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VORTICAL FLOW OF CONDUCTING FLUID DRIVEN BY AN ALTERNATING MAGNETIC FIELD IN A PLANE CHANNEL

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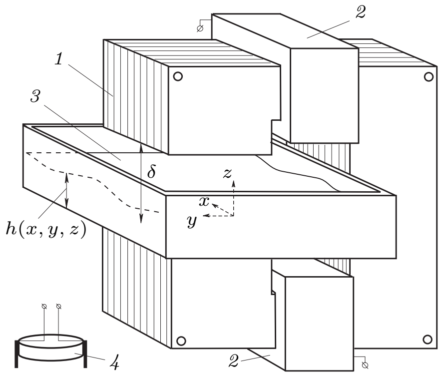
This paper deals with the study of a vortical flow in a plane layer of a conducting fluid driven by an alternating magnetic field normal to the layer and nonuniform in the layer plane. The layer with either a free or a solid upper boundary is considered. The topology of the vortical flow is assumed to be dependent on the geometry of the region subjected to the magnetic field and on the position of the magnetic field within the layer. It is found that in the case when process parameters exceed some critical value the flow becomes unstable that causes a temporary transformation of the vortical flow accompanied by the movement of vortical structures in the layer plane and its free surface oscillations. The investigated vortical flow can be used for stirring molten metals and, as a general operation principle, for designing MHD-pumps for foundry works.

Introduction. The study is focused on a vortical flow in a plane layer of a conducting fluid (hereinafter, liquid metal). Two cases are considered: a layer confined between solid boundaries, and a layer with a free surface capable of making oscillations. A vortical flow in the liquid metal is induced under the action of vortex electromagnetic forces, which can be generated in a number of ways. Thus, in [1], such forces were generated by an electric current of density $\mathbf{j}$ passing through a thin layer of liquid metal between vertical electrode surfaces. The magnetic field $\mathbf{B}$ of the current was enhanced by ferromagnetic C-cores or yokes. The electromagnetic forces $\mathbf{f}^{\mathrm{em}} = \mathbf{j} \times \mathbf{B}$ had a vortex component because the dimensions of the yokes were smaller than those of the layer, and the edges of the yokes became the sources for the electromagnetic force vortex component. This led to the development of an electrovortical flow (EVF), which was named from the examined phenomenon induced by the current passed through the layer and by its magnetic field.

In this paper, we consider a similar way for generating electromagnetic forces but with the help of an alternating magnetic field. To this end, we use an inductor consisting of a C-shaped ferromagnetic core 1 (Fig. 1*a*) and biasing coils 2. An alternating magnetic field $\mathbf{B}$ is induced in the core gap, in which we place a cuvette filled with a liquid metal. The field $\mathbf{B}$ generates an electric current $\mathbf{j}$ in the layer. The interaction of the field $\mathbf{B}$ with the electric current $\mathbf{j}$ gives rise to electromagnetic forces $\mathbf{f}^{\mathrm{em}} = \mathbf{j} \times \mathbf{B}$ directed towards the core center (Fig. 1*b*). Due to size limitation of the poles, the induced electromagnetic forces have a vortex component. For the problem under study, these forces drive a vortical flow in the rectangular cuvette by virtue of the fact that the flow generation mechanism in this case is similar to that of electrovortical flows with the difference that the alternating magnetic field rather than the electric current plays the role of a driving force. For brevity sake, this vortical flow is called a "magnetovortical" flow (MVF). In the case when some critical parameters are exceeded, this flow may lose its stability in much the same way as the vortical flows driven in a plane liquid layer by some other mechanisms [1]–[5].



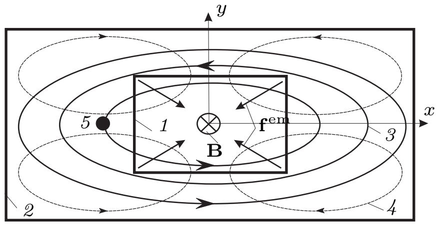


Fig. 1. (a) Diagram of the experimental setup: *1* – ferromagnetic C-core, *2* – biasing coils, *3* – plane layer of liquid metal, *4* – conductive velocity gauge; (b) schematic representation of four-vortex MVF generation: *1* – ferromagnetic core zone, *2* – layer zone, *3,4* – electric and hydrodynamic stream lines, *5* – location of the velocity gauge.

