The article presents results of an experimental study on damping an oscillatory system with incomplete sealing of the air cavity by a magnetic fluid. The sealing degree varies due to the process of gas pumping through capillaries of different radii. The proposed relaxation theory of a capillary oscillating gas flow predicts abnormally large values of the damping coefficient and almost complete damping of the oscillatory system with a magneto-fluid inertial element. The obtained data are of applied importance.

Introduction. The investigations of the physical properties of magnetizing fluids began as early as the 20th century with studying the magnetic and rheological properties of suspensions [1]. The application of magnetorheological suspensions (MR-Fluids) is based on a very strong dependence of the viscosity on the magnetic field strength, due to which magnetorheological suspensions are applied in brake devices [1–4]. The disadvantage of such dispersed systems is their instability, the irreversible separation of the magnetic and non-magnetic phases under the action of gravity or a non-uniform magnetic field.

A qualitative progress in the development of stable magnetically controlled fluids with high structural stability was achieved in the sixties as a result of the magnetic fluid (MF) development. They have found application in various fields of science and technology, e.g., magneto-fluid seals, magnetically controlled lubrication in friction units and bearings, separators of non-magnetic materials, oil surface cleaners, tilt and acceleration sensors, gap fillers for magnetic heads of loudspeakers.

However, MFs are characterized by a relatively weak dependence of their viscosity on the magnetic field [5, 6]. Meanwhile, many devices whose active element is MF need damping of possible oscillations. In this regard, the idea of additional damping of the oscillatory system by means of incomplete sealing of the air cavity with a magneto-fluid jumper (MF-jumper) is of interest. In this case, MF membranes described in [7–9] can serve as a research base. For this purpose, capillaries of different radii but of equal length, are one by one incorporated in the bottom of the tube, which makes it possible to vary the elasticity of the gas cavity and the damping of oscillations.

The use of capillaries allows one to apply the well-known conclusions of the theory of viscous gas flow. Capillaries can be part of the design of devices in which the MF is an active element. The obtained information is also important for ‘solid-state’ oscillatory systems.

This article presents the results of an experimental study of the elasticity and damping of an oscillatory system with incomplete sealing of the air cavity with an MF-jumper. The sealing degree of the air cavity varies due to the process of gas pumping through capillaries of different radii. To explain the obtained regularity patterns, a model theory
of viscous gas flow approximation according to the Poiseuille’s law is proposed, and the
deductions of the well-known theory of sound propagation in molecular acoustics and the
theory of sound ducts are considered.

1. Oscillatory system with lumped parameters. The presence of a massive
MF-jumper and a restoring force due to the ponderomotive action of the uniform mag-
netic field of an annular magnet on the MF and of the elasticity of the partially sealed
air cavity determined the main characteristics of the oscillatory system with lumped
parameters in the situation under consideration. Let us write the differential equation of
oscillatory motion in the standard form as

$$\frac{\partial^2 \xi}{\partial t^2} + 2\beta \frac{\partial \xi}{\partial t} + \frac{k_{mf}}{m_i} \xi = 0,$$

where $\xi$ is the displacement of the center of the jumper mass, $\beta$ is the damping coefficient
of oscillations, $k$ is the elasticity coefficient, $m_i$ is the mass of the MF-jumper. In the
schematic presentation of the experimental setup (see Fig. 1 bellow) the mass of the
jumper is concentrated in its center in the form of a black ball with the mass index $m_i$, and
the axis of vibrational displacements $\xi$ is shown by the arrow pointing upwards.

The coefficient of elasticity $k$ is the sum of

$$k = k_p + k_\sigma + k_g^*,$$

where $k_p$ is the coefficient of ponderomotive elasticity due to the interaction of the MF-
jumper with a magnetic field, $k_\sigma$ is the coefficient of elasticity due to the MF surface
tension, $k_g^*$ is the coefficient of elasticity of the gas cavity which depends on the gas pass
through the capillary.

The air cavity sealing under the jumper gives it a ‘normal’ gas-type elasticity, with
the elasticity coefficient $k_g$. The elasticity coefficient of the gas cavity was obtained in
[10]. If the insulated gas chamber is a part of a cylindrical tube of height $h_0$, then

$$k_g = \frac{\gamma \pi d^2 P_0}{4h_0},$$

where $P_0$ is the gas pressure in the cavity in the absence of oscillations; $\gamma$ is the Poisson
ratio for gas (for air $\gamma = 1.4$), $d$ is the tube diameter. The coefficient $k_g^* \leq k_g$ due to the
capillary gas leak.

