VELOCITY STRUCTURE OF A FLOW IN A MAGNETIC FIELD PERIODICALLY VARYING ALONG THE FLOW

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A numerical calculation is performed for a MHD flow in a magnetic field periodically varying along the flow. An M-shaped velocity structure is obtained which confirms the "cumulative effect" of distortions of the velocity profile when the flow passes through a number of successive amplitudes of the field. It is shown that in spite of the M-shaped velocity structure, the streamline of the fluid is practically straight.

The conditions for the emergence of M-shaped velocity structures in steady MHD flows in a constant magnetic field were analyzed in [1, 2]. In particular, it was shown that distortion of the velocity profile in the plane perpendicular to the field occurs when the y component of the curl of the electromagnetic field is nonzero: $\mathrm{curl}_y\, F_{em} \neq 0$ (the x axis is parallel to the flow, the y axis is parallel to the force lines of the field, and the z axis is perpendicular to the field and the velocity).

A similar situation occurs in the flow sections of MHD devices which regulate the flow rate of liquid metals through a substantial increase in the drag coefficient in a constant magnetic field periodically varying along the flow.

Theoretical and experimental investigations of the integral characteristics of the flow in such fields were carried out in [3-5]. There is also considerable interest in the velocity structure of such flows. This is primarily due to the fact that the formation of an M-shaped profile in a field periodically varying along the length should be especially strong as a result of the "cumulative effect" of the fluid passing repeatedly through regions with an appreciable field gradient along the length of the channel. We note that a similar pattern should also be observed in the traveling wave of induction machines.

In actual devices the presence of velocity profiles with sharp gradients in the flow can induce such phenomena as separation, liquefaction, substantial change in heat and mass exchange, etc. Therefore, even a qualitative analysis of the development of such velocity structures is worthwhile.

The present paper gives a numerical calculation for a two-dimensional flow in a magnetic field periodically varying along the length of the channel. The calculation is for a two-dimensional flow with Re =40 and N =40 (all parameters were determined for the flow dimension perpendicular to the force lines of the field) so that a comparison with the data of [1] is possible.

The magnetic field has a component perpendicular to the plane of the flow and dependent on the longitudinal coordinate in the form $B = \sin 2\pi x/\lambda$, where $\lambda = 1$. A field with this configuration exists, for example, in the magnetic system employed by [4, 5] and also in the gap of a two-dimensional inductor.

We calculate the system of equations for the stream function ψ, the vorticity ω, and the electric-field potential φ

$$\frac{1}{\mathrm{Re}} \nabla^2 \omega_y - \frac{D(\psi, \omega)}{D(x, z)} - N \left[\frac{\partial B}{\partial x} \frac{\partial \varphi}{\partial x} - \frac{\partial}{\partial x} (B)^2 \frac{\partial \psi}{\partial x} \right] = 0;$$
$$\nabla^2 \psi = -\omega; \qquad \nabla^2 \varphi = -B\omega. \tag{1}$$

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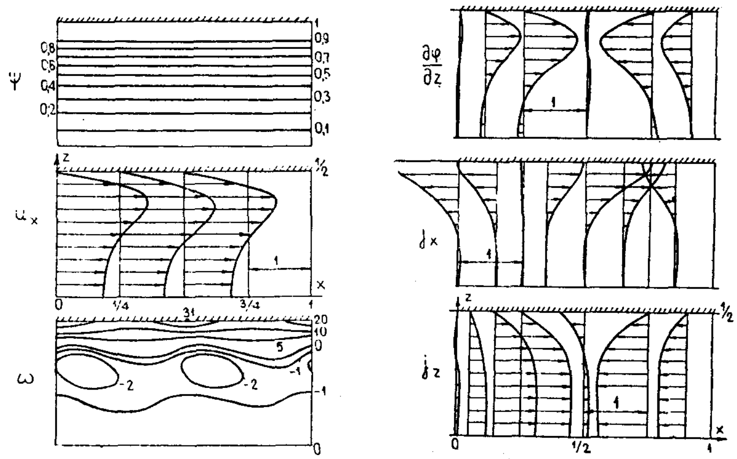

