MAGNETOHYDRODYNAMIC FLOW IN THE REGION OF A JUMP IN THE CONDUCTIVITY AT THE WALL

Yu. B. Kolesnikov and A. B. Tsinober

UDC 538.4

A study is made of the flow of an electrically conducting liquid with a sharply changing conductivity of the wall, perpendicular to the applied homogeneous magnetic field. It is established experimentally that, in this case, in a plane perpendicular to the field, a non-monotonic velocity profile is formed. It is shown that the reason for its formation is the presence of a gradient of the component of the electric current along the field.

In [1-3] visual experiments were used to disclose that the nonhomogeneity in the electrical conductivity at the flow boundaries perpendicular to the magnetic field leads to a very strong distortion of the flow. Thus, in [3], visual observation of the free surface with the flow of mercury in a trough showed that, in the zone above the electrically conducting region of the wall, perpendicular to the field, there is practically no flow, if the magnetic field is sufficiently great.

In the present article, an attempt is made at a more detailed analysis of the reasons for this type of behavior of the flow, as well as to investigate some of its characteristics (the field of the currents, the velocities).

To clarify the reasons leading to such a strong rearrangement of the structure of the flow, following Shercliff [4] and [5, 6], we go back to the vorticity equation for the velocity field. Here, since the rearrangement of the flow is most considerable in the plane perpendicular to the field, we write this equation for the component of the vorticity, ω_{y1} parallel to the field (Fig. 1):

$$(\mathbf{v}\cdot\mathrm{grad})\,\omega_{y1}=\nu\nabla^2\omega_{y1}+B\,\frac{\partial j_{y1}}{\partial y_1}. \tag{1}$$

It follows from Eq. (1) that there can be a significant distortion of the velocity field only in the presence of a gradient of the y component of the electrical current in the direction of the magnetic field.

We consider below a model problem, from whose solution it follows that, even in the case of a homogeneous velocity profile, a gradient $\partial j_{y1}/\partial y_1$ develops. Thus, even with the flow of a nonviscous liquid, there arises in the flow a nonpotential force, which generates a corresponding component of the vorticity, leading to a rearrangement of the structure of the flow, with the formation of a nonmonotonic velocity profile. In other words, it is precisely the appearance of the gradient $\partial j_{y1}/\partial y_1$ in the flow which leads to the generation of the component of the vorticity ω_{y1} and to a rearrangement of the velocity field.

Let us consider the problem of the distribution of the electrical current with the motion of a medium having a homogeneous velocity profile, in a half-space in a homogeneous magnetic field perpendicular to the plane which bounds the region of the motion (Fig. 1). Part of the plane $y_1=0$, in the form of a circle $r_1 \le R$ ($r_1^2=x_1^2+y_1^2$) is an ideal conductor, while the remaining part, where $r_1>R$, is an insulator.

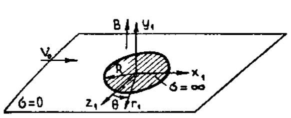

Fig. 1. Statement of the problem.

The boundary value problem of the distribution of the potential of the electrical field is formulated in the following manner:

$$\nabla^2\varphi_1=0, \tag{2}$$

Translated from Magnitnaya Gidrodinamika, No. 1, pp. 70-74, January-March, 1972. Original article submitted August 27, 1971.

$$\varphi_1=0 \text{ at } r_1<R, \quad \partial\varphi_1/\partial y_1=0 \text{ at } r_1>R, \tag{3), (4}$$

$$j_1|_{r_1, y_1=\infty}=0; \quad \frac{\partial\varphi_1}{\partial y_1}=\frac{\partial\varphi_1}{\partial x_1}=0, \quad \frac{\partial\varphi_1}{\partial z_1}=V_0B. \tag{5}$$

The problem (2)-(5) is three-dimensional. It can be easily reduced to two-dimensional, if the solution is sought in the form*

$$\varphi=[r+F(r, y)]\cos\theta. \tag{6}$$

Here the following boundary value problem is obtained for the function F(r, y):

$$\frac{\partial^2F}{\partial r^2}+\frac{1}{r}\frac{\partial F}{\partial r}-\frac{1}{r^2}F+\frac{\partial^2F}{\partial y^2}=0; \tag{7}$$

$$F(r, 0)=-r \text{ at } 0\leqslant r<1; \quad \frac{\partial F(r, 0)}{\partial y}=0 \text{ at } r>1; \tag{8), (9}$$

$$F(r, \infty)=0. \tag{10}$$

This problem can be solved using a Hankel transform:

$$f(\xi, y)=\int_0^\infty rF(r, y)\cdot J_1(r\xi)\,dr. \tag{11}$$

For the Hankel transform, we obtain the equation

$$\frac{d^2f}{dy^2}-\xi^2f=0.$$

Its solution, by virtue of conditions (9), must be taken in the form

$$f(\xi, y)=A(\xi)e^{-\xi y}. \tag{12}$$

To find the function $A(\xi)$, from boundary conditions (8), (9), as well as from (11) and (12) we obtain the following system of pairwise integral equations:

$$\int_0^\infty \xi^{-1}B(\xi)J_1(r\xi)\,d\xi=-r \quad (0\leqslant r<1),$$

$$\int_0^\infty B(\xi)J_1(r\xi)\,d\xi=0 \quad (1<r<\infty), \tag{13}$$

where $B(\xi)=\xi^2A(\xi)$.