The formula for the coefficient of elasticity due to surface tension is known [9]:

$$k_\sigma = 16\pi \sigma,$$

where $\sigma$ is the coefficient of the MF surface tension; $k_\sigma/k_g < 1$, which allows not to take
this parameter into account in the future.

The expression for the ponderomotive elasticity coefficient $k_p$ of the MF column in
a strong magnetic field was derived in [11, 12] and for the jumper in the approximation
model in [7–9]. It is significant that the value of $k_p$ does not depend on the frequency
of oscillations. For the MF-jumper, $k_p$ can be determined experimentally by measuring
the oscillation frequency of the system with the jumper held by the forces of magnetic
levitation, $\omega_0$, and by calculating this parameter using the formula:

$$k_p = m_i \omega_0^2$$

The general solution of Eq. (1) has a known form $\xi = \xi_0 e^{-\beta t} \cos(\omega t + \psi)$, where $\xi_0$, 
$\psi$ are the initial values of the amplitude and phases of the oscillation, $\beta$ is the oscillation
damping coefficient.
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If the MF-jumper experiences only the influence of the ponderomotive force, the oscillation frequency $\omega$ is expressed as

$$\omega = \sqrt{\omega_0^2 - \beta_0^2},$$

where $\omega_0^2 \equiv k_p/m_f$, $\beta_0$ is the damping coefficient in an oscillatory system without an attached cavity.

However, if a partially sealed air cavity with the elasticity coefficient $k_g^*$ is included in the oscillatory system, then

$$\omega = \sqrt{\omega_1^2 - \beta_1^2},$$

where $\omega_1^2 = (k_p + k_g^*)/m_f$, $\beta_1$ is the damping coefficient in an oscillatory system with an attached cavity.

2. Equilibrium processes of the capillary oscillating gas flow. Assuming the gas flowing through the capillary to be incompressible (the velocity of its laminar flow is much less than the sound velocity), let us use the well-known Poiseuille formula:

$$\frac{\Delta V^e}{\Delta t} = \frac{\pi r^4 \Delta P}{8 l \eta_g},$$

where $\Delta V^e$ is the amount of the air through the capillary for the time $\Delta t$ during the equilibrium process; $r$ is the capillary radius ($r \ll d/2$); $l$ is the capillary length; $\eta_g$ is the air viscosity; $\Delta P$ is the pressure drop at the ends of the capillary. In this case and hereinafter, the term ‘equilibrium’ is assumed as a model of laminar gas flow by layers without mixing with a parabolic velocity distribution over the capillary cross-section (Poiseuille’s law).

To analyze the formulas used and their application in the discussion of the obtained experimental data, it is necessary to give explanations for the term ‘capillary tube’ or ‘capillary’. In [13] it was shown that for capillary tubes the oscillatory flow of a viscous medium obeys the Poiseuille’s law. The criterion is the inequality $r < 2 \sqrt{2 \eta_g/(\omega \rho_g)}$ which, in our case, is satisfied for both ‘narrow’ and ‘wide’ capillaries. In tubes of a ‘large’ radius, this condition may not be satisfied. In these tubes, the flow of a viscous medium is a piston-like flow when the movement of particles is concentrated in a narrow wall region (Helmholtz model). It is the circumstance that allows us to use formula (7) and all the subsequent expressions under the assumption of ‘equilibrium’ capillary air flow.

Let the pressure in the gas cavity vary according to the harmonic law $\Delta p = \Delta p_0 \cdot \cos \omega t$. The alternating pressure $\Delta p$ is superimposed on the constant external pressure $P_0$, and $\Delta p_0 \ll P_0$. Then the amplitude value of the equilibrium volume fluctuations is as

$$\delta V_0^e = \frac{\pi r^4 \Delta p_0}{8 l \eta_g \omega}.$$

The volumetric enlargement of the gas cavity $V_0$ when the MF-jumper is displaced due to the compression and gas leak through the capillary reads as

$$\Delta V_0 = -\frac{\Delta V_0}{\gamma P_0} \Delta p_0 - \frac{\pi r^4}{8 l \eta_g \omega} \Delta p_0.$$

Since

$$\Delta V_0 = S \xi_0,$$
V.M. Polunin, P.A. Ryapolov, E.V. Shel’deshova, G.V. Karpova, V.M. Paukov

where $S$ is the tube cross-sectional area, $\xi_0$ is the jumper displacement amplitude,

$$S^2\xi_0 = -\left(\frac{V_0}{\gamma P_0} + \frac{\pi r^4}{8 l_\eta \omega}\right) \Delta F_0,$$

(10)

where $\Delta F_0$ is the amplitude of the force pressing the cavity.

