TWO-DIMENSIONAL TURBULENT FLOW BEHIND A CIRCULAR CYLINDER

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The flow of mercury in the wake behind a circular cylinder with axis parallel to a magnetic field is investigated. We give measurements of the mean and the fluctuating characteristics of the flow by means of a thermoanemometer and a conduction anemometer with a three-electrode pick-up. It is shown that, in a sufficiently strong magnetic field, the flow acquires properties characteristic of two-dimensional turbulent flows.

Interest has recently increased in problems of two-dimensional turbulence [1-7] (see also the references in [7]). This interest is due to the fact that processes are observed in various natural phenomena that are characteristic of two-dimensional turbulent flows. These are large-scale motions in the atmosphere of the Earth and of other planets, large-scale ocean currents, motion of the photosphere of the Sun, and also motions in a number of other astrophysical objects [7-9] (see also references in [8, 9]).

There is great difficulty in the investigation of two-dimensional turbulence in natural objects, and it gives limited information. Therefore it is of interest to obtain two-dimensional turbulent flows under laboratory conditions and to study their laws experimentally.

Until recently the point of view was held that two-dimensional turbulent flows could not be realized because of three-dimensional nonstability [3-6]. However, in [10] it has been shown on the basis of an analysis of known experimental results that two-dimensional turbulent flow can nevertheless be realized under laboratory conditions if an electrically conducting fluid and a sufficiently strong magnetic field are employed for this purpose.

In a recently completed experiment, whose data were processed, measurements of spatial correlations in a magnetohydrodynamic flow through a rectangular channel were made (see [11]) which confirm the basic conclusions of [10]. In particular it was shown that the correlation coefficient in the direction of the magnetic field approaches unity.

Rather than proceeding immediately to the experimental results of the present study, we discuss the question of why, when the magnetic field is increased, the structure of the flow approaches that of a two-dimensional one in the plane perpendicular to the magnetic field.

A rigorous demonstration of this fact, based on the equations of motion, does not exist; we have only heuristic results (see, for example, the references in [10]).

In favor of a tendency toward two-dimensionality thermodynamic considerations based on Le Chatelier's principle [12] may also be adduced. We consider some perturbation with a characteristic time τ (this time is, for instance, inversely proportional to the characteristic frequency of the perturbation, i.e., $\tau \sim \omega^{-1}$). In a magnetic field of sufficient strength the magnetohydrodynamic interaction time $(\sigma B^2/\rho)^{-1}$ can be considerably less than the characteristic time τ or ω^{-1}, i.e., $\sigma B^2\omega/\varrho \gg 1$. Then during the magneto-interaction time the perturbation can be regarded as a closed thermodynamic system. According to Le Chatelier's principle, the perturbation will tend to be reconstructed in such a way that interaction with the magnetic field is reduced to a minimum. This will occur when the Joule dissipation is a minimum, that is, when the electric current vanishes (if this is compatible with the boundary conditions). Then from the

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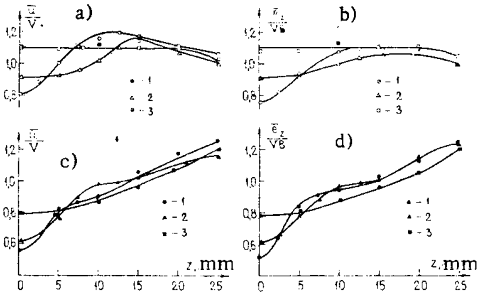

Fig. 1. Profiles of the mean velocity and the constant z component of the electric field at various distances from the cylinder. 1) x = 25 mm; 2) x = 100 mm; 3) x = 400 mm. Measured by: a,c) thermoanemometer; b,d) conduction anemometer; a) B = 0; b) B = 0.08 T; c,d) B = 0.8 T.

equation rot $\mathbf{j} = (\mathbf{B}\nabla)\mathbf{v}$ it follows that $(\mathbf{B}\nabla)\mathbf{v} = 0$, that is, the flow becomes two-dimensional.

