# EXPERIMENTAL STUDY OF THE MAGNETOHYDRODYNAMIC WAKE BEHIND A CIRCULAR CYLINDER

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Experimental data on the effect of a transverse magnetic field on flow in the wake behind a circular cylinder in a rectangular channel are presented. Some peculiarities of this effect are explained. It is found, in particular, that for a fixed wake cross section there exists a value of the Stewart number $N_*$ such that when $N < N_*$ the magnetic field not only does not equalize the velocity distribution but increases the velocity defect in the wake. For a fixed Stewart number there exists a wake cross section $x = x_*$ such that when $x < x_*$ the velocity defect in the wake is greater with a magnetic field than without.

1. The experiments were performed in the closed mercury circuit described in [1]. The working part of the circuit was a 46 × 70-mm rectangular channel 1000 mm long. The side walls of the channel, which were perpendicular to the magnetic-induction vector, were made of plastic. The other walls were of copper. A plastic cylinder with diameter d = 8 mm was installed parallel to the side walls 470 mm from the channel input. This channel length was sufficient to stabilize the flow with or without a field. As experiments show, the length of the stabilization part with a magnetic field is greatly dependent upon the shape of the input profile. If very nonuniform flow (a jet) enters a magnetic field, the length of this part may be considerably greater than without a magnetic field [1]. But if the input profile is almost uniform (with the exception of the regions near the channel walls), it will terminate at a distance that exceeds the channel width by a factor of about five [2].

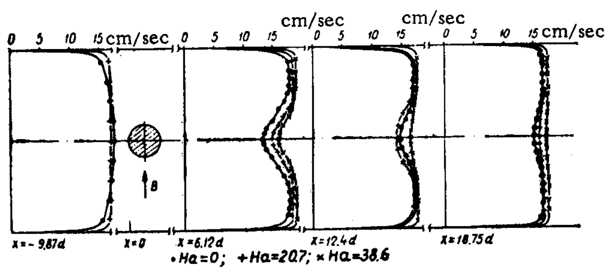


Fig. 1

The procedure for measuring the flow rate and velocity distribution is similar to that described in [1]. Measurements were made at distances of 6.12d, 12.4d, and 18.75d from the leading critical point of the cylinder 35 mm from the bottom of the channel, which was equal to one-half the height of the cylinder. Velocity was measured with a Pitot-Prandtl tube; the Stewart number, which was calculated using the outside diameter of the nose of the tube, did not exceed 0.05. In accordance with [3], therefore, precalibration of the tube for the magnetic field was not necessary.

2. It is known that the velocity profile in developed turbulent flow in rectangular channels is greatly dependent upon the magnetic-field induction [2]. Therefore, the velocity profile in the channel in front of the cylinder was measured before the velocity distribution was measured. These profiles are shown on the left in Fig. 1. It can be seen that the incident-flow profile is a function of the magnetic induction.

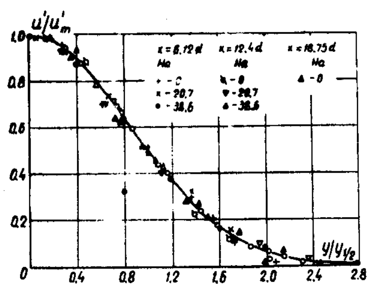


Fig. 2

Figure 1 also shows the results of velocity-profile measurements in three wake cross sections for three Hartmann numbers and Re = 10 500. It can also be seen from the figure that all profiles are entirely symmetric relative to the wake axis x (here and below we shall let x be the coordinate along the wake axis, y the wake-width (i.e., field-line) coordinate, and z the cylinder-height coordinate).

In processing the experimental results, the maximum velocity was used as the wake velocity. As follows from Fig. 1, this velocity increases with an increase in magnetic induction.

Note that in the described experiments the wake width was comparable with the channel width (Fig. 1). Thus, the side walls distorted the flow pattern in the wake. In particular, the velocity defect on the wake axis does not obey the well-known relation [4]

$$u' = C_x/\sqrt{x}, \tag{1}$$

where $C_x$ is drag coefficient of the cylinder.

Figure 2 shows the results of all measurements in coordinates $u'/u'_m = f(y/y_{1/2})$, where $u' = (u_0 - u)/u_0$, $u'_m$ is the value of u' on the wake axis, $u_0$ is the wake velocity, and $y_{1/2}$ is the value of the y coordinate for which $u' = u'_m/2$.

The solid curve in this figure corresponds to

$$u'/u'_m = e^{-\ln 2\, y^2/y_{1/2}^2}. \tag{2}$$

As is apparent from Fig. 2, all experimental data are well described by relation (2).* This means that the velocity-defect relation in the wake has the form

$$u' = u'_m(x)\, e^{-y^2/\delta^2(x)},$$

where the function $\delta(x)$ characterizes the wake width. The specific form of the functions $u'_m(x)$ and $\delta(x)$ is determined by the dependence of the turbulent viscosity $\nu_T$ upon x.

