# SELF-OSCILLATIONS IN A SINGLE-PHASE CONDUCTION MHD GENERATOR

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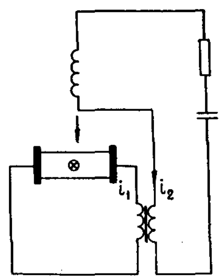
We study a self-excited single-phase ac conduction MHD generator with an intermediate transformer. A modification of the Krylov-Bogolyubov method of equivalent linearization makes it possible to compile and analyze the nonlinear equations of oscillation build-up. We investigate the hard and soft regimes of oscillation build-up, the achievement of steady states, and their stability. We analyze the effect of the intermediate transformer on the oscillation conditions in the circuit.

The self-oscillation regime of conduction MHD-generator operation in the purely linear formulation




was covered in [1, 2]. As is well known, a self-oscillating system must include a nonlinear element—an oscillation-amplitude limiter. The possible states of oscillation, their stability, the oscillation amplitude, and the nature of the oscillation build-up cannot be determined from the linear formulation. The asymptotic methods of the theory of oscillations make it possible to solve the corresponding nonlinear problem and to determine the above-cited parameters. Let us consider the oscillations that are close to the harmonic. In this case, sufficient accuracy is offered by the solutions which correspond to the first-approximation equations of the theory of oscillations [3, 4].

We will employ the modification of the Krylov-Bogolyubov method of equivalent linearization which makes it possible to equivalently linearize the existing nonlinear parameters, and to compile and solve the system of linearized algebraic equations and to derive the first-approximation equations from this solution, these equations determining the nature of the oscillation build-up. The equivalently linearized parameters are determined by means of the principle of harmonic balance.

Assuming $i = I\cos(\omega t + \varphi)$ and knowing $\Psi(i)$, for example, we can determine the linearized inductance

$$L=\frac{1}{\pi\omega I}\int_0^{2\pi}\frac{d}{dt}\{\Psi I\cos(\omega t+\varphi)\}\times \times\sin(\omega t+\varphi)\cdot d(\omega t+\varphi). \quad (1)$$

Here $\Psi(i)$ is the relationship between the flux linkage and the current.

The problem of oscillation build-up in circuits employing an intermediate transformer is solved under the following assumptions:

1. The fluid velocity V is constant through the cross section of the channel.
2. The effect of vortex and terminal currents, as well as the influence of electrically conductive walls, are neglected.
3. The reaction of the current flowing in the channel is compensated. The figure shows a circuit for a self-exciting conduction generator. The incompressible electrically conductive fluid flows perpendicularly to the plane of the figure, and the magnetic field is produced by an excitation winding with inductance L.

To analyze this circuit, we use the Kirchhoff equations for complex amplitudes:

$$\dot{\mathcal{E}}+j\omega w_1\dot{B}_{tr}Q_{tr}+\dot{I}_1(R_1+r_1+j\omega L_1),$$
$$-j\omega w_2\dot{B}_{tr}Q_{tr}=\dot{I}_2[r_2+r_t+R+ + j\omega(L+L_s+L_2)-j(\omega C)^{-1}]; \quad (2)$$

the equation relating the induction $\dot{B}_{tr}$ in the transformer core (the core section $Q_{tr}$) with the magnetizing current $\dot{I}_\mu$ is

$$\dot{B}_{tr}=\dot{I}_\mu L_\mu/w_1Q_{tr}; \quad (3)$$

the equation of ampere-turn equilibrium is

$$\dot{I}_\mu=\dot{I}_1+\dot{I}_2w_2/w_1; \quad (4)$$

the feedback-circuit equation for the emf $\dot{\mathcal{E}}$ in the channel as a function of the excitation current is

$$\dot{\mathcal{E}}=-\dot{I}_2LV/wl. \quad (5)$$

Subscript 1 pertains to the parameters of the primary transformer circuit; subscript 2 pertains to the secondary transformer circuit; $L_s$ is the leakage inductance of the excitation winding; $L_\mu$ is the magnetization inductance of the transformer; w and $r_t$ are the number of turns and the resistance of the excitation winding, respectively; $l$ is the length of the generator channel; $R_i$ is the internal resistance of the channel.

Equations (1)–(5) reduce to a system of two linear homogeneous algebraic equations:

$$[j\omega(L_\mu+L_1)+(R_i+r_1)]\dot{I}_1+\left[j\omega L_\mu\frac{w_2}{w_1}+\frac{L}{wl}V\right]\cdot\dot{I}_2=0,$$
$$j\omega\frac{w_2}{w_1}L_\mu\dot{I}_1+\left[(r_2+r_t+R)+ + j\omega\left(\frac{w_2^2}{w_1^2}L_\mu+L+L_s+L_2-\frac{1}{\omega^2C}\right)\right]\cdot\dot{I}_2=0. \quad (6)$$