In the present work we carried out an experimental investigation of the turbulent flow in the wake behind a circular cylinder whose axis was parallel to the magnetic field, and it is shown that in a strong magnetic field the flow acquires properties that are characteristic of two-dimensional turbulent flows. This flow was the object of our investigation for the following reasons. First, the energy of the pertubations in the wake behind a body is two orders of magnitude higher than that in the case of flow in a channel or a tube. This makes possible a significant improvement in the accuracy of the results. Secondly, the orientation of the axis of the cylinder parallel to the magnetic field facilitates the transition of the flow to a two-dimensional structure. Thirdly a large part of the energy of the perturbations – up to 95% – enters the spectrum in a narrow frequency interval, a circumstance that makes it possible to detect in the spectrum not only the interval with the -3 law, but also the interval with the $-5/3$ law in the low-frequency region [1].

Experimental Procedure

The experiments were carried out on a horizontal mercury circuit similar to that described in [13, 14]. The operative portion was a channel with isolated walls having a 20×60 mm^2 rectangular cross section and a length of 1200 mm. The channel was placed in the 40 mm gap of a constant-current electromagnet with 90×900 mm^2 poles, enabling one to obtain a field B = 1.1 T in the gap. A cylinder with a diameter of 5 mm whose axis was aligned with the magnetic field was located in the channel at a distance of 400 mm downstream from the front section of the magnet. The special construction of the channel made it possible to change the sources of the perturbations (a body, system of bodies, etc.). The top of the channel was covered with special mountings with the exception of a narrow slot for the introduction of the pick-up.

We suppose that the fluid moves along the x axis and that the magnetic field is directed along the y axis.

The measurements were made with a conduction anemometer with a three-electrode pick-up [15] and a thermoanemometer with a pick-up, the technology of whose manufacture is described in [16]. In the experiments we measured the profiles of the mean velocity $\overline{u}$, the profiles of the components $\overline{e}_y$ and $\overline{e}_z$ of the constant electric field, the intensity $\tilde{u}'$ of the longitudinal velocity fluctuations, and the intensities $\tilde{e}'_y$ and $\tilde{e}'_z$ of the components of the electric field fluctuations in transverse sections of the channel situated at various distances from the cylinder, and also the correlation functions.

We used a type F-116 microvoltammeter for the measurement of the constant components of the electric field, and for the measurement of the intensity of fluctuations of the components of the electric field we used an amplifier with a pass band from 1 Hz to 10 kHz and a background noise level up to 0.4 μV and a voltmeter giving root-mean-square readings.

The correlation function in the x direction was obtained from the autocorrelation function, measured with the two-channel magnetic store of an ÉASP-S electronic analyzer for stationary random processes and a type 55D70 correlometer. The magnetic store made it possible to reproduce the signal of the same process with a given time shift simultaneously with the original signal.

The one-dimensional Taylor spectra given in this paper were obtained with a type FSP-80 low-frequency spectrum analyzer.

Part of the spectra were also calculated by Fourier cosine transformations of the correlation functions.

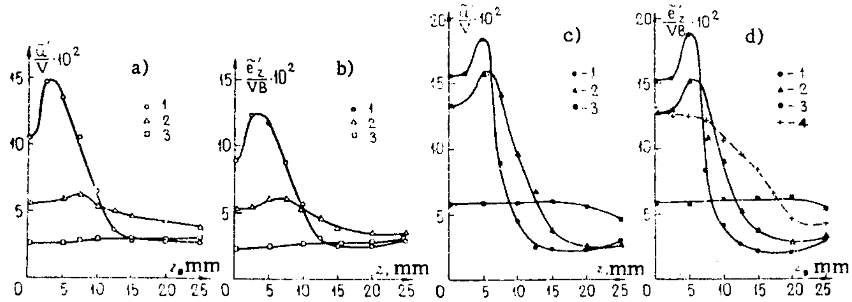

Fig. 2. Profiles of velocity fluctuations and fluctuations of the z component of the electric field at various distances from the cylinder. 1) x = 25 mm; 2) x = 100 mm; 3) x = 400 mm; 4) x = 200 mm. Measured by: a,c) thermoanemometer; b,d) conduction thermoanemometer; a) B = 0; b) B = 0.08 T; c,d) B = 0.8 T.