A possibility for the onset of self-oscillatory regime in such vortical flow was examined in [2], where a vortical flow in a liquid layer was driven by thermogravitational convection. In [3], the effect of electromagnetic forces resulting from the interaction of a direct electric current, passing through the layer, with the magnetic field of permanent magnets is investigated. In this particular case, the electromagnetic forces give rise to an unstable oscillatory mode of a four-vortex flow. In [1], it has been demonstrated that in a layer with a free surface the above-mentioned EVF with four vortices is destroyed and replaced by a large-scale single-vortex EVF. In [5], the emphasis is made on studying the instability of one- and two-vortex EVF in a plane layer of liquid metal with solid boundaries. The present work intends to continue the study undertaken in [7] to support experimentally the existence of instability in a plane layer of similar configuration. The objective of our work is theoretical and experimental investigations of this process.

1. Problem formulation. Mathematical model. Here consideration is given to a plane layer of a conducting fluid (gallium alloy 2%Zn + 87.5%Ga + 10.5%Sn with the density $\rho = 6256\,\mathrm{kg/m^3}$, kinematic viscosity, $\nu = 3{\cdot}10^{-7}\,\mathrm{m^2/s}$, and electric conductivity $\sigma = 3.56{\cdot}10^6\,\mathrm{Sm}$) with a solid or a free upper surface. The thickness of the layer at rest was $d_0 = 0.006\,\mathrm{m} \div 0.015\,\mathrm{m}$ and its dimension in the plane was $0.2 \times 0.1\,\mathrm{m}$. The magnetic field in the gap $\delta = 0.05\,\mathrm{m}$ had the frequency $\omega = 2\pi{\cdot}50\,\mathrm{s^{-1}}$ and the maximum value $B_{\max} = 0.02\,\mathrm{T}$. The maximum value of the flow velocity developed in the layer was $V_{\max} = 0.1\,\mathrm{m/s}$. For the considered problem, the maximum hydrodynamic Reynolds number was $\mathrm{Re} = V_{\max} d_0/\nu = 4322$, the magnetic Reynolds number was $\mathrm{Rm} = V_{\max} d_0 \sigma \mu_0 = 8.18 \cdot 10^{-3}$ ($\mu_0 = 4\pi \cdot 10^{-6}\,\mathrm{Hn/m}$), and the Hartmann number was $\mathrm{Ha} = B_{\max}\delta\sqrt{\sigma/\rho\nu} = 34$.

The mathematical model is constructed using the equations of magnetohydrodynamics, i.e., Navie–Stokes, continuity, Maxwell and Ohm equations. Here, as in [1], [4]–[6], reasoning from the previous estimations, we adopt some approximations, which allow us to reduce the number of equations. The fulfillment of the condition $\mathrm{Rm} \ll 1$ allows us to use an induction-free approximation. Since the layer thickness d_0 is by order of magnitude smaller than its planar dimensions, we employ a shallow layer approximation. We approximate the horizontal three-dimensional velocity components using the profile function and perform integration over the layer thickness from the bottom to the top surface, whose location is described by the function $h(x,y,t)$. Upon completing these manipulations, we obtain a system of two-dimensional equations for local flow rates $V_i(x,y,t)$ $(i,j=1,2)$ and the location of the free surface h for the layer with the free surface

$$\frac{\partial V_i}{\partial t} + \frac{\partial (V_i V_j q)}{\partial x_j} + V_i r \frac{\mathrm{d}h}{\mathrm{d}t} = -\mathrm{G}h\frac{\partial h}{\partial x_i} + \frac{\partial^2 V_i}{\partial x_j^2} + \kappa V_i + \mathrm{S}hf_i^{\mathrm{em}}; \tag{1}$$

$$\frac{\partial h}{\partial t} + V_j \frac{\mathrm{d}h}{\mathrm{d}x_j} = -\frac{\partial V_j}{\partial x_j}. \tag{2}$$

