SOME NONSTATIONARY PROBLEMS OF MAGNETIC RHEOLOGY

I. S. Granik and L. K. Martinson

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Two-dimensional nonstationary problems of shear flow are investigated for a conducting incompressible liquid with power rheologic law in a transverse magnetic field whose induction changes with time. It is shown that in dilatant liquids the nature of the change of the magnetic field affects the velocity of propagation of the shear-wave front.

The effect of the rheologic properties of a conducting medium on the nature of magnetohydrodynamic flows was investigated in [1-5]. In particular, as shown in [3-5], in dilatant liquids pertaining to the class of nonlinear-viscous media with power rheologic law, shear perturbations propagate with a finite velocity. An external transverse magnetic field causes a decrease in the velocity of propagation of the shear-wave front and, in some cases, results in complete stoppage of the front. In these cases shear perturbations penetrate into a magnetized medium only to a finite distance [3, 5].

Below we consider some nonstationary shearing flows of conducting liquids with power rheologic law in a transverse magnetic field, for which the magnitude of induction B(t) depends on time.

In the noninduction approximation ($\mathrm{Re_m} \ll 1$) the equation of magnetic rheology describing such motions can be written in the form

$$u_t = \mathfrak{L}u - \gamma(t)u; \qquad \mathfrak{L} = a\frac{\partial}{\partial z}\left[\left|\frac{\partial}{\partial z}\right|^{n-1}\frac{\partial}{\partial z}\right], \tag{1}$$

where u(z, t) is the velocity of the liquid, $a = k/\rho = \mathrm{const} > 0$, $\gamma(t) = \sigma\rho^{-1}B^2(t)$ is a known function of time, ρ is the density, σ is the conductivity of the medium, and k and n are the rheologic constants of the medium.

Let a weightless nonconducting plate be located in the plane $z = 0$ in a liquid at rest. At the initial instant $t = 0$ a momentum $\mathfrak{P}$ per unit area is imparted instantaneously to the plate and the liquid is set into motion.

In accordance with the statement above, we formulate the problem for determining the velocity profile of the liquid u(z, t) at any time $t > 0$

$$\begin{cases} u_t = \mathfrak{L}u - \gamma(t)u; & -\infty < z < \infty, \quad t > 0, \\ u(z, 0) = P\delta(z). \end{cases} \tag{2}$$

Here $P = \mathfrak{P}/\varrho$, and $\delta(z)$ is Dirac's delta function.

The solution of this problem for the case $\gamma = \gamma_0 = \mathrm{const}$ was obtained in [3]. Below we investigate the solution of problem (2) for arbitrary $\gamma(t)$ corresponding to an arbitrary law of variation of the external magnetic field with time.

We express the solution of problem (2) in the form

$$u(z, t) = w(z, t)\exp\left[-\int_0^t \gamma(t)\,dt\right]. \tag{3}$$

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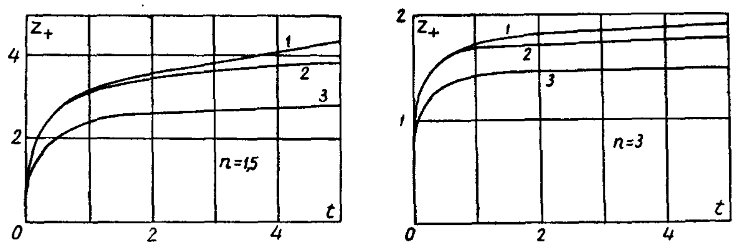

Fig. 1. Nature of motion of shear-wave front z + (t) with time: 1) $\gamma(t) = (1+t)^{-1}$; 2) $\gamma(t) = 1$; 3) $\gamma(t) = 1+t$.

Then after some simple operations for the function w we get

$$\begin{cases} w_\tau = \mathfrak{L} w, \\ w(z,0) = P\delta(z), \; \tau(t) = \int\limits_0^t \exp\left[-(n-1)\int\limits_0^\xi \gamma(t)\,dt\right] d\xi. \end{cases} \tag{4}$$

Thus transformation (3) has enabled us to reduce problem (2) about the motion of a conducting liquid in a transverse magnetic field to the corresponding problem (4) describing analogous motion without the magnetic field. However, in this process the time variable has been transformed.

The solution (4) has been investigated in [3] (problem (3) for $\lambda = 0$). For $n \le 1$ the solution corresponds to instantaneous involvement of all the layers of the liquid in the motion. In other words, in such media the propagation velocity of shear perturbations is infinite.