The spectra obtained in both these ways differed by no more than 10%.

All measurements were made under the conditions of a constant flow velocity V = 20 cm/sec and $0 \leq B \leq 0.8$ T.

Results and Discussion

1. Mean Characteristics. Figure 1 shows profiles of the mean velocity, measured with the thermoanemometer, and profiles of the constant constituent of the z component of the electric field, measured with the conduction anemometer.

As is evident from Fig. 1c and d, in a strong magnetic field both methods give good qualitative agreement. This agreement deteriorates only near solid boundaries of the flow. From Fig. 1a and b one sees that qualitative measurements of the velocity profile can also be made with the conduction anemometer even with weak magnetic fields ($B \sim 0.08$ T), although the accuracy of these measurements is less than that of measurements made with a strong magnetic field. Similar results were obtained in [16] for $B \geq 0.23$ T.

2. Profiles of Fluctuation Intensities. Figure 2 shows profiles of the intensity of the longitudinal velocity fluctuations, measured with the thermoanemometer and of the fluctuations of the z component of the electric field, measured with the conduction anemometer.

As in the case of mean profiles, one concludes from a comparison of these results that in a weak magnetic field ($B \sim 0.08$ T) the conduction anemometer can be successfully employed for rough qualitative measurements of the intensity of longitudinal velocity fluctuations (Fig. 2a and b). In a strong magnetic field ($B \sim 0.8$ T) results obtained with the thermoanemometer and the conduction anemometer are in good qualitative agreement (Fig. 2c and d). The largest disparity (~20%) occurs near the walls of the channel. Similar results were obtained in [16] for $B \geq 0.23$ T.

We remark that Figs. 1 and 2 give only the z dependence of the profiles at y = 0. Measurements along the y axis showed that with no magnetic field and with B = 0.08 T there was a weak dependence on y (with a maximum disparity of approximately 20%) of the mean values and the fluctuations. With B = 0.8 T all these quantities are independent of y within an accuracy of ~5%.

From Fig. 2 it is also evident that with a strong magnetic field present the intensity of the perturbations in the wake behind the cylinder is twice as large as it is with no field or a weak field. This result is similar to that obtained in [17].

The maintenance of a significantly larger intensity of the perturbations with a strong magnetic field present is explained by the transfer of energy to the higher part of the spectrum in consequence of the transition of the structure of the flow to a two-dimensional one [10].

A two-dimensional flow in the plane perpendicular to the magnetic field has the property that the electric current in it vanishes. In particular, the component j_y vanishes, a matter that is easily tested by experiment, since $j_y = \sigma e_y$. For this reason measurements of the fluctuations $\tilde{e}'_y$ were made in our

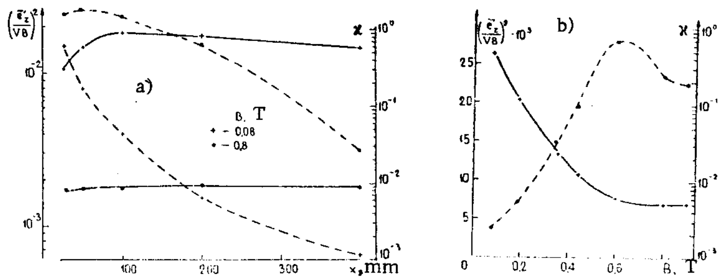

Fig. 3. Dependences of the energy of the fluctuations and the coefficient of three-dimensionality: a) on the coordinate x; b) on the magentic field induction at x = 100, y = 0, z = 5 mm. Dashed lines correspond to the energy of the fluctuations and solid lines to the coefficient of three-dimensionality.