At the boundary of the plane layer we assume the following conditions: $V_i = 0$, $\partial h/\partial x_i(\mathbf{e}_i \cdot \mathbf{n}) = 0$, where $\mathbf{e}_i$ is the unit vector and $\mathbf{n}$ is the normal vector to a relevant boundary. Conversion to a non-dimensional form is performed using the following non-dimensional units: d_0 for length, ν/d_0 for velocity and $\sigma\omega\mu_0 B_0^2$ for electromagnetic forces. As a result of this transformation, the equations acquire new terms, i.e., a Galilean parameter $\mathrm{G} = gd_0^3/\nu^2$ and a parameter $\mathrm{S} = d_0\delta B_0^2/\rho\mu_0\nu^2$. Integration of the equations results in appearance of new functions $q(x,y,t)$, $r(x,y,t)$, $\kappa(x,y,t)$ described in [1, 5]. The values of these functions are determined by local numbers Re and Ha: $\kappa = \kappa_1(\mathrm{Ha}) + \kappa_2|\mathbf{V}|$. For a layer with a solid upper boundary, the Navie–Stokes equation is written in the same way as in [5], [6]:

$$\partial_t \omega^h + V_x \partial_x \omega^h + V_y \partial_y \omega^h = \partial_x^2 \omega^h + \partial_y^2 \omega^h + \kappa\omega^h + \mathrm{S}(\partial_y f_x^{\mathrm{em}} - \partial_x f_y^{\mathrm{em}}) \ , \tag{3}$$

namely, in terms of vorticity $\omega^h = \triangle\psi^h$ and hydrodynamic stream function ψ^h, which is related to the flow velocity by the equations $V_x = \partial_y \psi^h$, $V_y = -\partial_x \psi^h$.

The electromagnetic forces are defined using only the vertical component of the magnetic field B_z, which is essentially larger than its horizontal components, so that $\mathbf{B} = (0,0,B_z)$, $B_z \equiv B(x,y,t)$. The distribution of the initial magnetic field $B_0(x,y,t)$ generated by the inductor is defined by multiplying the maximum value of the transverse component of the magnetic field (both, the initial and the induced one) in the gap between the inductor poles by the scattering function φ. The scattering function φ of the magnetic field generated by the inductor takes into account a decrease of the magnetic field transverse component due to the scattering near the edges of the inductor poles (Fig. 1*a*). It can be defined by solving the problem of magnetic potential distribution between the poles having the same shape like the poles of the inductor.

The alternating magnetic field generates an eddy current with two components $\mathbf{j} = (j_x, j_y, 0)$ in the layer, which in turn generates its own magnetic field $B_{\mathrm{ind}}(x,y,t)$. Using the Maxwell equation $\mathrm{rot}\,\mathbf{j} = -\sigma\partial_t \mathbf{B}$ and introducing a stream function $j_x = -\partial_y \psi$, $j_y = \partial_x \psi$ for the induced electric current yields

$$\triangle\psi = -\sigma\frac{\partial B}{\partial t}. \tag{4}$$

Applying the Ampere law, we write down the relation between the stream function and the induced magnetic field as $B_{\mathrm{ind}} = \psi d_0 \mu_0/\delta$. The time-dependence of the stream function of the induced alternating current and the total alternating magnetic field is defined from the expressions: $\psi = \psi' \mathrm{e}^{i\omega t}$, $B = B'\mathrm{e}^{i\omega t}$, where ψ', B' are the amplitude values. After substituting these expressions into (4), we obtain an equation for the stream function of the electric current

$$\triangle\psi' = -i\sigma\omega B'. \tag{5}$$

The boundary of the plane layer meets the condition $\psi' = 0$. In order to define the magnetic field and the electric current fields, it is necessary to perform iteration with $B' = B'_0$ taken as an initial value. After this, from Eq. (5) we derive ψ' and $B' = B'_{\text{ind}}$. For the following iterations, we take $B' = B'_0 + B'_{\text{ind}}$, and when the iteration process converges, we define vector components of the current density and electromagnetic forces $f_x^{\text{em}} = \Re\text{e}\{j_y B^*\}/2$, $f_y^{\text{em}} = -\Re\text{e}\{j_x B^*\}/2$. For the problem under discussion, the vector of electromagnetic forces lies in the layer plane.

