ANALOGY BETWEEN A TRANSITION TO TWO-DIMENSIONAL TURBULENCE AND ORIENTATIONAL MAGNETIZATION

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Independent measurements by different authors have quantitatively [1, 2] and qualitatively [3] confirmed the hypothesis of the transformation of three-dimensional turbulence into two-dimensional turbulence in the flow of a conducting fluid in a channel in a strong transverse magnetic field. Some theoretical considerations [4, 5] have been used to support this hypothesis. In particular, it has been noted [5] that for

$$\rho\omega/\sigma B^2 \ll 1 \tag{1}$$

(ω is the frequency of turbulent fluctuations, ρ is the density, σ the electrical conductivity, and B the induction), the transformation of three-dimensional turbulence into two-dimensional follows from the Le Chatelier principle (minimum Joule dissipation). However, the dynamics of this transformation, just as the possibility of calculating the transition to two-dimensional turbulence as the field is increased, remains an open question.

Below, a physical analogy is proposed which allows us to answer these questions approximately (in the spirit of the usual semiempirical theories of turbulence).

Experiment indicates that when a strong magnetic field is applied transversely to the mean velocity of the flow, there is a significant (orders of magnitude) decrease in fluctuations along the field, while they remain at the previous level across the field. This process is interpreted at a 90° turn of the axes of those turbulent vortices which would be perpendicular to it in a weak magnetic field.

A turbulent vortex (see the diagram in Fig. 1) with an axis perpendicular to the magnetic field is identical to a magnetic dipole or a ferromagnetic particle in an external field. The closed current of a turbulent vortex has a magnetic moment identical to the dipole magnetic moment. The external field **H** hinders rotation and tries to turn the axis of the magnetic dipole in the direction of the force line with a mechanical rotational moment $\mathbf{K} = \mathbf{m} \times \mathbf{H}$. Simultaneously with the orienting action of the field in the theory of magnetism (polarization), the disorienting action of thermal motion or the Brownian motion of particles in ferromagnetic suspensions is taken into account.

If we neglect the momentum of the turbulent velocity vortices and the interaction between the fields of such vortices (in experiments the magnetic Reynolds number is small), then the rotation of the axes of turbulent vortices in the field can be described approximately as the orientational magnetization [6] or polarization [7] by using the Langevin formula:

$$\frac{M(a)}{M_0} = L(a) = \operatorname{cth} a - \frac{1}{a}; \qquad a = \frac{\mathbf{m} \cdot \mathbf{H}}{kT}. \tag{2}$$

Here, M is the magnetization (the sum of magnetic moments per unit volume) which is dependent on the field **H** through a, and M_0 is the maximum magnetization when the magnetic material is saturated (all the dipoles are along the field, and thus there are no dipoles across the field). In the numerator a is the orientation energy and in the denominator it is the thermal motion energy.

In our case Eq. (2) is transformed thusly: the damping component of the fluctuations along the field with respect to fluctuations without the field is $1 - L(a)$.

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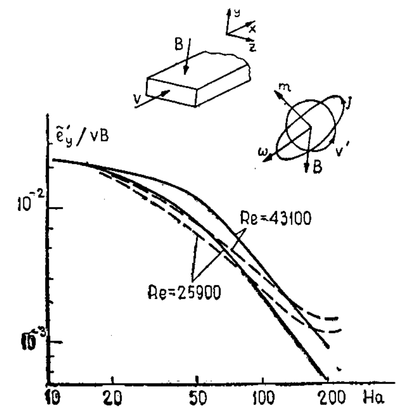

Fig. 1

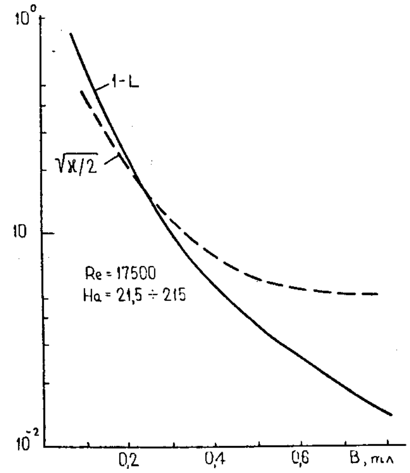

Fig. 2

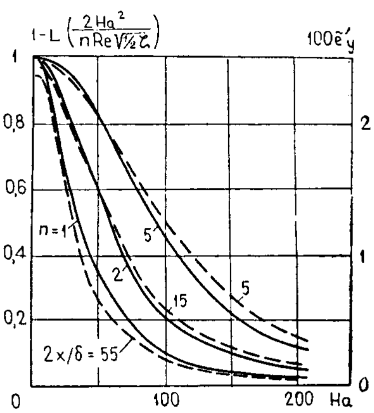

Fig. 3

If the average dimension of the vortex is l and the velocity fluctuation is v', then in a vortex perpendicular to the field, as in a rotating wire frame, a closed current arises with density $I = \sigma v'B$ and strength $\sigma v'Bl^2$ with magnetic moment $m = \mu_0 \sigma v'Bl^4$. The orientation energy is $mH = \sigma v'B^2 l^4$. Instead of thermal motion, in our case there is turbulent mixing with energy $\rho l^3 v'^2$ from which