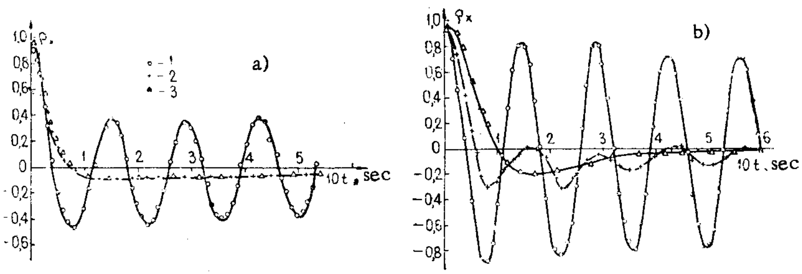

Fig. 4. Correlation functions for three distances from the cylinder. 1) x = 25 mm; 2) x = 100 mm; 3) x = 400 mm. a) B = 0; b) B = 0.8 T; y = 0, z = 5 mm.

experiments in order to judge the degree to which the structure of the flow is approximated by a two-dimensional one. With this aim in mind we introduce a coefficient of three-dimensionality (or anisotropy) $\varkappa = 2\overline{e'_y{}^2}/\overline{e'_z{}^2}$. The appropriateness of introducing this quantity stems from the fact that for three-dimensional homogeneous and isotropic turbulence $\varkappa = 1$ and for two-dimensional flow in a plane perpendicular to the magnetic field $\varkappa = 0$.

We shall show that for three-dimensional, homogeneous, isotropic turbulence $\varkappa = 1$. To this end we appeal to the equation for the potential of an electric field

$$\nabla^2 \varphi_2 = B\omega_2, \qquad \omega = \mathrm{rot}\,\mathbf{v}$$

and consider formally the vector field

$$\varphi = (\varphi_1, \varphi_2, \varphi_3).$$

Because of the isotropicity of the vector field ω and of the Laplace operator, the vector field φ will also be isotropic.

Now consider the tensor

$$\mathbf{G}_{ijkl} = \overline{\frac{\partial \varphi_i}{\partial x_k}\frac{\partial \varphi_j}{\partial x_l}}.$$

The quantities $e'_y{}^2$ and $e'_z{}^2$ are expressed in terms of this tensor in the following way

$$\overline{e'_y{}^2} = \overline{e'_2{}^2} = \mathbf{G}_{2222}, \qquad \overline{e'_z{}^2} = \overline{e'_3{}^2} = \mathbf{G}_{2233} = \overline{e'_x{}^2}.$$

From the isotropicity properties we obtain the fact [18] that $\mathbf{G}_{2222}/\mathbf{G}_{2233} = 1/2$, whence it follows that $\varkappa = 1$.

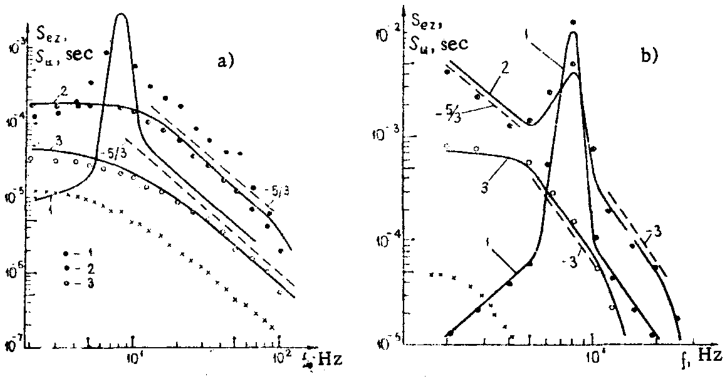

Fig. 5. Spectra of longitudinal velocity fluctuations and of fluctuations of the z component of the electric field at various distances from the cylinder (y = 0, z = 5 mm). The solid curves correspond to measurements with the conduction anemometer. 1) x = 25 mm; 2) x = 100 mm; 3) x = 400 mm. a) B = 0 and B = 0.08 T; b) B = 0.8 T; ×'s denote background flow noise.