From Eq. (10) follows an expression similar to Hooke’s law:

$$\Delta F_0 = -\left(\frac{V_0}{\gamma P_0} + \frac{\pi r^4}{8 l_\eta \omega}\right)^{-1} S^2 \xi_0.$$

(11)

If the gas cavity has the shape of a cylindrical tube, then Eq. (11) can be represented as

$$\Delta F_0 = -\frac{\gamma \pi d^2 h_0}{4 h_0} \left(1 + \frac{\gamma P_0 r^4}{2 d^2 h_0 l_\eta \omega}\right)^{-1} \xi_0.$$

(12)

The equilibrium value of the coefficient of gas elasticity of the system with the capillary $k_g^e$ is derived from expression

$$k_g^e = \frac{\gamma \pi d^2 P_0}{4 h_0} \left(1 + \frac{\gamma P_0 r^4}{2 d^2 h_0 l_\eta \omega}\right)^{-1},$$

(13)

or, considering Eq. (4),

$$k_g^e = k_g \left(1 + \frac{\gamma P_0 r^4}{2 d^2 h_0 l_\eta \omega}\right)^{-1}.$$

(14)

In the proposed approximation theory, the coefficient of ‘equilibrium’ elasticity $k_g^e$ replaces the parameter $k_g^\ast$ a priori included in expression (2).

If we set the frequency $\omega$, then for a ‘wide’ capillary it yields

$$\frac{\gamma P_0 r^4}{2 l_\eta d^2 h_0 \omega} \gg 1,$$

$$\Delta F_0 = -k_g^e \xi_0 = -k_g \frac{2 l_\eta d^2 h_0 \omega \xi_0}{\gamma P_0 r^4},$$

(15)

and for a ‘narrow’ capillary,

$$\frac{\gamma P_0 r^4}{2 l_\eta d^2 h_0 \omega} \ll 1,$$

$$\Delta F_0 = -k_g^e \xi_0 = -k_g \left(1 - \frac{\gamma P_0 r^4}{2 l_\eta d^2 h_0 \omega}\right) \xi_0.$$

(16)

In the first and second limiting cases, $k_g^e = 0$ and $k_g^e = k_g$, respectively.

Neglecting the damping coefficient in Eq. (6) and taking into account the transition to the accepted approximation yield

$$\omega_1^2 - \omega_0^2 = \frac{k_g}{m_f} \left[1 + \frac{\gamma P_0 r^4}{2 l_\eta d^2 h_0 \omega_1}\right]^{-1}.$$

(17)
Damping of an oscillatory system with incomplete sealing of the air cavity by magnetic fluid

Considering formula (15) provides the expression for an oscillatory system with a ‘wide’ capillary:

\[
\frac{\omega_1^2 - \omega_0^2}{\gamma P_0 r^4} = \frac{k_g \cdot \frac{2l \eta_g d^2 h_0 \omega_1}{\gamma P_0 r^4}}{m_f},
\]

from where the quadratic equation for finding the parameter \(\omega_1\) follows:

\[
\omega_1^2 - 2k_g \frac{l \eta_g d^2 h_0 \omega_1}{m_f \gamma P_0 r^4} \cdot \omega_1 - \omega_0^2 = 0.
\]

By assuming \(r\) as an unknown term in Eq. (18), after simple transformations we have

\[
r = \left[\left(\frac{k_g}{m_f (\omega_1^2 - \omega_0^2)} - 1\right) \frac{2l \eta_g d^2 h_0 \omega_1}{\gamma P_0 r^4}\right]^{1/4}.
\]

Considering Eqs. (3) and (15), let us derive the expression for the reaction force

\[
\Delta F_0 = -\frac{\pi d^4 \eta_g \omega \xi_0}{2 r^4},
\]

For the force acting per the unit mass of the MF-jumper, we derive

\[
\frac{\Delta F_0}{m_f} = -\frac{\pi d^4 \eta_g}{2 m_f r^4} \cdot \frac{\partial \xi}{\partial t}.
\]