The solution of problem (4) for $n > 1$ is of different nature. For such media the solution of problem (4) corresponds to a shear-wave whose front propagates in the liquid with finite velocity going away from the plate, the source of the shear perturbations [3]. With transformation (3) taken into consideration the solution of problem (2) for $n > 1$ will also have the form of a shear wave, for which the position of the front $z_\pm(t)$ is determined by the following time dependence:

$$z_\pm(t) = \pm\sqrt{\frac{n+1}{n-1}}\,(NP^{n-1}a\tau)^{\frac{1}{2n}} \equiv \pm S(n)\tau^{\frac{1}{2n}}(t),\; N = 2n\left\{\frac{2n}{n+1}\,\mathfrak{B}\left(\frac{n}{n+1},\,\frac{2n-1}{n-1}\right)\right\}^{1-n}, \tag{5}$$

$\mathfrak{B}(\xi, \theta)$ is a beta function.

Since the transformed variable τ depends on $\gamma(t)$, the nature of variation of the induction of the external magnetic field with time affects the speed of the shear-wave fronts.

In (5), passing on again to variable t with the dependence $\tau(t)$ taken into consideration, it can be noted that if the dependence $\gamma(t)$ is such that $\tau \to \tau_{max} < \infty$ for $t \to \infty$, then the stoppage of the shear-wave fronts will occur and the shear perturbations from the plate will penetrate into the liquid to a finite distance z_{max}, i.e.,

$$|z_\pm(t)| \le z_{max} = S(n)\tau_{max}^{\frac{1}{2n}}. \tag{6}$$

If for certain $\gamma(t)$ $\tau \to \infty$ for $t \to \infty$, then the stoppage of the shear-wave fronts will not occur and the perturbations from the plate will penetrate into the liquid to unrestricted distance. According to the conclusions of [3] it should be expected from physical considerations that for magnetic fields increasing with time, the effect of spatial localization of shear perturbations will always occur. For the induction of the external magnetic field decreasing with time ($\gamma(t) \to 0$ for $t \to \infty$) the stoppage of shear-wave fronts may not be observed.

In order to verify this conclusion we write the laws of motion of shear-wave fronts for some specific functions $\gamma(t)$ with (5) taken into consideration:

$$z_\pm(t) = \pm S(n)\left\{\frac{1-\exp\left[-(n-1)\gamma_0 t\right]}{\gamma_0(n-1)}\right\}^{\frac{1}{2n}} \quad \text{for } \gamma(t) = \gamma_0 = \text{const}, \tag{7}$$

$$z_\pm(t) = \pm S(n)\left\{\frac{\sqrt{\pi}}{2}\,\frac{\exp(\delta^2T^2)}{\delta}\,[\Phi\{\delta(t+T)\} - \Phi\{\delta T\}]\right\}^{\frac{1}{2n}} \quad \text{for } \gamma(t) = \gamma_0 + \alpha t, \tag{8}$$

$$z_{\pm}(t) = \pm S(n)\left\{\frac{1}{\gamma_0(n_k-n)}\left[(1+\beta t)^{\frac{n_k-n}{n_k-1}}-1\right]\right\}^{\frac{1}{2n}} \text{ for } \gamma(t)=\gamma_0(1+\beta t)^{-1}, \tag{9}$$

where $\delta=\sqrt{\frac{(n-1)\alpha}{2}}$, $T=\frac{\gamma_0}{\alpha}$, $n_k=1+\frac{\beta}{\gamma_0}$, $\Phi(\theta)$ is the error integral.

Figure 1 shows the dependence $z_+(t)$ for $P=\alpha=1$, $\alpha=\beta=\gamma_0=1$ for two values of $n=1.5$ and $n=3$. The effect of spatial localization of shear perturbations is observed for the cases (7) and (8). If the induction of the external magnetic field decreases with time according to the law $B(t) \sim (1+\beta t)^{-1/2}$, then it follows from (9) that the stoppage of the shear-wave front will occur only for $n > n_k$. If $n \leq n_k$, the penetration of the shear perturbations into the liquid will be unrestricted ($|z_{\pm}(t)| \to \infty$ for $t \to \infty$).

In conclusion we note that the uniqueness of the solution of the problem can be proved making use of the monotone property of operator $\mathfrak{L}$ in the same way as in [6] for the case $\gamma(t)=0$.

LITERATURE CITED

1. L. K. Martinson and K. B. Pavlov, Magnitn. Gidrodinam., No. 4, 61 (1965).
2. L. K. Martinson and K. B. Pavlov, Magnitn. Gidrodinam., No. 3, 69 (1966).
3. L. K. Martinson and K. B. Pavlov, Magnitn. Gidrodinam., No. 2, 50 (1971).
4. Ya. G. Sapunkov, Zh. Prikl. Mekhan. i Tekh. Fiz., No. 2, 50 (1970).
5. K. B. Pavlov, Magnitn. Gidrodinam., No. 3, 142 (1972).
6. J. L. Lions, Some Methods of Solution of Nonlinear Boundary-Value Problems [Russian translation], Mir, Moscow (1972).