System (13) is a partial case of a system of pairwise integral equations described, for example, in monograph [7]. The solution for $B(\xi)$ has the form

$$B(\xi)=\frac{4}{\pi}\left(\cos\xi-\frac{1}{\xi}\sin\xi\right). \tag{14}$$

From (13) and (10) we find [8]:

$$F(r, y)=\frac{2}{\pi}\left[\operatorname{tg}\frac{\varphi}{2}\ (\cos\psi+y\sin\psi)-\psi r\right], \tag{15}$$

where

$$\sin\psi=\frac{1}{a}, \quad \sin\varphi=\frac{r}{a}, \quad 0\leqslant\psi, \quad \varphi\leqslant\frac{\pi}{2}.$$

$$a^2=\frac{1}{2}(r^2+1)+\left[\frac{1}{4}(r^2-1)^2+y^2\right]^{1/2}.$$

In Figs. 2a and 4 the dashed lines show, respectively, the distribution of the component of the current $j_y=-\partial\varphi/\partial y$ and the gradient $\partial j_y/\partial y$, calculated starting from (6) and (15). As is evident from these

*In what follows, we shall consider dimensionless quantities, taking R for the characteristic dimension, and V_0B for the characteristic value of the electrical field.

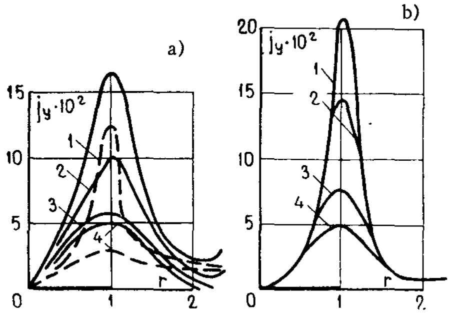

Fig. 2

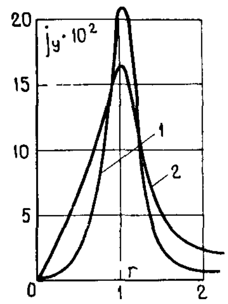

Fig. 3

Fig. 2. Dependence $j_y(r)$ in the direction $\theta=0$: a) $B=0.23$ T, b) $B=0.92$ T. 1) $y=0.16$; 2) $y=0.32$; 3) $y=0.64$; 4) $y=0.8$. Solid lines denote experiment, dashed lines theoretical calculation.

Fig. 3. Dependence $j_y(r)$ in the plane $y=0.16$, $\theta=0$: 1) $B=0.92$ T; 2) $B=0.23$ T.

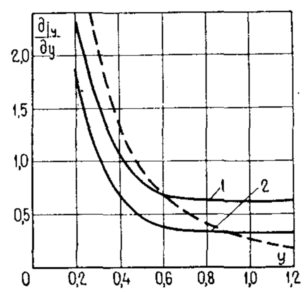

Fig. 4

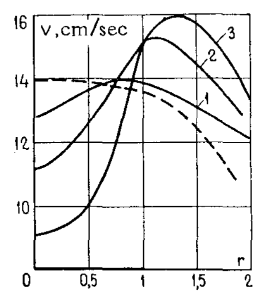

Fig. 5

Fig. 4. Distribution of gradient of y component of electric current along the direction of the magnetic field at $r=1$ and $\theta=0$: 1) $B=0.92$ T; 2) $B=0.23$ T. Solid curves denote experiment, dashed curves theoretical calculation.

Fig. 5. Profiles of the velocities, measured in the plane $y=0.8$ with $B=0.92$ T: 1) at the leading boundary of the insert; 2) in the middle part of the insert; 3) at rear boundary of insert. The dashed curve denotes the velocity profile without insert.

figures, in the neighborhood $r=1$ there are considerable currents, j_y, and gradients, $\partial j_y/\partial y$, which, in the final calculation, lead to the appearance of a nonmonotonic velocity profile. These same figures give also certain experimental results, described below.

2. The experiments were carried out in a mercury loop, described in [9]. The working section was a nonconducting channel with a cross section of 20×60 mm^2. On the wall with a height of 60 mm, perpendicular to the field, there was installed a flush copper insert with a radius of $R=12.5$ mm and a thickness of 0.4 mm. The velocity was measured with a Pitot-Prandtl tube, while the component of the electric current $j_y=-\partial\varphi/\partial y$ was measured using a conduction-type anemometer with a double-electrode pickup, which permitted direct measurement of $\partial\varphi/\partial y$. The arrangement of the coordinate axes is shown in Fig. 1. All the measurements were made at a Reynolds number $Re=11{,}300$, calculated from the radius of the copper insert, in magnetic fields of 0.23 and 0.92 T.