It follows from formula (20) that for ‘narrow’ capillaries, the increment of force per unit mass is

\[
\frac{\delta F}{m_f} = -\frac{\pi \gamma^2 P_0^2 r^4}{8 m_f h_0^2 \eta_g \omega} \cdot \xi_0,
\]

or

\[
\frac{\delta F}{m_f} = -\frac{\pi \gamma^2 P_0^2 r^4}{8 m_f h_0^2 \eta_g \omega^2} \cdot \omega \xi_0.
\]

3. Equilibrium and dynamic oscillation damping coefficients. The formula for the damping coefficient due to the ‘equilibrium’ pumping of gas through the capillary follows from expression (22), in the terminology used – through the ‘wide’ capillary:

\[
\beta_P = \frac{\pi d^4 \eta_g}{4 m_f r^4}.
\]

As shown below, the value of the ‘equilibrium’ oscillation damping coefficient \(\beta_P\) is comparable with the damping coefficient in case of the absence of the bottom in the tube.

At the same time, formula (24) leads to an absurd conclusion that as \(r \to 0\), \(\beta_P \to \infty\).

To derive an expression for the ‘non-equilibrium’ damping coefficient due to energy losses during the gas transfer through the capillary ‘with delay’, as assumed in the theory of acoustic relaxation [14, 15], in expression (24), it is necessary to use the dynamic viscosity \(\eta_{dn}\) instead of the static viscosity \(\eta_g\),

\[
\eta_{dn} = \frac{\eta_g}{1 + \omega^2 \tau^2},
\]

where \(\tau\) is the duration of the relaxation process.
The ‘dynamic’ damping coefficient of the oscillatory system is expressed as

\[ \beta_P = \frac{\pi d^4 l}{4 m_f r^4} \cdot \frac{\eta_g}{1 + \omega^2 \tau^2}. \]  \hspace{1em} (26)

In the framework of the model theory, we assume that the time to determine the equilibrium value of the gas flow velocity \( \tau \) is commensurate with the transit time of a gas portion \( V_0 \) through the capillary. This time \( \Delta t \) can be derived from the Poiseuille formula (7), in which the change \( \Delta V_e \rightarrow V_0 \) is made and the numerical coefficient \( \psi \) is introduced:

\[ \tau = \frac{2 \psi l \eta_g d^2 h_0}{r^4 \gamma P_0}. \]  \hspace{1em} (27)

The value of the numerical coefficient \( \psi \) is determined by selection to match the experimental data.

In an expanded form, formula (26) reads as

\[ \beta_P = \frac{\pi d^4 l}{4 m_f r^4} \cdot \frac{\eta_g}{1 + \omega^2 \left( \frac{2 \psi l \eta_g d^2 h_0}{r^4 \gamma P_0} \right)}. \]  \hspace{1em} (28)

Expression (28), as a function of \( r \), has an extremum. To obtain the value of \( r_{ex} \), we introduce the notation \( \tau = kr^{-4} \), where

\[ k = \frac{2 \psi l \eta_g d^2 h_0}{\gamma P_0}, \]

so Eq. (28) takes the form:

\[ \beta_P = \frac{\pi d^4 l}{4 m_f} \cdot \frac{\eta_g}{r^4 + \omega^2 k^2/r^4}. \]  \hspace{1em} (29)

Performing standard procedures when finding the extremum of the expression in parentheses in formula (29) with a fixed value of \( \omega \) yields

\[ r_{ex} = 4 \sqrt[4]{\omega k} \]  \hspace{1em} (30)

In an expanded form, it reads as

\[ r_{ex} = \sqrt[4]{\frac{2 \psi l \eta_g d^2 h_0 d \omega}{\gamma P_0}}. \]  \hspace{1em} (31)

According to formulas (31) and (28), it is possible to calculate the maximum value of the damping coefficient of an oscillatory system with capillaries:

\[ \beta_{P_{max}}^c = \frac{\pi d^4 l \eta_g}{4 m_f r_{ex}^4}. \]  \hspace{1em} (32)

From Eq. (29), it is possible to derive the formula for the ‘narrow’ capillary:

\[ \beta_P = \frac{\pi d^4 l \eta_g r^4}{4 m_f \omega^2 k^2}. \]  \hspace{1em} (33)
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Upon substituting the expressions for $k$ into Eq. (33), we derive the formula for calculating the damping coefficient:

$$\beta_P = \frac{\pi \gamma P_P^2 r^4}{16 m_f \eta g h_0 l \omega^2 \psi^2 \omega^2}.$$  

(34)

Thus, in contrast to the equilibrium model, it follows from the relaxation theory that when $r \to 0$, $\beta_P \to 0$.

