ELECTROHYDRODYNAMIC FLOW OF A NON-NEWTONIAN FLUID IN A CIRCULAR PIPE

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The gradient flow of a non-Newtonian fluid with a rheological power law and a spatial density of electric charges in a circular pipe under the action of a homogeneous external electric field is analyzed. Distribution functions are obtained for the space charge density, the induced field, and the velocity of the medium. The possibility of the formation of a reverse-flow zone is demonstrated. Passage to the limits of a Newtonian fluid and an uncharged fluid with a rheological power law are realized.

Electrohydrodynamic (EHD) flows of Newtonian fluids have been investigated in [1-7]. It is instructive in connection with certain problems in biomechanics and the mechanics of polymers to analyze EHD flows of fluids having more complex mechanical properties than a viscous Newtonian fluid.

In the present study we analyze the EHD flow of a non-Newtonian fluid with a rheological power law [8], which adequately approximates the properties of a number of fluids and is widely used in theoretical rheology.

Suppose that an incompressible fluid with a rheological power law is moving in a cylindrical pipe of radius R with nonconducting walls under the influence of a constant pressure gradient $P = -\partial p/\partial z > 0$. The fluid in this case is assumed to be electrically charged with a certain space charge density $\rho(r)$, where the charge per unit length of the pipe

$$q_0 = 2\pi \int_0^R \varrho(r) r dr \tag{1}$$

is one of the known parameters of the problem.

A homogeneous external electric field $E_z = E_0 = \text{const}$ is directed along the axis of the pipe and creates an additional volume force acting on the fluid.

The charge distribution $\rho(r)$ over the pipe cross section and the induced radial electric field $e(r)$, which are related by the familiar expression

$$\varrho = \operatorname{div} \mathbf{D} = \frac{\varepsilon\varepsilon_0}{r} \frac{d}{dr}(re), \tag{2}$$

can be determined by assuming that the radial component of the electric current is zero in the steady state:

$$j_r = \mu\varrho e - D(\operatorname{grad}\varrho)_r = 0. \tag{3}$$

Here μ is the charge mobility, and D is the diffusivity of the charged particles in the fluid. Transforming to the dimensionless variables

$$r' = \frac{r}{R}; \quad e' = \frac{\varepsilon\varepsilon_0 R}{q_0} e; \quad \varrho' = \frac{R^2}{q_0}\varrho,$$

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we can transform relation (3) with allowance for (2) as follows (dropping the primes on the dimensionless variables):

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(re)\right]-\Pi\frac{e}{r}\frac{d}{dr}(re)=0;\quad \Pi=\frac{\mu q_0}{\varepsilon\varepsilon_0 D}. \tag{4}$$

Here Π is the characteristic dimensionless parameter of the problem. Equation (4) is integrated with regard for the obvious condition $e(0)=0$ and the integral relation (1). We obtain as a result

$$e(r)=\frac{4r}{8\pi+\Pi(1-r^2)};\quad \varrho(r)=\frac{64\pi+8\Pi}{[8\pi+\Pi(1-r^2)]^2}. \tag{5}$$

Calculating the hydrodynamical part of the problem, we write the dimensionless equation for the EHD flow of a power-law rheological fluid:

$$\frac{1}{r}\frac{d}{dr}\left[r\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr}\right]=-1-\Pi_1\frac{1}{r}\frac{d}{dr}(re). \tag{6}$$

We adopt the quantity $u_0=(k^{-1}R^{n+1}P)^{1/n}$ as the characteristic velocity in this case, where k and n are the rheological constants of the medium; the dimensionless electrical interaction parameter $\Pi_1=q_0E_0/PR^2$ characterizes the ratio of the electric forces exerted by the external field on the fluid to the pressure drops. We note that the sign of Π_1 depends on the direction of the external electric field.

Integrating (6) once with regard for the condition $du/dr=0$ at $r=0$, we have

$$\left|\frac{du}{dr}\right|^{n-1}\frac{du}{dr}=-\frac{r}{2}-\frac{4\Pi_1 r}{8\pi+\Pi(1-r^2)}. \tag{7}$$

We now investigate the sign of the derivative du/dr. It follows from (7) that three flow regimes are distinguished, depending on the value of the parameter Π_1.