2. Experiment. The above-mentioned experimental setup includes an inductor with a C-core *1* (Fig. 1*a*) made from transformer steel plates, electric coils *2*, generating an alternating magnetic field in the core gap, and a cuvette *3* with liquid gallium. In experiments with a free-surface layer the free surface of the melt was covered with an aqueous hydrochloric acid solution to remove an oxide film from the metal surface. The reaction of gallium with acid produces bubbles of hydrogen, which are entrained by a moving metal, indicating thereby the trajectories and velocity of its motion. As a test problem, we investigated theoretically and experimentally the most stable two-vortex MVF driven by a core located at the longer wall of the cuvette. In the course of experiment, we found patterns and velocities of the planar flows at the metal surface by taking pictures of the MVF at a known exposure and by measuring the length of the bubble tracks. Fig. 2 compares the calculated and the experimental profiles of the planar surface velocities to validate the applied mathematical model.

The electrodynamic part of the model was checked during special experiments, which allowed to define the total electromagnetic force acting on the aluminum plate $0.2\,\text{m}\times 0.1\,\text{m}\times 0.004\,\text{m}$ placed in the core gap (Fig. 2*b*). The results of the test showed that theoretical predictions were in good agreement with the experimental results.

3. Instability of equilibrium MVF. The stability analysis was performed for the most interesting case of MVF generation, when the region of the layer was overlapped by the inductor (the region of magnetic "spot") located at the center, as shown in Fig.1. At low values of the external magnetic field the MVF induced in the layer exhibited a four-vortex pattern (Fig. 3*a*), which did not change until the magnetic field in the gap of the inductor reached a certain value. A further increase of the magnetic field revealed the existence of three stages in the system behaviour. At the first stage, which is difficult to differentiate from

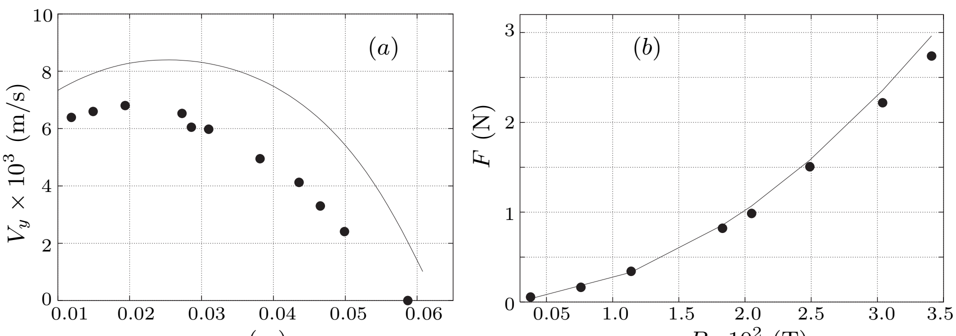


Fig. 2. (*a*) Velocity profile at $y = 0$ (solid line – calculation, points – experiment); (*b*) total electromagnetic force as a function of the initial magnetic field of the inductor (solid line – calculation, points – experiment).