From Eq. (28), the process of static viscosity deviation from dynamic viscosity starts at the oscillation frequency at which $\omega^2 \tau^2 = 1$ in Eq. (28). Referring to this frequency as a ‘critical’ one and denoting it by $\nu_{cr}$ yield

$$\nu_{cr} = \frac{\gamma P_0 r^4}{4 \pi \psi l \eta g d^2 h_0}.$$  

(35)

4. Measurement technique and physical and chemical properties of the studied sample. Schematically, the experimental setup is shown in Fig. 1. A portion of magnetic fluid 1 with a volume of 1.5 cm$^3$ (with the mass $m_f = 0.00187$ kg) introduced using a medical syringe into a glass tube 2 into the region of the maximum magnetic field of the annular magnet 3 overlaps the tube section with a diameter of 12 mm (MF-jumper).

A neodymium annular magnet (NdFeB alloy) sized $60 \times 24 \times 10$ mm was used in the study. The magnetic field strength in the center of the magnet, measured by a TPU-01 milliteslameter with an error of 2.5%, is $220$ kA/m.

During the measurements, three options for the operation of the setup were implemented: (i) the MF-jumper is hung up due to magnetic levitation (‘bottomless’ experiment); (ii) the MF-jumper isolates an air cavity formed by a plug in the bottom (‘bottom’ experiment); (iii) a plug with a capillary is inserted into the bottom of the tube. Thus, the lower base of the tube either remains open or closed by a plug 4 with or without a capillary 5.

![Fig. 1. Schematic presentation of the experimental setup: 1 – magnetic fluid, 2 – plexiglass tube, 3 – annular magnet, 4 – plug with a capillary, 5 – capillary, 6 – rubber plug with a hole to excite oscillations, 7 – inductance coil, 8 – GVT-427B amplifier, 9 – GwInstek GDS-72072 oscilloscope.](image)
Between the lower free MF surface and the plug, an air volume is formed; its sealing degree depends on the linear dimensions of the capillary and becomes maximum when the capillary is plugged. The height of the isolated cavity and of the ‘incompletely sealed’ cavity in the tube is set at three fixed levels $h_0$: 20 mm, 30 mm, and 40 mm.

To excite oscillations of the MF-jumper, a rubber plug with a hole $b$ is used; the plug is inserted into the upper end of the tube. There is an air cavity between the plug and the free surface of the fluid. Pulling the plug with a closed hole out of the tube causes a pressure jump in the gas cavity. During the measurements, no jumper break is allowed.

An inductance coil 7, a GVT-427B amplifier 8 and a GwInstek GDS-72072 digital oscilloscope 9 were used to trace oscillograms. The inductance coil has 1,900 turns of copper wire with a diameter of 0.071 mm around a plexiglass frame. The frame is rigidly placed into the hole of the annular magnet, but there is a narrow gap between the magnet and the surface of the tube that allows moving the magnet and the MF-jumper to the desired position. Oscilloscope functionality to determine the oscillation frequency allows two decimal places in the frequency range $\leq 100 \text{ Hz}$. The damping coefficients of the oscillatory system $\beta$ were obtained using the enveloping curves of the oscillograms, i.e. the trend lines.

The capillary radius was measured using a Lyncee tec R2100 digital holographic microscope which together with the attached LEICA 10x lens provided a lateral resolution of 0.8 $\mu$m.

In this work, we studied a MF sample based on highly dispersed magnetite Fe$_3$O$_4$ stabilized by a surfactant (oleic acid C$_8$H$_{17}$CH = CH(CH$_2$)$_7$ – COOH). Aviation kerosene TS-1 was used as a dispersion medium, i.e. a carrier fluid. The MF sample was synthesized in the Fundamental Scientific Research Laboratory of Applied Ferrohydrodynamics at the Ivanovo State Power Engineering University. The density of the test sample is $\rho = 1.245 \text{ kg/m}^3$.

The MF magnetization curve was obtained in the Laboratory of Nanoscale Acoustics of the Southwest State University by the ballistic method. Based on the data obtained, the saturation magnetization $M_s = 39.5 \text{ kA/m}$ was calculated. Shear viscosity $\eta$ was measured using a Brookfield DV2T viscometer; its value is 34.8 MPa·s at a shear rate of 79.200 1/s.