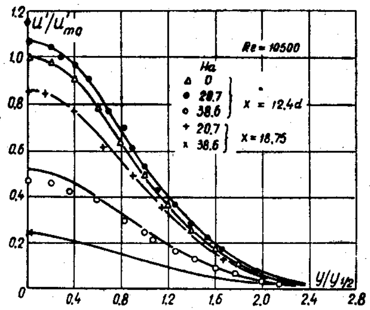


Fig. 3

Let us assume that the turbulent-viscosity coefficient is independent or x, i.e., $\nu_T$ = const, where this constant is, generally speaking, a function of the magnetic induction. Then the velocity defect in the wake is determined by [5]

$$u' = \frac{A(N)}{\sqrt{x}}\, e^{-Nx} e^{-\mathrm{Re}_T \frac{y^2}{4x}}. \tag{3}$$

Here and below, u' refers to the incident-flow velocity $u_0$, and x and y refer to the diameter of the cylinder; $N = \sigma B^2 d/\rho u_0$ is the Stewart number; $\mathrm{Re}_T = u_0 d/\nu_T$ is the turbulent Reynolds number; and A(N) is a constant. It is known [4] that A(0) is proportional to the drag coefficient of the body. It is natural to assume that this relation is preserved at least for small N, i.e., $A(N)/A(0) = C_x/C_{x0}$, where $C_x$ and $C_{x0}$ are the drag coefficients with an without a magnetic field, respectively.

Since the wake flow is laminar at a sufficient distance from the body, it is advisable to represent the viscosity coefficient as

$$\nu_\Sigma = \nu_T + \nu$$

where $\nu$ is the molecular-viscosity coefficient.

If we assume that $\nu_T \sim u'_m(x)\delta(x)$, $N(\nu/\nu_\Sigma) \ll 1$, and $u' = u_0 u'_m(x) f(y/\delta(x))$, then from the linearized wake motion equation we obtain

$$u' = A\, e^{-Nx}\left(\frac{1-e^{-Nx}}{N} + \frac{\nu}{\nu_{T0}}\right)^{-1/2} \times$$

$$\times \exp\left\{-\mathrm{Re}_{T0}\, y^2/4\left(\frac{1-e^{-Nx}}{N} + \frac{\nu}{\nu_{T0}}\right)\right\},$$

where $\nu_{T0}$ is the value of $\nu_T$ at N = 0, $\mathrm{Re} = u_0 d/\nu_T$, and $\mathrm{Re} = u_0 d/\nu$.

At $N \ll 1$, $\nu = 0$, and not very high x, we obtain Eq. (3).

It is experimentally known [6] that $C_x/C_{x0} = 1 + 3.4N^{1/2}$ for a circular cylinder. Now relation (3) can be represented as

$$u'/u'_{m0} = (1 + 3.4N^{1/2})\, e^{-Nx} e^{-\mathrm{Re}_T \frac{y^2}{4x}}, \tag{4}$$

where $u'_{m0}$ is the value of $u'_m$ at N = 0.

On the wake axis

$$u'_m/u'_{m0} = (1 + 3.4N^{1/2})\, e^{-Nx}. \tag{5}$$

It is easy to see that the function on the right side of Eq. (5) has a maximum with respect to N when

$$N = N_* = \left[\frac{(1 + 26.32x^{-1})^{1/2} - 1}{6.8}\right]^2. \tag{6}$$

Thus, in any fixed wake cross section, as the field is increased $u'_m$ increases at first and decreases only when $N > N_*$. This conclusion is experimentally confirmed. Figure 3 shows the measurement results in coordinates $u'/u'_{m0} = f(y/y_{1/2})$ for cross section x = = 12.4.

The solid curves there correspond to relation (4):

$$u'/u'_{m0} = (1 + 3.4N^{1/2})\, e^{-Nx} e^{-\ln 2\, y^2/y_{1/2}^2} \tag{4'}$$

and are in good agreement with the experimental data. Specifically, the upper curve in Fig. 3 corresponds to the case when the velocity defect is greater with a magnetic field than without it. The figure also shows that point corresponding to the maximum of $u'_m/u'_{m0}$ in cross section x = 12.4

$$(u'_m/u'_{m0})_{\max} = 1.17 \quad \text{at} \quad \mathrm{Ha}_* = 11.57.$$

3. The increased velocity defect in the wake for low Stewart numbers can be explained as follows.

In a transverse magnetic field, the velocity distribution in the wake at a sufficient distance from the body is determined by the drag of the body and the electromagnetic body forces.

The drag increases in a transverse magnetic field, and this must increase the velocity defect in the wake. On the other hand, electromagnetic body forces in the wake tend to equalize the velocity profile. The relationship between these two factors also determines the increase or decrease of the velocity defect in the wake.

With low Stewart numbers, the drag-increase effect dominates, since the drag and, therefore, the

*Appreciable deviations from this relation are observed near the wake boundaries. This can be explained by the low measurement accuracy in this region, which is confirmed by the increased spread of points near the wake boundary.

velocity defect increase linearly with $N^{1/2}$, while the decrease due to electromagnetic forces in the wake is proportional to N.

At high Stewart numbers, these effects exchange roles.

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