If $\Pi_1>-\pi$, the derivative du/dr is negative everywhere in the domain $0<r\le 1$ and is equal to zero only in the center of the pipe ($r=0$). The fluid in this case moves through the pipe in the direction of the z axis ($u\ge 0$).

If $-\pi-\Pi/8<\Pi_1<-\pi$, the derivative du/dr is equal to zero, not only at the center of the pipe, but also at the point

$$r_0=\sqrt{1+(\Pi_1+\pi)8/\Pi},$$

where $du/dr>0$ for $r_0<r\le 1$ and $du/dr<0$ for $0<r<r_0$. Reverse flow takes place in the given regime (see Fig. 1), the fluid moving oppositely to the pressure forces on account of the action of the spatial electric forces.

If, on the other hand, the retarding electric field is large, $\Pi_1<-\pi-\Pi/8$, then du/dr is positive everywhere except at the center of the pipe $r=0$, where it is zero. In this case the fluid velocity is negative throughout the entire flow domain.

Taking into account the sign of the velocity derivative in the various flow regimes and requiring satisfaction of the fluid sticking condition $u(1)=0$, we write the solution (7) in quadratures. For example, in the case of a flow such that $u\ge 0$ ($\Pi_1>-\pi$) everywhere in the pipe the corresponding quadrature has the form

$$u(r)=\int_r^1\left(\frac{\xi}{2}+\frac{4\Pi_1\xi}{8\pi+\Pi(1-\xi^2)}\right)^{\frac{1}{n}}d\xi, \tag{8}$$

whereas for the flow regime $-\pi-\Pi/8<\Pi_1<-\pi$ the velocity distribution is given by the expression

$$u(r)=\begin{cases}\displaystyle\int_1^r\left|\frac{\xi}{2}+\frac{4\Pi_1\xi}{8\pi+\Pi(1-\xi^2)}\right|^{\frac{1}{n}}d\xi, & r_0\le r\le 1,\\ \displaystyle\int_r^{r_0}\left(\frac{\xi}{2}+\frac{4\Pi_1\xi}{8\pi+\Pi(1-\xi^2)}\right)^{\frac{1}{n}}d\xi-\int_{r_0}^1\left|\frac{\xi}{2}+\frac{4\Pi_1\xi}{8\pi+\Pi(1-\xi^2)}\right|^{\frac{1}{n}}d\xi, & 0\le r<r_0.\end{cases} \tag{9}$$

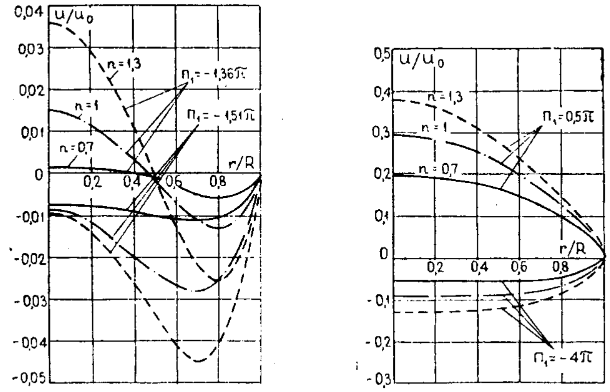

Fig. 1

The velocity distributions of the fluid in a cylindrical pipe for three values of the power exponent n in the rheological law, calculated for $\Pi = 8\pi$ and different values of the parameter Π_1 corresponding to the indicated regimes are shown in Fig. 1.

The relations derived above make it possible to realize two passages to limits: $n \to 1$ and $\Pi_1 \to 0$. The first corresponds to the transition to a Newtonian fluid, where the quadratures are expressed in terms of elementary functions and we have

$$u(r) = \frac{1-r^2}{4} + 2\frac{\Pi_1}{\Pi} \ln\left|1 + \frac{\Pi}{8\pi}(1-r^2)\right|.$$

The second limiting transition $\Pi_1 \to 0$ corresponds to the flow of an uncharged non-Newtonian fluid with a rheological power law in a circular pipe under the influence of a constant pressure gradient [8]. In this case we obtain from (8)

$$u(r) = \left(\frac{1}{2}\right)^{\frac{1}{n}} \frac{n}{n+1}\left(1 - r^{\frac{n+1}{n}}\right).$$

We note in conclusion that the problem of the gradient EHD flow of a power-rheological medium in an annular duct can be similarly analyzed.

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