SELF-SIMILAR PROBLEMS IN THE MAGNETOHYDRODYNAMICS OF NON-NEWTONIAN FLUIDS

L. K. Martinson

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Certain self-similar solutions are found to the problem of the shear MHD flow of a non-Newtonian fluid with a power rheological law for injection of the fluid by means of a nonconducting plate lying on the fluid. These solutions have the form of shear waves, the fronts of which propagate with the velocity of motion of the medium in the direction of injection.

Stationary and nonstationary problems in the magnetohydrodynamics of non-Newtonian fluids with a power rheological law are considered in [1-5]. In the present communication we cite some self-similar solutions to problems describing shear MHD flows of such fluids, in which the velocity has a transverse component due to injection through the bounding surface.

Let a conducting fluid with a power rheological law occupy the half-space $z>0$. An external uniform magnetic field with induction B_0 is directed along the z axis; there is no electric field. A nonconducting plate lying on the fluid injects the latter with a velocity $v_z = v(t)$. A shear flow arises in the fluid due to the motion of the plate. In the zero-induction approximation the corresponding problem describing this sort of flow can be written in the form

$$\begin{cases} \dfrac{\partial u}{\partial t} + v(t)\dfrac{\partial u}{\partial z} = a\dfrac{\partial}{\partial z}\left[\left|\dfrac{\partial u}{\partial z}\right|^{n-1}\dfrac{\partial u}{\partial z}\right] - \lambda u; \\ u(0,t) = U(t); \quad u(z,0) = 0, \end{cases} \tag{1}$$

where $a = k/\varrho$, $\lambda = \sigma B_0^2/\varrho$, and the rest of the symbols have their usual meanings.

We restrict the discussion to media for which the rheological constant $n>1$. As was shown in [2-4], shear perturbations in such media propagate with a finite velocity, forming the front of a shear wave.

No analytic solution to nonlinear problem (1) is possible for arbitrary functions v(t) and U(t). Self-similar solutions to problem (1) can, however, be found in certain special cases when a definite relationship exists between the injection velocity v(t) and the velocity of motion of the plate U(t). Indeed, let

$$U(t) = A\left\{\int_0^t v(\theta)\,d\theta\right\}^{\frac{n+1}{n-1}}, \qquad A = \left\{\frac{\lambda}{2a}\,\frac{n-1}{n}\left(\frac{n-1}{n+1}\right)^n\right\}^{\frac{1}{n-1}}. \tag{2}$$

Problem (1) then has the generalized self-similar solution

$$u(z,t) \equiv u(\xi) = \begin{cases} A\xi^{\frac{n+1}{n-1}}, & \xi>0; \\ 0, & \xi \leq 0, \end{cases} \tag{3}$$

where $\xi(z,t) = \int_0^t v(\theta)\,d\theta - z.$

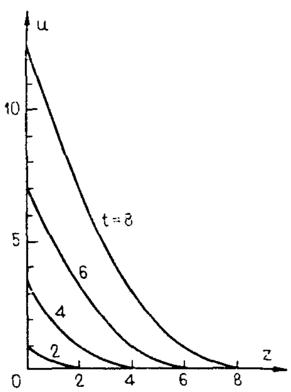

Fig. 1

Solution (3) corresponds to a shear wave whose front propagates with the velocity of motion of the medium v(t) in the direction of the z axis. The

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position of the front at any moment of time, $z_0(t)$, is obtained from the condition $\xi = 0$. The point $z = z_0(t)$ is a point of weak continuity. At this point u and $\partial u/\partial z$ are continuous. This assures that the physical requirements of continuity of velocity and tangential stress at any point are met. Derivatives of higher orders may have a discontinuity at the point $z = z_0(t)$.

Let us consider the particular case when the injection velocity and the plate velocity are power functions of time:

$$v(t) = ct^{p-1}; \quad U(t) = u_0 t^{\frac{p(n+1)}{n-1}};$$
$$u_0 = A\left(\frac{c}{p}\right)^{\frac{n+1}{n-1}} = \text{const}; \quad p \geq 1. \tag{4}$$

Solution (3) now has the form

$$u(z,t) = \begin{cases} A\left[\frac{c}{p}t^p - z\right]^{\frac{n+1}{n-1}}, & z < z_0(t) = \frac{c}{p}t^p; \\ 0, & z \geq z_0(t). \end{cases} \tag{5}$$

The figure shows the velocity distribution u(z) in the fluid for various moments of time for $n = 3$ and $a = \lambda = c = p = 1$, when the shear front propagates at constant velocity.

LITERATURE CITED

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