Figures 2-7 illustrate the experimental results. The measurements of the component of the electric current j_y were made with respect to r, at $\theta=0$ and $\theta=\pi$, in planes perpendicular to the induction of

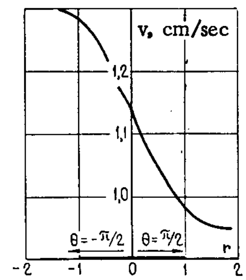

Fig. 6

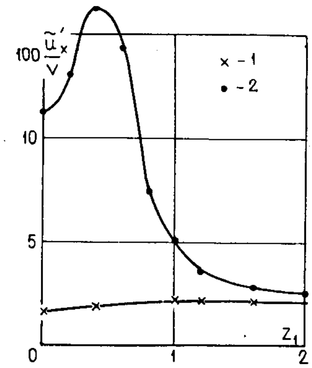

Fig. 7

Fig. 6. Distribution of velocities in the direction of the flow at a distance $y=0.8$ from the insert. $B=0.92$ T.

Fig. 7. Profiles of the intensity of the pulsations of the longitudinal velocity: 1) in the plane $r_1=R$ ($\theta=-\pi/2$); in the plane $r_1=4R$ ($\theta=\pi/2$). $B=0.92$ T.

the magnetic field. In view of the fact that values measured at $\theta=0$ and $\theta=\pi$ differ only in sign, Figs. 2 and 3 give results corresponding only to $\theta=0$.

As is evident from Fig. 2, for both values of the magnetic field, the currents j_y are concentrated mainly in the zone of the jump in the conductivity at the wall, and attain maximal values at $r=1$. This is in agreement with the theoretical result (dashed curves) of the preceding part of the work (we can, of course, speak only of a qualitative comparison between the experimental results and the results obtained analytically in Sect. 1); in this same zone, there are considerable gradients, $\partial j_y/\partial y$. In strong magnetic fields (Figs. 2b), the flow in the region of the insert can be sharply divided into a core where $\partial j_y/\partial y=0$ and a magnetohydrodynamic layer where $\partial j_y/\partial y \neq 0$.

The two curves shown in Fig. 3 exhibit a tendency toward a narrowing of the zone of this layer with an increase in the magnetic field.

It is evident from the course of the curves in Fig. 4 that, at relatively small distances from the wall, there is a sharp drop in the gradient $\partial j_y/\partial y$, with the subsequent establishment of a constant value. The sharp change in the gradient $\partial j_y/\partial y$ indicated that, in the immediate vicinity of the wall, the flow has a three-dimensional character. At distances from the wall where $\partial j_y/\partial y$ has a constant value, the flow can probably be assumed to be plane, in a plane perpendicular to the magnetic field. For purposes of comparison, this same figure plots a dashed curve, corresponding to the solution of the electrodynamic problem in the first part of the article.

Figure 5 gives velocity profiles, measured along r, with $\theta=0$ (with $\theta=\pi$, the picture of the velocity distribution is given symmetrically). It is evident from the figure that the flow in the region of the conducting insert is impeded and, in the zone of the jump of the conductivity at the wall a sharply expressed nonmonotonic profile is formed. It must also be noted that with increasing distance away from the cross section corresponding to the leading edge of the insert, the defect in the velocity increases downstream. The flow then becomes of the wake-type.

It may be concluded from Fig. 6 that a substantial deformation of the velocity profile takes place in the region between cross sections corresponding to the leading and trailing edges of the conducting insert.

The presence of points of inflection on the velocity profiles (Fig. 5) should lead to a rapid loss of stability and to the generation of perturbations with a considerable degree of intensity.

The results of measurements of the intensity of the component of the electrical field e'_z (which, in a strong magnetic field, coincides with a high degree of accuracy with the longitudinal pulsation of the velocity $\tilde{u}'_x$ [10]), using a conduction-type anemometer and a three-electrode, are given in Fig. 7. As is obvious from this figure, immediately behind the copper insert, i.e., where there is a sharply nonhomogeneous velocity profile with points of inflection, there are actually observed perturbations, whose intensity attains values on the order of 16% (curve 2). At the same time, near the leading boundary of the insert, where the velocity profile is distorted considerably more weakly, the intensity of the perturbations is only about 2% (curve 1). The measurements of the intensity of the pulsations of the electric current $\tilde{j}'_y$ showed that this value is an order of magnitude less than the intensity $\tilde{e}'_z$, which bears witness to the tendency of the pulsation field toward a two-dimensional structure [11].

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