(*a*)

(*b*)

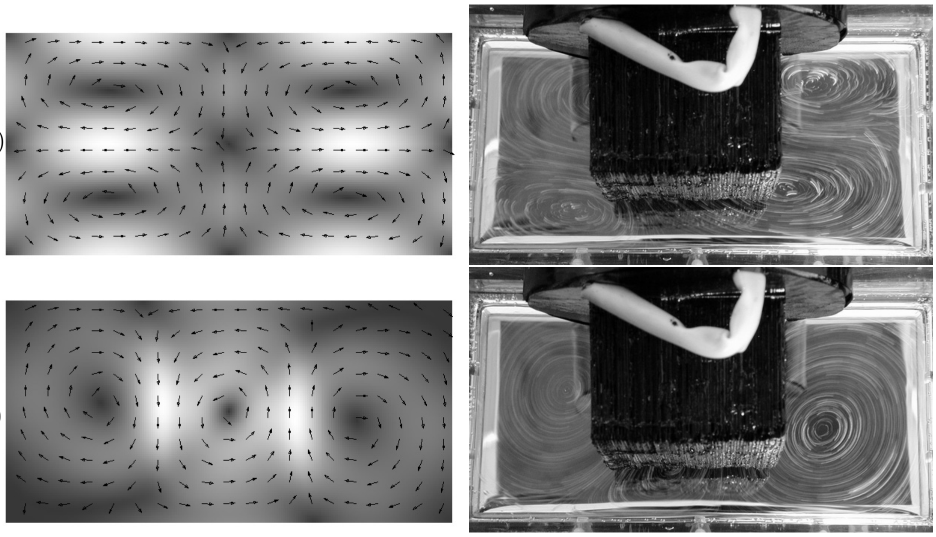


Fig. 3. (*a*) Velocity field of equilibrium MVF; (*b*) MVF pattern after transformation of a four-vortex pattern into a three-vortex flow. Left – calculation results (arrows indicate the direction and gradations of grey – velocity modulus), right – experimental results.

a steady state, an increase of the magnetic field from the inductor enhanced the MVF intensity resulting in an increase of surface deformation. The surface deformation, in its turn, influenced the value and spatial distribution of the vortex electromagnetic forces. Variations in the vortex electromagnetic forces also exerted an immediate effect on the MVF. Thus, there occurred alternating gains in intensity of the diagonal vortex motion. At the second stage, the oscillatory motion of the surface became more and more intensive and the MVF periodically changed from a four- to a three-vortex pattern. Such behaviour involving generation of three similar vortices during an intermediate oscillation period (Fig. 3*b*) was determined by a rectangular shape of the layer. At the third stage, at large values of the external field the MVF oscillations became less regular and the surface near a longer layer side began to execute long-wave oscillations.

To find the threshold value of the external field, at which the MVF begin to oscillate, we measured the velocity field at point *5* (Fig. 1*b*) with coordinates (0.05 m; 0.05 m). The measurements were performed by a conductive velocity gauge *4* consisting of a permanent magnet and two electrodes (Fig. 1*a*). The potential difference between the electrodes is proportional to a mean value of the velocity component normal to the line connecting these electrodes. Typical gauge signals for stable and unstable cases are shown in Fig. 4*a*.

Fig. 4*b* illustrates the dependence of the oscillation energy on the parameter S determined in a fixed time interval, which allows to define the stability threshold. These measurements were used to define neutral curves for the examined layer (both for a free and a solid upper surface (Fig. 5). The layer with the free surface proved to be less stable than the layer with the upper solid boundary. The neutral curve shows that with the increase of the layer thickness the MVF becomes more stable. In this sense, the MVF stability can also be improved by modulating the amplitudes of the external magnetic field by a frequency approaching the frequency of long-wave oscillations of the layer surface [7].