The experimental data was obtained in a temperature range of $\pm 0.2^\circ\text{C}$. The required temperature was set and maintained during the experiment using the LEBERG LS / LU-09OL split system. The temperature of the MF specimen was measured on its surface by a Fluke 62 MAX infrared thermometer, which makes it possible to determine the surface temperature in the range of $-30^\circ\text{C}$ to $+500^\circ\text{C}$ with a resolution of 0.1$^\circ\text{C}$. To analyze the oscillograms recorded in the measurements, the NI LabVIEW software was used.

With Picasso 3D Designer 3D printing technology and laser material tailoring by the Raylogic 11G 1290 instrument, an inductance coil frame, tube holders, and other structural elements were cut, which made it possible to reduce the elastic energy losses and improve the noise immunity of the experimental setup.

5. Experimental results and discussion.

5.1. Oscillatory frequency. The observed oscillograms of the oscillations of the MF-jumper in the tube with no bottom, with a bottom, and with a capillary have a typical form of damped oscillations of a system with lumped parameters. A rather high reproducibility of the experimental results should be noted: with the same filling of the
Damping of an oscillatory system with incomplete sealing of the air cavity by magnetic fluid

Table 1. Experimental data values.

<table>
<thead>
<tr>
<th>$h_0 = 20$ mm</th>
<th>With capillary</th>
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<tbody>
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<td></td>
<td>With no</td>
</tr>
<tr>
<td></td>
<td>bottom</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$, [1/s]</td>
<td>27</td>
</tr>
<tr>
<td>$\nu$, Hz</td>
<td>55.0</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>$h_0 = 30$ mm</th>
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<tr>
<td>$\beta$, [1/s]</td>
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<td>$\nu$, Hz</td>
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<tr>
<th>$h_0 = 40$ mm</th>
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<tr>
<td>$\beta$, [1/s]</td>
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<td>$\nu$, Hz</td>
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tube with the fluid, each of 12 pulls of the plug out of the tube excites oscillations, whose frequency and damping coefficient differ by $\leq 0.1\%$ and $\leq 1\%$, respectively.

In the experiments ‘with no bottom’, the oscillation frequency and the damping coefficient had the same ratio before and after the measurements.

Table 1 shows the oscillation frequency $\nu$ measured in the experiments ‘with the bottom’ and in experiments ‘with no bottom’ as well as with the use of capillaries no. 1–5. The table also presents the corresponding damping coefficient values.

It is noteworthy that at all values of $h_0$ a sharp decrease in oscillation frequency was observed with a transition from the ‘narrow’ capillaries no. 1–3 to the ‘wide’ capillaries no. 4 and 5 as well as the coincidence of the oscillation frequency of the setup ‘with the bottom’ and with the capillary no. 1. The latter can be explained by the fact that the presence of the capillary no. 1 scarcely affects the sealing of the air cavity with the MF-jumper. This conclusion is consistent with the theory of sound wave passing through the boundary of the connection of pipes of different radii. So, in [16], an expression for the ratio of the energy flux from a tube with a larger cross-section surface $S_1$ to a tube with a smaller cross-section $S_2$ (in our case, it is a capillary), the value of which is

$$D = 1 - \left( \frac{S_1 - S_2}{S_1 + S_2} \right)^2 \approx 4 \frac{S_2}{S_1} = 4 \left( \frac{d_2}{d} \right)^2,$$

where $d_2$ is the capillary diameter, i.e. very small.

Using formula (20), for a cavity with $h_0 = 2$ cm, the calculated values of $r$ were obtained in the frequency range of 55 Hz to 112 Hz with 2 Hz increments (in the vicinity of 55 Hz, an increment was of 1 Hz). In this case, the value of $\omega_0$ was determined from the experiment ‘with no bottom’ neglecting the damping coefficient. Considering this data set as a set of frequencies corresponding to a certain value of the capillary radius, we can graphically represent the $\nu(r)$ dependence in Fig. 2. The calculated results are plotted in cubes, whereas the sequence of calculated frequencies is approximated by a smooth thin line. The dashed line shows the oscillation frequency when the tube is open (‘with no bottom’ experiment), the dash-dotted line shows the oscillation frequency when the tube was closed (‘with the bottom’ experiment).
In accordance with the data listed in Table 1, the calculated dependence $\nu(r)$ shown in Fig. 2 is characterized by a sharp decrease in frequency with a transition from ‘narrow’ capillaries to ‘wide’ capillaries, and in its lower part it tends to approach a dashed line representing the oscillation frequency with the tube being open. As expected, the solutions of the quadratic equation (19) (we have saved only the solution with a positive sign of the two solutions) indicated by black circles belong to an approximating thin line.