For two-dimensional flow, as already mentioned, j = 0 and in particular $j_y = \sigma e_y = 0$. Consequently in this case $\varkappa = 0$.

We note that the vanishing of j_y, and consequently of $\varkappa$ as well, is, generally speaking, only a necessary condition for the two-dimensionality of the flow. A sufficient condition would be, for example, the vanishing of all components of the electric field if the magnetic field is different from zero.

Figure 3 shows the dependences of the coefficient $\varkappa$ on the magnetic field induction and on the coordinate x. In particular, from Fig. 3a one sees that with B = 0.08 T, $\varkappa = 0.65 \div 0.9$ ($\chi \geq 100$ mm), and with B = 0.8 T, $\varkappa \simeq 10^{-2}$ (for all x).

Moreover, $\varkappa$ is a monotonically decreasing function of B (Fig. 3b). This is in agreement with the idea that the flow becomes two-dimensional as the magnetic field induction increases, i.e., the quantity $\tilde{e}'_z$ remains bounded away from zero (and even increases – Fig. 3b), while the quantity $\tilde{e}'_y$ decreases rapidly.

3. Correlation Functions and Spectra. Figure 4 shows the correlation functions obtained from thermoanemometric measurements for three distances from the cylinder at the point y = 0, z = 5 mm. In a field B = 0.8 T at a distance x = 25 mm from the cylinder practically all the energy of the perturbations is concentrated in a narrow frequency band in the neighborhood of 8 Hz. Without the magnetic field only ~40% of the energy is contained in the neighborhood of this frequency. This is explained by the fact that in the absence of a field the original two-dimensional perturbations generated by the cylinder rapidly break up into three-dimensional perturbations, and their energy dissipates owing to transfer to the upper part of the spectrum; i.e., turbulent dissipation takes place. In a strong magnetic field the two-dimensional structure of the perturbations is maintained, and consequently a flow of the energy of the perturbations into the upper part of the spectrum does not occur. In consequence of this the intensity of the perturbations remains considerably larger (Fig. 3b) and the perturbations maintain their regularity for a longer time. This is quite evident from a comparison of curves 1 and 2 in Fig. 4a and b.

From Fig. 4 it follows that as the distance from the cylinder is increased, without a field the integrated scale of the turbulence hardly changes at all, while in a strong field it increases.

We turn to a consideration of the spectra of the longitudinal velocity fluctuations and of the z component of fluctuations of the electric field (Fig. 5) at the point y = 0, z = 5 mm at various distances from the cylinder.

First of all, we note that the results of measurements with the conduction anemometer and the thermoanemometer in a strong magnetic field (B ~ 0.8 T) are in good qualitative agreement throughout the entire range of frequencies that were observed.

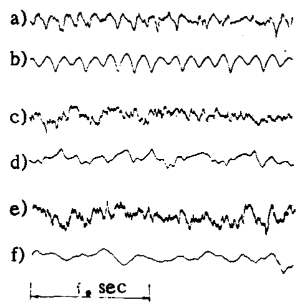

Fig. 6

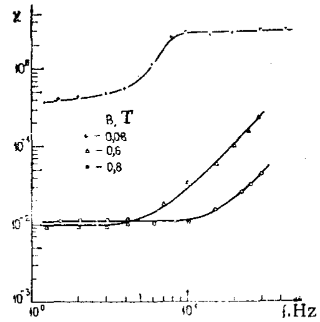

Fig. 7

Fig. 6. Oscillograms of longitudinal velocity fluctuations. a,b) x = 25 mm; c,d) x = 100 mm; e,f) x = 400 mm; a,c,e) B = 0; b,d,f) B = 0.8 T; y = 0, z = 5 mm.