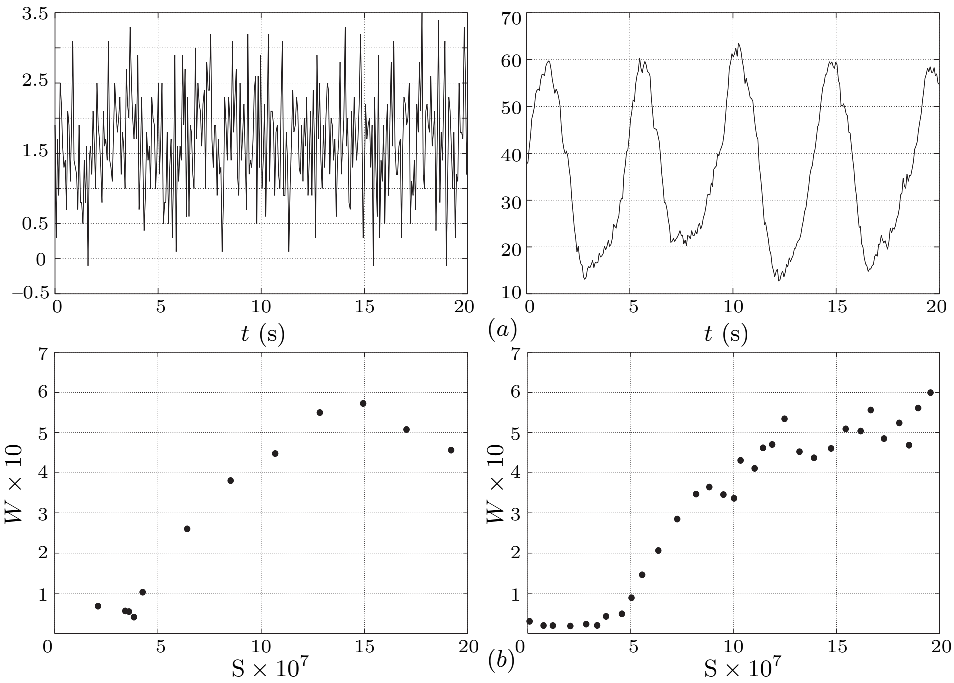


Fig. 4. (*a*) Time-dependence of the velocity gauge signal for a random steady flow (left) and at the onset of instability (right); (*b*) total energy of the oscillatory motion vs. the parameter of MHD-interaction ($G = 10^8$, left – layer with the free top surface, right – layer with the solid top boundary).

4. The use of MVF for liquid metal stirring in technological processes. Our physical and numerical experiments have demonstrated that by creating a spot of the alternating magnetic field in a plane layer of liquid metal we can generate a planar vortical liquid metal flow. Thus, it may be concluded that the MVF can be applied for stirring liquid metals in flat rectangular containers in various technological processes This flow can be made more complicated and hence technologically more effective, if it is excited by several magnetic "spot" (Fig. 6*a*). The degree of stirring can be conveniently determined by the integral parameter

$$\tau = \int_{\Omega} \sqrt{(\partial_x V_y)^2 + (\partial_y V_x)^2}.$$

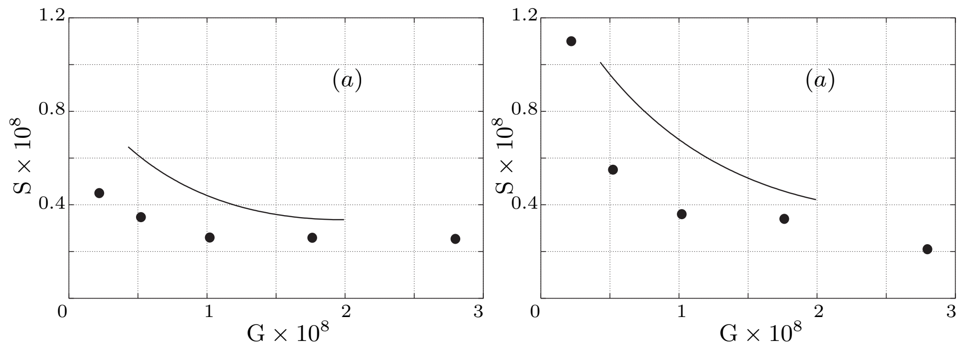


Fig. 5. Neutral curves separating the region of parameters, at which a four-vortex MVF is unstable. (*a*) Layer with the free surface, (*b*) layer with the solid top surface. Solid line – calculation, points – experiment.