The sharp decrease in oscillation frequency indicates a strong decrease in elasticity of the gas cavity with the transition from ‘narrow’ capillaries to ‘wide’ capillaries.

5.2. Damping coefficient. In the considered oscillatory system with an inertial element in the form of an MF column in the tube, the dissipation of elastic energy is mainly caused by the simultaneous action of three physical mechanisms:

(i) energy loss due to vibrations of the membrane consisting of a viscous magnetic fluid;

(ii) emission of energy of elastic oscillations to the structural elements and environment;

(iii) loss of energy for gas pumping through the capillary.

From the experiment on oscillations, the energy losses associated with the vibrations of the MF-jumper as a membrane and emission of elastic energy can be estimated by the value of the damping coefficient at a frequency of about 55 Hz when the cavity is open (‘with no bottom’ experiment) and at a frequency of about 100 Hz by the value of the damping coefficient when the cavity is closed (‘with the bottom’ experiment).

The mechanism of electromagnetic energy emission (due to which oscillations are registered by an induction sensor) also contributes to the energy dissipation process; however, its contribution makes up a small portion of the total losses.

It can be assumed that the contribution of the mechanism of elastic energy emission with minimal structural changes during the measurement remains constant [17].
Let us denote the damping coefficient resulted from all types of losses, except for the third one, as $\beta_s$. The resulting damping coefficient is represented as the sum of

$$\beta = \beta_s + \beta_p$$  \hspace{1cm} (36)

Moreover, let us take the damping coefficient of the oscillatory system ‘with the bottom’ for ‘narrow’ capillaries that practically seal the air cavity, and the damping coefficient of the oscillatory system ‘with no bottom’ for ‘wide’ capillaries that almost completely remove the cavity sealing.

The relaxation time $\tau$ is calculated by formula (27), which is presented in Table 2.

For the ‘narrow’ capillaries, the relaxation time is relatively long, for the ‘wide’ ones, it is relatively short, which physically seems obvious. A tendency of $\tau$ increasing with increasing $h_0$ is also expected.

In Fig. 3, the crosses show the experimental data for the damping coefficient of the system with a gas cavity $h_0 = 2$ cm and capillaries no. 1–5 in the order of $r$ increasing along
the $x$-axis. The dashed line represents the damping coefficient for an open tube (‘with no bottom’ experiment), and the dash-dotted line represents the damping coefficient for an open tube (‘with the bottom’ experiment). The numerical values for the damping coefficient for air cavities with $h_0 = 2$ cm, $h_0 = 3$ cm and $h_0 = 4$ cm as well as for the setups with an open and closed tube are listed in Table 1.

The $\beta(r)$ dependences for the air cavities with $h_0 = 3$ cm, $h_0 = 4$ cm are characterized by a qualitatively similar dependence for the air cavity $h_0 = 2$ cm. Moreover, the maximum value of $\beta$ is for a capillary with a radius of 0.3 mm.

In Fig. 3, curves 1, 2, and 3 are the result of the relaxation theory. They are plotted using expression (28) and considering the relationship between the parameters $\nu$ and $r$, previously used to plot the graph in Fig. 2. The presented $\beta(r)$ dependences differ in their numerical value $\psi$. So, curve 1 is for $\psi = 1$, curve 2 for $\psi = 1.5$, curve 3 for $\psi = 1.8$. Curve 2 passes through three crosses (experimental values of $\beta$), which allows us to prefer $\psi = 1.5$ as the reference value.

All $\beta(r)$ curves have maxima.

The deviation of static viscosity from dynamic viscosity begins with the ‘critical’ oscillation frequency determined by formula (35). The calculated values of $\nu_{cr}$ are collected in Table 3. The relaxation delay for the ‘narrow’ capillaries begins already in the range of infrasonic frequencies, whereas for the ‘wide’ capillaries this zone is in the kHz frequency range, i.e. outside the investigated frequency range.