Fig. 1

The boundary conditions used are: $z=0$ and $\psi=\omega=\varphi=0$ at the channel axis; $z=1/2$, $\psi=1/2$, and $\partial\varphi/\partial z=0$ at the channel wall. the Dorodnitsyn condition $\omega^{n+1}|_s=\omega^n|_s=\sigma\frac{\partial\psi^n}{\partial n}\Big|_s$ is given for ω; and $\sigma=3$ since the use of Thom or Woods conditions leads, as in [1], to a divergence of the process. Moreover, the periodicity conditions are given as

$$\psi|_{x=x_a}=\psi|_{x=x_b};\quad \omega|_{x=x_a}=\omega|_{x=x_b};\quad \varphi|_{x=x_a}=\varphi|_{x=x_b};$$

$$\frac{\partial\psi}{\partial x}\Big|_{x=x_a}=\frac{\partial\psi}{\partial x}\Big|_{x=x_b};\quad \frac{\partial\omega}{\partial x}\Big|_{x=x_a}=\frac{\partial\omega}{\partial x}\Big|_{x=x_b};\quad \frac{\partial\varphi}{\partial x}\Big|_{x=x_a}=\frac{\partial\varphi}{\partial x}\Big|_{x=x_b}.$$

The difference equations for system (1) (the first and second derivatives were replaced by central differences) were calculated in the following manner: a relaxation scheme $\partial u/\partial\tau=Lu$ was used for the nonlinear equation, and an upper relaxation scheme was used for the linear equations. The computation was set up in the form of Zeidel interactions and the values of all three functions were calculated at once at the given point, but not the entire range of values for each function separately. This scheme is illustrated by a model system of equations

$$\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial z^2}\right)+A(\psi)\frac{\partial\omega}{\partial x}+B(\psi)\frac{\partial\omega}{\partial z}+f(\psi)=0,\quad \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}+\omega=0. \tag{2}$$

The first equation is nonlinear; the second is linear. We assume that the difference equations have the form

$$\alpha_1\omega_{i-1,j}+\alpha_2\omega_{i+1,j}+\alpha_3\omega_{i,j-1}+\alpha_4\omega_{i,j+1}-\alpha_0\omega_{i,j}+f_{i,j}=0;$$

$$\beta_1\psi_{i-1,j}+\beta_2\psi_{i+1,j}+\beta_3\psi_{i,j-1}+\beta_4\psi_{i,j+1}-\beta_0\psi_{i,j}+\omega_{i,j}=0,$$

where the coefficients α_i and $f_{i,j}$ depend on ψ. The iteration process is as follows: successive approximations $\omega^{n+1}_{i,j}$ and $\psi^{n+1}_{i,j}$ at point (x_i, y_i) are calculated from

$$(\omega^{n+1}_{i,j}-\omega^n_{i,j})/\tau=\alpha^n_1\omega^{n+1}_{i-1,j}+\alpha^n_3\omega^{n+1}_{i,j-1}+\alpha^n_2\omega^n_{i+1,j}+\alpha^n_4\omega^n_{i,j+1}-\alpha^n_0\omega^{n+1}_{i,j}+f^n_{i,j};$$

$$\psi^{n+1}_{i,j}=(1-\zeta)\psi^n_{i,j}+(\zeta/\beta_0)(\beta_1\psi^{n+1}_{i-1,j}+\beta_3\psi^{n+1}_{i,j-1}+\beta_2\psi^n_{i+1,j}+\beta_4\psi^n_{i,j+1}+\omega^{n+1}_{i,j}),$$

where τ and ζ are certain parameters ($\zeta>1$ is the upper relaxation parameter).

In this scheme the calculations begin from the lower left corner of the region and continue so that the values of ω^{n+1} and ψ^{n+1} below and to the left of the given point are either calculated or given in the form of boundary conditions.

The figure shows the streamlines ψ, the constant vortex lines ω, and also the profiles of u_x, $\partial\varphi/\partial z$, j_x, and j_z in different sections of the channel for Re = 40 and N = 40. It can be seen from the figure that in such a flow the profile has an M-shaped structure, and the ratio of the maximum velocity to the velocity in the center of the channel is roughly 1.5-2 times larger than when the velocity profile is distorted by fringe effects [1]. This is all the more important since in this case the variation of the field is considerably less sharp than in the boundary region of the magnet, confirming the role of the "cumulative effect" which was mentioned earlier. It is obvious that summation of the "tongues" in the velocity profiles takes place only up to a certain value, and then magnification of the effect in the region next to the one where the effect originates is balanced by viscous dissipation to the farthest extent of the flow.

We note also that in our case, in contrast to [1], the streamlines in the fluid are practically straight, although the velocity profile has a well-defined M-shaped structure. That the streamlines are very nearly straight indicates the feasibility of constructing an analytic solution for the problem by the small perturbation method.

In conclusion, we emphasize that the qualitative picture of the rearrangement of the velocity structure of the flow, which was calculated for small Re, is completely valid for turbulent conditions with large Reynolds numbers, although sufficiently high Stuart numbers N must be attained.

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