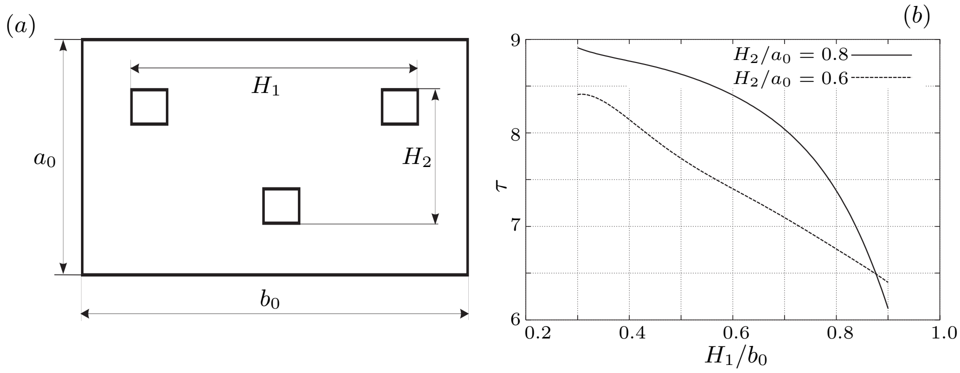


Fig. 6. (*a*) Location of inductors providing metal stirring; (*b*) dependence of the parameter characterizing the effectiveness of stirring on the horizontal relevant location of two upper inductors of the stirrer.

This parameter points out the existence and the magnitude of flow inhomogeneity in the plane layer. It is analogous to the so-called shear rate parameter, which is sometimes used to define the degree of metal stirring in a cylindrical volume under the action of the rotating magnetic field of the electromagnetic stirrer. With the increase of this parameter the quality of stirring essentially improves, i.e., the distribution of introduced dopes and the existing inclusions through the volume of the metal become more homogeneous. As the results of numerical experiments show (Fig. 6*b*), the value of the parameter τ strongly depends on the relevant position of the magnetic "spot". From Figs. 6*b*, 7 it follows that the stirring effect produced by the MVF in a rectangular layer increases when the inductors are located close to the long sides of the layer and at the possible greatest distance from its short sides.

5. Conclusion. In this paper, we have shown that a planar vortical stirring flow can be induced by generating a spot of the alternating magnetic field in a plane liquid layer. Topology and intensity of the excited MVF depend on the shape of the magnetic spot and the boundaries of the plane layer, within which the vortical flow is excited. Due to instability, this flow changes its pattern in an oscillatory manner, which can induce oscillation of the layer surface, provided it is free. By changing the number of magnetic spots and their relevant positions one can alter the topology and the intensity of the melt motion and generate a complicated stirring vortical flow in the layer.

In summary, it may be said that the MVF can be applied for stirring liquid metals in flat rectangular containers in various technological processes of metal-

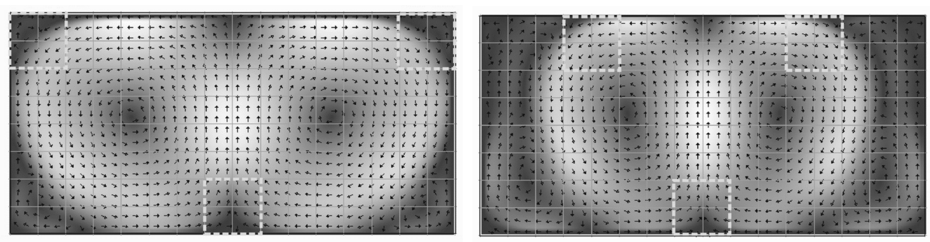


Fig. 7. Different flow patterns during metal stirring in a flat rectangular container (0.8 m × 0.5 m × 0.4 m). The MVF is excited by C-shape inductors (the pole projections are denoted on the upper diagram by three squares) generating an alternating (50 Hz) magnetic field. The figure shows two versions of inductor arrangement.

lurgy. By using several inductors for the generation of the magnetic spots and by changing their relevant positions and the magnetic field strength it is possible to affect the character and the intensity of the flow and hence to control the efficiency of the technological process.

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