System (6) has a nontrivial solution if its determinant is equal to zero. Having separated the real and imaginary parts of the determinant, from the above condition we can obtain the following oscillation conditions:

$$\frac{L}{wl} V\left(\frac{w_2}{w_1}\right) = \left(1+\frac{L_1}{L_\mu}\right)(r_2+r_t+R) + \\ +\frac{R_i+r_1}{L_\mu}\left(\frac{w_2^2}{w_1^2}L_\mu+L+L_s+L_2-\frac{1}{\omega^2 C}\right), \quad (7)$$

$$\omega^2\left[L_1\frac{w_2^2}{w_1^2}L_\mu+(L_\mu+L_1)(L+L_s+L_2)\right]= \\ =\frac{L_\mu+L_1}{C}+(R_i+r_1)(r_2+r_t+R). \quad (8)$$

Note that the nonlinear parameters in Eqs. (7) and (8) have been equivalently linearized, i.e., they are current functions. Equation (7) therefore makes it possible to determine the possible steady states, i.e., the stable and unstable limit cycles, while Eq. (8) makes it possible to determine the frequency of the steady-state oscillations.

In purely linear formulation, Eqs. (7) and (8), respectively, determine the critical velocity and the oscillation frequency.

When there are many nonlinearities, the equations for the oscillation build-up—derived from (7) and (8)—are not easily analyzed.

Let us only note that the saturated transformer for which $L_\mu(I_\mu)$ is substantially nonlinear may exert a strong effect on the oscillations, as far as to produce a pronounced distortion of their shape. With sufficiently large $L_\mu$, i.e., when the transformer is not saturated, Eqs. (7) and (8) are simplified and assume the form

$$k_{tr}VL/wl-R^*=0; \quad (9)$$

$$\omega=k_{tr}(L^*C)^{-1/2}. \quad (10)$$

Here $k_{tr} = w_2/w_1$ is the transformation factor, while $R^*$ and $L^*$ are the total resistance and inductance introduced into the primary circuit:

$$R^*=R_i+r_1+(r_2+r_t+R)k_{tr}^2, \quad L^*=L_1+(L+L_2+L_s)k_{tr}^2.$$

Using (9) and (10), the settling equations for the current amplitude and the frequency are, according to [3]:

$$\frac{dI_1}{dt}=I_1\frac{1}{2k_{tr}^2}\left(\frac{1}{wl}Vk_{tr}-\frac{R^*}{L}\right), \quad (11)$$

$$\omega+\frac{d\varphi}{dt}=\frac{k_{tr}}{\sqrt{L^*C}}. \quad (12)$$

We examine two cases of a nonlinear function L(i) corresponding to

$$\Psi=b_1 i-b_2 i^3, \quad (13)$$

$$\Psi=b_1 i+b_2 i^3-b_3 i^5. \quad (14)$$

The equivalently linearized inductances for (13) and (14), determined from (1), are

$$L=b_1-{}^3/_4 b_2 I^2, \quad (15)$$

$$L=b_1+{}^3/_4 b_2 I^2-{}^5/_8 b_3 I^4. \quad (16)$$

Having substituted (15) into (11), we obtain the settling equation for the current:

$$\frac{dI}{dt}=\mu I\frac{1-(\alpha/\mu)I^2}{b_1(1-\gamma I^2)}=\mu\Phi(I), \quad (17)$$

where

$$\mu=\frac{1}{2}\left(\frac{V}{wlk_{tr}}-\frac{R^*}{b_1k_{tr}^2}\right); \quad \alpha=\frac{3b_2V}{wb_1lk_{tr}}; \quad \gamma=\frac{3}{4}\frac{b_2}{b_1}.$$

The possible steady states correspond to $\Phi(I) = 0$. The equilibrium, according to Lyapunov, is stable if $\mu\Phi'(I_k) < 0$, where $I_k$ denotes the roots of the equation $\Phi(I) = 0$. Equation (17) has two possible steady states. The first—$I = I_1 = 0$, $dI/dt = 0$—is the state of stable equilibrium when $\mu < 0$ but is unstable when $\mu > 0$, since $\mu\Phi'(I_1) = \mu/b_1$. The second possibility—$I = I_2 = 2\sqrt{\mu/\alpha}$, $dI/dt = 0$ when $\mu > 0$—yields the limit cycle corresponding to the real self-oscillation process. The limit cycle is stable because $\mu\Phi'(I_2) = -2\mu/b_1 < 0$. In this case we have a soft-excitation regime; regardless of the initial conditions, on reaching the critical parameters, in this case $\mu > 0$, an oscillation regime with the amplitude $I = 2\sqrt{\mu/\alpha}$ is established in the system. In addition, we note that Eq. (17) can easily be integrated. The solution, with arbitrary constant $C_0$ is

$$I^2\left(1-\frac{\alpha}{\mu}I^2\right)^{\frac{\gamma R^*}{\mu}}=C_0 e^{\frac{2\mu}{b_1}t}. \quad (18)$$

The steady-state value of the frequency is determined from the relationship

$$\omega=\left[b_1C\left(1-\frac{R^*}{\mu+R^*}\right)\right]^{-1/2}. \quad (19)$$