Fig. 7. Ratio of the spectral densities of the fluctuations $\varkappa = 2S_{ey}/S_{ez}$ for x = 100 mm (y = 0, z = 5 mm) and various intensities of the magnetic field.

Without a field and also with B = 0.08 T a significant disparity occurred only for x = 25 mm (Fig. 5a); however, the qualitative agreement remained.

The fundamental results, shown in Fig. 5, lead to the following conclusions. In a weak magnetic field (B ~ 0.08 T) the flow behind the cylinder has the same structure it has without a field; i.e., the greater part of the energy is contained in the high-frequency region, an inertial interval with the −5/3 law being formed, which is characteristic of three-dimensional turbulence (Fig. 5a).

In a sufficiently strong magnetic field (Fig. 5b) at a distance x = 100 mm from the cylinder both inertial intervals that are typical of two-dimensional turbulence [1-7] are observed in the spectrum: in the high-frequency part of the spectrum the −3 law, which corresponds to the absence of a flow of energy and the presence of a flow of vorticity into the upper part of the spectrum, and at low frequencies the −5/3 law, corresponding to the presence of a flow of energy and the absence of a flow of vorticity into the lower part of the spectrum.

As the distance from the cylinder is increased, starting at x = 100 mm (the spectrum for x = 200 mm is not shown in the figure), all the lower frequencies are preserved in the spectrum.

This is also very evident from oscillograms of the fluctuations shown in Fig. 6.

Above we discussed how the degree to which the structure of the flow is approximated by a two-dimensional one can be judged by measuring the intensity of the fluctuations $\tilde{e}'_y$. However, such measurements do not enable one to judge in what scales (or frequencies) the structure remains three-dimensional.

Such information is given by a comparison of the spectra of $\tilde{e}'_y$ and $\tilde{e}'_z$. Figure 7 shows the ratio of the spectral densities S_{ey} and S_{ez}, from which it is evident that with an increase in the magnetic field the three-dimensional structure of the field is maintained in all the higher frequencies. This agrees with the idea that the stronger the magnetic field, the more the three-dimensionality is "disentangled" by the smaller scales. We emphasize that the nonvanishing of $\tilde{e}'_y$ is due to the presence of some three-dimensionality since for ideal two-dimensional flow $\tilde{e}'_y = 0$. A similar conclusion was reached in [19] for flow in a channel. We remark that in a strong magnetic field, when $\tilde{e}'_y$ becomes very small, the accuracy of measurements of $\tilde{e}'_y$ deteriorates for reasons described in [19]. Therefore these measurements must be regarded more as qualitative than quantitative.

All the data on correlation functions and spectra given here refer to the point of a cross section of the channel with the coordinates y = 0, z = 5 mm. Measurements made at various points of a cross section of the channel showed that the basic properties of the correlation functions and spectra, described above, are maintained over the entire cross section of the wake behind the cylinder.

TABLE 1

L \ r_0	1	0,5	0,25	0,125	0,0625
200	0,212	0,39	0,52	0,44	0,22
400	0,054	0,18	0,29	0,22	0,08

4. Flow of Energy into the Lower Part of the Spectrum. The preservation of low frequencies and the vanishing of high frequencies in the spectrum with B ~0.8 T as the distance from the cylinder is increased indicates the existence of a flow of energy from high frequencies to low ones. Here it should be explained on the basis of viscous dissipation.

In fact, an estimate of viscous dissipation of perturbations with a frequency of 8 Hz, which at a distance of 25 mm from the cylinder contain about 95% of all the energy of the fluctuations, shows that in transversing a distance of 400 mm the perturbations lose only ~3% of their energy through viscous dissipation, i.e., the entire dissipation at such a distance leads to practically no noticeable loss of energy and does not distort the form of the spectrum.