$$a = \sigma B^2 l / \rho v', \tag{3}$$

i.e., a is the interaction parameter constructed from the turbulent fluctuation frequency v'/l. Obviously, for $a \ll 1$ the field does not affect the turbulence, and for $a \gg 1$ the turbulence, if it exists at all, is only two-dimensional since the axes of all the vortices are along the field (saturation magnetization); this is in agreement with estimate (1).

To obtain formulas and make comparisons with experiment we use the following estimates (standard for semiempirical theories of turbulence):

$$v' = V_* = \sqrt{\tau_w/\rho} = V\sqrt{{}^1/_2\zeta}; \qquad l = \delta. \tag{4}$$

Here, ζ is an experimental coefficient of surface friction in the channel, and δ is the dimension of the channel along the field (the other channel dimension is several times larger).

Ultimately for the damping of fluctuations in the direction of the field as the magnetic field increases, i.e., Ha, we obtain

$$\frac{e'(\mathrm{Ha}, \mathrm{Re})}{e'(0, \mathrm{Re})} = 1 - L\left(\frac{2\,\mathrm{Ha}^2}{\mathrm{Re}\sqrt{{}^1/_2\zeta}}\right), \tag{5}$$

where $\mathrm{Ha} = {}^1/_2 B\delta\sqrt{\sigma/\eta}$; $\mathrm{Re} = V\delta\rho/2\eta$.

In Fig. 1 a calculation using (5) is compared with experiment (see Fig. 1 in [1]) (the solid lines are calculation and the dashed lines experiment). The fluctuations were measured by a conduction anemometer with a three-electrode sensor at a distance of 20δ from the entrance to the magnetic field at a mercury velocity 0.3-0.5 m/sec.

For Ha < 10 we found $a < 0.1$ and the fluctuations independent of the field. For Ha > 200 calculation gives stronger damping of the fluctuations than experiment. In all the calculations the same value is taken for the drag coefficient $\zeta = 0.01$, i.e., $\lambda = 4\zeta = 0.04$; this agrees approximately with the known experimental data for insulated rectangular channels with $\mathrm{Re} = (2\text{-}4) \cdot 10^4$.

In another experiment (Fig. 3 in [2]) $\varkappa$ is given as twice the ratio of the mean square of electric field fluctuations in a mercury flow along and across the field as a function of induction. If the turbulence is isotropic in the absence of a magnetic field then 1-L, according to (5), corresponds to $\sqrt{\varkappa/2}$ from experiment. The calculation is compared with experiment in Fig. 2. The greatest discrepancy is noted here also in the strong field, where the fluctuations are more strongly damped in the calculation than in experiment.

Heretofore, the calculations have been compared with measurements of fluctuations far from the entrance to the field. We take our third experiment from [8], where measurements were carried out at different distances from the entrance. It indicates that with an increasing field fluctuations are damped near the entrance (a distance of 2.5δ) considerably more slowly than deep in the channel $(17.5–27.5)\delta$. One can suppose that this is due to an increase in scale as the vortices pass through the channel, and the larger the vortex the stronger the orientational action of the external field, since the vortex scale enters into the numerator of argument a.

To explain this effect within the framework of our model we introduce the coefficient $n = \delta/l$ which indicates how many vortices fit along the height of channel.

Figure 3 shows a calculation using (5), where the argument is $2Ha^2/nRe\sqrt{^1/_2\xi}$ and the values taken are n = 1, 2, and 5. Such a variation in the scale of turbulent vortices is encountered in nonmagnetic hydrodynamics. It corresponds roughly to the measurements of Cont-Bello in tubes, but with a larger Reynolds number than in the experiments under discussion [9].

For ease of comparison, the right-hand scale in Fig. 3 gives the fluctuations in the direction of the field.

The calculation must be compared to the dashed curves in Fig. 3, which gives damping of the fluctuations along the field at different distances from the entrance to the magnetic field (expressed in half-heights of the channel) [8].

The dependence of dimensional fluctuations along the field on the Hartmann number with a characteristic maximum, which is given in Fig. 3 in [8], can also be obtained from Eq. (5), in which the limit $1 - L$ is a factor proportional to induction. At first, the dimensional fluctuations grow due to the growth in the field and then decline because of the stronger influence of the rotation of the vortices. If we prescribe an increasing distance along the channel and increasing vortex scales, then quantitative agreement can also be obtained between calculation and experiment.

The similarity between the calculated and experimental curves for the three different experimental papers is evidently not random, so that the above-noted analogy and the model of the phenomenon deserve attention.

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