Substitution into (11) of the inductance in the form of (16) yields the settling equation for the current, which describes both the soft and the hard regimes of oscillation build-up

$$\frac{dI}{dt}=\frac{I[\alpha_1+\alpha_2I^2-\alpha_3I^4]}{2(b_1+{}^3/_4b_2I^2-{}^5/_8b_3I^4)}=\Phi(I). \quad (20)$$

Here

$$\alpha_1=\frac{V}{wlk_{tr}}-\frac{R^*}{b_1k_{tr}^2}, \quad \alpha_2=\frac{3}{4}\frac{b_2V}{b_1wl}, \quad \alpha_3=\frac{5}{8}\frac{b_3V}{b_1wl}.$$

Let us examine the possible steady state (20). Here $dI/dt = \Phi(I) = 0$ yields

$$I_1=0; \quad I_2=\sqrt{\frac{\alpha_2}{2\alpha_3}+\sqrt{\frac{\alpha_2^2}{4\alpha_3^2}+\frac{\alpha_1}{\alpha_3}}}; \quad I_3=\sqrt{\frac{\alpha_2}{2\alpha_3}-\sqrt{\frac{\alpha_2^2}{4\alpha_3^2}+\frac{\alpha_1}{\alpha_3}}}.$$

Depending on the relationships of the parameters, the system may have either a single real root, $I = I_1$, if $\alpha_1 < 0$ and $|\alpha_1\alpha_3| \geq \alpha_2^2/4$, or two real roots $I = I_1$ and $I = I_2$, if $\alpha_1 \geq 0$, or all three real roots $I_1$, $I_2$, and $I_3$, if $\alpha_1 < 0$ and $|\alpha_3\alpha_1| < \alpha_1^2/2$.

*In the first case, the system exhibits only a state of equilibrium* $I = I_1 = 0$, $dI/dt = 0$. The equilibrium is stable, since $\Phi'(I_1) = \alpha_1/2L < 0$. In the second case, the system exhibits a state of unstable equilibrium $I_1 = 0$, $dI/dt = 0$, since $\Phi'(I_1) = \alpha_1/2L > 0$, and the steady state $I = I_2$ corresponding to the limit cycle, which is stable since

$$\Phi'(I_2)=\frac{\alpha_1}{2L}+\frac{1}{2L}\left[\frac{\alpha_2}{2\alpha_3}+\sqrt{\frac{\alpha_2^2}{4\alpha_3^2}+\frac{\alpha_1}{\alpha_3}}\right]\times \\ \times\left[\frac{\alpha_2}{2}\left(1-5\sqrt{1+\frac{4\alpha_3\alpha_1}{\alpha_2^2}}\right)\right]<0,$$

because the first set of brackets is always positive, the second set of brackets is always negative, and $\alpha_1$ differs little from zero. Thus, as before, we have the soft regime of oscillation build-up. In the third case, the system exhibits a state of stable equilibrium $I_1 = 0$; $dI/dt = 0[\Phi'(I_1) = \alpha_1/2L < 0]$, the steady state $I = I_3$, $dI/dt = 0$, corresponding to the unstable limit cycle $[\Phi'(I_3) > 0]$, and the steady state $I = I_2$, $dI/dt = 0$, corresponding to the stable limit cycle $[\Phi'(I_2) < 0]$. Thus, to obtain stable self-oscillations with the amplitude $I = I_2$ we need certain initial conditions and, namely: $I_0 > I_3$, since otherwise the oscillations will attenuate. Here we have the hard regime of oscillation build-up.

Note that Eq. (20) can also be integrated directly; however, the solution is rather cumbersome and is not given here. The oscillation frequency is determined from the equation

$$\omega=\left[\left(b_1+\frac{3b_2}{4}I_2^2-\frac{5}{8}b_3I_2^4\right)C\right]^{-1/2} \tag{21}$$

In a similar manner we can investigate the effect of the nonlinearities C(I), $R^*(I)$, and $L_\mu(I)$ or their combination.

When dI/dt = 0 and $k_{tr} = 1$ for an ideal transformer in linear formulation the conditions of oscillation coincide with the conditions given in [2].

Note that self-oscillations are possible also in circuits similar to the one presented here, but with the load connected in parallel with the excitation winding or in parallel to the capacitance. The corresponding equations for oscillation build-up are given in [5].

## REFERENCES


1. P. R. Smy, J. Appl. Phys., **32**, 10, 1946, 1961.
2. H. Burkhardt, "Time characteristics of self-excited Faraday generators and their use in oscillators," Magnetohydrodynamic Energy Conversion [in Russian], Part I, Moscow, 1966.
3. N. M. Krylov and N. N. Bogolyubov, Introduction to Nonlinear Mechanics [In Russian], UkrSSr, 1937.
4. N. N. Bogolyubov and Yu. A. Mitropol'skii, Asymptotic Methods in the Theory of Nonlinear Oscillations [in Russian], Fizmatgiz, Moscow, 1962.
5. S. E. Dvorchik, Trudy seminara In-ta teplofiziki UkrSSR, (in press).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