Expression (23) allows us to derive the equation for the additional damping due to the air friction during the equilibrium gas flow through the ‘narrow’ capillary:

$$\beta_2 = \frac{\gamma^2 \pi P^2_0 r^4}{16 m_I \eta g h_0^2 \omega^2}. \quad (37)$$

Eqs. (34) and (37) describing the damping coefficient differ only in the $\psi$ coefficient, which is a consequence of the reasonable choice of Eq. (27) for the relaxation time. It can be assumed that the relaxation process imposes a restriction on the type of the gas flow through the capillaries, considering the capillaries’ ‘throughput’. In this case, apparently, the velocity ‘parabolic’ distribution along the radius of the capillary remains unchanged. Only the maximum value of the velocity in the center of the capillary and, accordingly,

### Table 3. Critical values of the frequency $\nu_{cr}$.

<table>
<thead>
<tr>
<th>$h_0$, [m]</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$ of a capillary, [mm]</td>
<td>$\nu_{cr}$, [Hz]</td>
<td>1.585</td>
<td>1.8</td>
</tr>
</tbody>
</table>

### Table 4. Capillary radius $r_{ex}$ and maximum damping coefficient $\beta_{P_{max}}$.

<table>
<thead>
<tr>
<th>$h_0$, [m]</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{ex}$, [mm]</td>
<td>0.432</td>
<td>0.478</td>
<td>0.514</td>
</tr>
<tr>
<td>$\beta_{P_{max}}$, [s$^{-1}$]</td>
<td>225</td>
<td>150</td>
<td>112</td>
</tr>
</tbody>
</table>
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the average velocity over the cross-section decrease.

The calculation of \( r_{ex} \) according to Eq. (31) yields the values presented in Table 4. When calculating, \( \psi = 1.5, \nu = 100 \text{ Hz}, \gamma = 1.4; P_0 = 10^5 \text{ Pa} \) are assumed. With increasing \( h_0 \), \( r_{ex} \) increases, whereas the value of \( \beta_{P\text{max}}(r) \) decreases.

Thus, both experimentally and theoretically, a nonmonotonic dependence of the damping coefficient on the capillary radius has been found, which shows a sharp increase and, upon reaching a maximum, a smooth decrease in value of the damping coefficient with an increase in capillary radius.

By equating the expressions for the damping coefficients (24) and (37)

\[
\frac{\pi d^4 l \eta g}{4 m r^4} = \frac{\gamma^2 \pi P_0^2 r^4}{16 m l \eta h_0^2 \omega^2},
\]

it is possible to derive a formula for calculating the corresponding radius of the capillary:

\[
r_{ex} = \sqrt[4]{\frac{2d^2 l \eta g h_0 \omega}{\gamma P_0}}
\]

Assuming \( \nu = 100 \text{ Hz} \) and \( h_0 = 2 \text{ cm} \) yields \( r = 0.39 \text{ mm}, \beta = 339 \text{ s}^{-1} \).

So, the proposed model theory of the oscillating gas flow through capillaries in addition to experimental data predicts anomalously large values of the damping coefficients, i.e. almost complete damping of the oscillatory system with an MF as an inertial element.

The use of the data obtained is advisable in designing new shock absorbers, since the magneto-fluid damper with capillaries can damp low-frequency oscillations.

Some conclusions can be applied to other systems, including ‘solid-state’ ones. A vibrator converter with a cylindrical shell made of a high-temperature superconductor (HTSC) can be an example [18]. In the presence of two air chambers at both ends of the magnetic system, oscillation damping is possible due to the gas flow in the gap between the shell and the inertial element.

Conclusions. The results of the research are as follows:

- experiments on the damping of an oscillatory system with incomplete sealing of the air cavity by a magnetic fluid using capillaries were carried out;
- a model theory in the approximation of viscous gas flow according to the Poiseuille’s law is proposed to explain the obtained regularity patterns; the conclusions of the well-known relaxation theory of molecular acoustics and theory of sound ducts are also applied;
- the relaxation mechanism imposes a restriction on the type of the oscillating gas flow through capillaries, considering the capillaries’ ‘throughput’ capacity;
- the ‘parabolic’ distribution of the gas velocity along the radius of the capillary apparently remains unchanged, but the maximum value of the velocity in the center of the capillary and, accordingly, the average velocity over the cross-section decrease;
- the proposed model explains the presence of a maximum in the dependence of the damping coefficient on the capillary radius and its decrease with increasing volume (height) of the gas cavity;
- the relaxation theory of the model of an oscillating gas flow through capillaries, which predicts almost complete damping of an oscillatory system with capillaries of a certain radius, does not claim to provide high-precision calculated data.

Acknowledgements. The authors are sincerely grateful to I.M. Arefiev, the head of the Fundamental Research Laboratory of Applied Ferrohydrodynamics of the Ivanovo State Power Engineering University, for the magnetic fluid sample.
The publication has been prepared as part of the state assignment (No. 0851-2020-0035) and grant of the RF President (MK-1393.2019.8).

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Received 19.06.2020