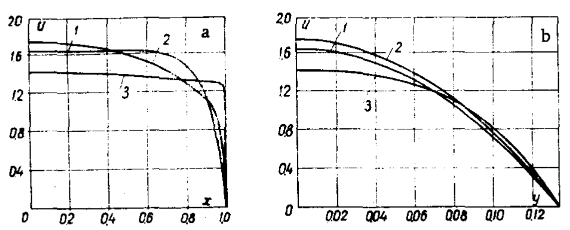

Fig. 1a, b. Velocity distribution in two mutually perpendicular directions in a pipe with side ratio $\beta = 0.133$ (coordinate origin situated in the center of the pipe, the x axis in the direction of the magnetic field induction vector, scale length chosen to be $a/2$; 1) Ha = 0; 2) Ha = 5.8; 3) Ha = = 49.2; u is the ratio of the absolute velocity to the mean flow velocity).

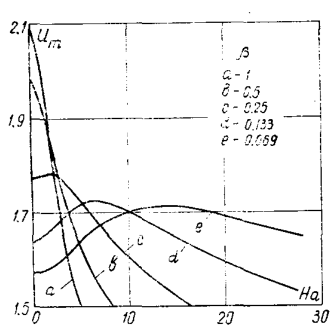

Fig. 2. Relative velocity at the center of the pipe as a function of the Hartmann number.

Since the velocity distribution in this interesting case of flow had not yet been studied the present investigation was carried out.

Computer calculations were made of the velocity distribution for laminar flow on the basis of the theoretical solution of [4]. For turbulent flow both experimental investigations were made as well as theoretical calculations from the semiempirical theory of [5].

LAMINAR FLOW

According to Shercliff [4], the flow velocity of an electrically conducting fluid in a pipe of rectangular cross section situated in a transverse magnetic field may be calculated from the function

$$U = \frac{8\lambda \operatorname{Re} \beta^2}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \left\{ 1 + \frac{\operatorname{ch} m_1 x \operatorname{sh} m_2 a - \operatorname{ch} m_2 x \operatorname{sh} m_1 a}{\operatorname{sh}(m_1 - m_2) a} \right\} \cos \frac{(2n+1)\pi y}{2b}, \quad (1)$$

where Re = Va/ν is the Reynolds number (V is the mean flow velocity); a is the side of the pipe parallel

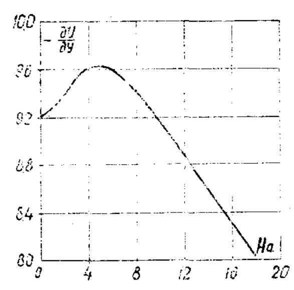

Fig. 3. Effect of the magnetic field on the behavior of the velocity gradient at a point in the stream with coordinates x = 0; y = 0.05 (β = 0.133).

to the magnetic field; b is the side perpendicular to the magnetic field; $\beta = b/a$; x is the axis of symmetry of the cross section parallel to the magnetic field: y is the axis of symmetry of the cross section perpendicular to the magnetic field; m_1 and m_2 are the roots of the quadratic equation $m^2 + \mathrm{Ha}(m/a) - (2n+1)^2\pi^2/4b^2 = 0$; Ha = $= Ba(\sigma/\eta)^{1/2}$ is the Hartmann number; λ is the resistance coefficient given by

$$\lambda = \frac{\pi^2}{16\,\mathrm{Re}} \left[\sum_{n=0}^{\infty} \frac{1}{(2n+1)^4} \times \right.$$

$$\left. \times \left\{ 1 - \frac{2\,\mathrm{Ha}_1\,\beta^2(\mathrm{ch\,Ha}_1 - \mathrm{ch\,Ha})}{\pi^2(2n+1)^2\,\mathrm{sh\,Ha}_1} \right\} \right]^{-1} \quad (2)$$

(here $\mathrm{Ha}_1^2 = \mathrm{Ha}^2 + (2n+1)^2\pi^2/\beta^2$).

Up until the present calculations from Shercliff's formulas have been usually carried out for $\beta \geq 1$. We are interested in the case when $\beta \ll 1$. Calculations made for $\beta = 1$; 0.5; 0.25; 0.133; 0.069 showed that the velocity distribution for small values of β is different in a series of interesting properties. Firstly, when a transverse magnetic field is applied to the flow of a conducting fluid the velocity profile does not begin to flatten out immediately. On the contrary, it becomes markedly extended at first the larger the Hartmann number (Fig. 1), and then, beginning at some value of Ha, which is larger as β is smaller, it flattens out (Fig. 2).

It also follows from Fig. 1 that in this case the velocity gradient at both walls increases continuously, which is in complete agreement with the familiar increase of the resistance coefficient. It is clear from Fig. 2 that an extension of the profile may be observed for $\beta = 0.25$ and this increases as β decreases. Calculations of local friction at a point in the stream with coordinates x = 0, y = 0.05 (Fig. 3) confirm this phenomenon.

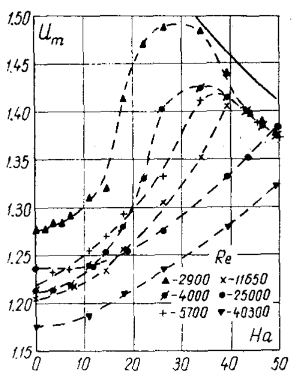

Fig. 4. Experimental data for the relative velocity at the center of the pipe as a function of the Hartmann number (the full line corresponds to the exact solution of Shercliff [4] for laminar flow).

TURBULENT FLOW

Experimental measurements were made of the local velocity (velocity at the center of the flow core) in a pipe with non-conducting walls (plastic). The dimensions of the transverse cross section were $0.4 \times 3\,\mathrm{cm}^2$ ($\beta = 0.133$). The apparatus used is described in detail in paper [1]. The overall length of the pipe was 42.5 cm, of which the last 38 cm were situated between the poles of an electromagnet. The total pressure was measured with the help of a Pitot tube with inner and outer diameters of 0.8 and 1.1 mm. The static and total pressures were sampled at a single cross section situated at a distance of 23 cm from the place where the stream entered the magnetic field. The Hartmann and Reynolds numbers calculated from the hydraulic radius of the pipe varied within the limits $0 \leq \mathrm{Ha} \leq 49.2$; $2900 \leq \mathrm{Re} \leq 40\,300$ in the experiment.

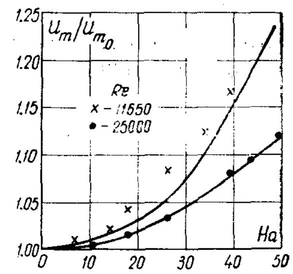

Fig. 5. Comparison of experimental data and semi-empirical theory [5] U_{m_0} is the velocity at the center of the stream in the absence of a magnetic field).

We see from Fig. 4 that as the Hartmann number increases the velocity at the center of the stream increases at first, attaining a maximum value, and subsequently decreases roughly in agreement with the exact solution of Shercliff for laminar flow. The discrepancy between experimental data and theory does not exceed 4% here.

We note that in the region far removed from laminar flow our results are in satisfactory agreement with the semi-empirical theory of [5], and the larger the Reynolds number the closer is the agreement (Fig. 5).

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