As the same time, as is seen from Fig. 5b, at a distance of 400 mm only a small portion of the energy of the perturbations remains in the neighborhood of the frequency 8 Hz. Moreover, at a distance of 400 mm from the cylinder a much greater part of the energy is contained in the region of the spectrum from 2 to 6 Hz than is contained in the same region at a distance of 25 mm. The conclusion can be made from this that a flow of energy into the lower part of the spectrum occurs, i.e., from higher to lower frequencies.

Note that in strong magnetic fields, despite the absence of turbulent dissipation of the energy of the perturbations and the very small role of viscous dissipation, the intensity of the perturbations is nevertheless substantially diminished with increasing distance from the cylinder (see Fig. 2), remaining, however, considerably higher than is the case in the absence of the field.

The reason for the sizeable decrease in the intensity of the perturbations is their inhibition by the walls of the channel that are perpendicular to the field, which is more marked the stronger the field. The greatest braking effect is manifested in the large-scale vortices. Such a conclusion is confirmed by numerical calculation of a model problem on the damping of a vortex between two parallel planes in a transverse magnetic field, directed along the axis of the vortex.

In line with this scheme we calculated the following nonstationary problem in cylindrical coordinates under the assumption that $v = (0, u, 0)$ and $\partial/\partial\theta = 0$:

$$\frac{\partial u}{\partial t} = \nabla^2 u - \frac{u}{r^2} + \mathrm{Ha}^2\left\{\frac{\partial \varphi}{\partial r} - u\right\},$$

$$\nabla^2\varphi = \frac{1}{r}\frac{\partial}{\partial r}(ru), \qquad u\Big|_{z=\pm 1} = \frac{\partial\varphi}{\partial z}\Big|_{z=\pm 1} = 0,$$

$$u_{t=0} = f(r, z) = f_1(r) f_2(z),$$

where $f_1(r) = r \cdot \exp(-r^2/r_0^2)$, $f_2(z)$ is the Hartmann profile.

Here we took as characteristic quantities: the distance L between the planes, the velocity V, equal to the mean flow velocity (20 cm/sec), the time L^2/ν, and the potential of the electric field VB.

The ratio W/W_0 was found, where

$$W = \int_\tau u^2 d\tau$$

for two moments of dimensionless time, $t = 10^{-3}$ and $t = 2 \cdot 10^{-3}$, corresponding, under the experimental conditions, to the passage of a perturbation through distances of 200 and 400 mm. Results of the calculation for Ha = 50 are exhibited in Table 1.

From the table it is evident that the rate of damping of the perturbations depends nonmonotonically on their dimensions. The largest-scale vortices die out comparatively rapidly, owing to the indicated braking at the walls because of the Hartmann effect.

As the dimensions are decreased, the role of this braking is diminished, and the perturbations die out more slowly. Finally, when the dimensions of the perturbations become sufficiently small, ordinary viscous damping becomes important and the perturbations again begin to die out more rapidly.

Conclusions

On the basis of the above discussion we formulate the basic conclusions concerning the flow of an electrically conducting fluid in the wake behind a circular cylinder whose axis is parallel to a sufficiently strong magnetic field.

1. The flow acquires a structure that is close to two-dimensional.

2. In the spectrum of the perturbations both inertial regions, characteristic of two-dimensional turbulence, appear.

3. There is a flow of energy from small to large scales.

4. The conduction anemometer can be successfully used not only for the measurement of the intensity of velocity fluctuations, but also in the analysis of their spectral constitution.

In conclusion the authors thank Kh. É. Kalis for performing numerical calculation of the problem indicated in the text, A. L. Tseskis, who pointed out to the authors the possibility of applying Le Chatelier's principle in the explanation of the tendency of the flow toward a two-dimensional structure, and O. A. Lielausis for a discussion and critical remarks.

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