SHAPE OF FREE SURFACE OF HEAVY FERROFLUID SUSPENDED IN ELECTROMAGNET GAP OF MAGNETOSTATIC SEPARATOR

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The development of cheap methods of obtaining ferrofluids in recent years [1] has increased the feasibility of constructing magnetostatic devices based on "weighting" of a ferrofluid in a nonuniform magnetic field [2-4]. A promising version of such a device is shown in Fig. 1.

The magnetic system 5 contains special pole pieces that produce a magnetic gradient in the direction of the z axis below the line D–D' and a magnetic gradient in the opposite direction above the line D–D'. When a current flows through the exciting coils 3 the ferrofluid 6 is suspended between the pole pieces, as shown in Fig. 1. The feed device 1 pours the material for separation onto the surface of the fluid: The light fraction 4 remains on top of the ferrofluid, while the heavy fraction 7 falls through onto the conveyer 8.

Below we investigate the position of the ferrofluid and the shape of its free surface for effective operation of such a device.

1. Equation of Free Surface. Method I. The most frequently used method of calculating the body forces acting on a magnetic material is that based on a consideration of the resultant field obtained after introduction of the magnetic material. In this case the body force density has the form [5]

$$\mathbf{f} = -\nabla p + \frac{1}{2}\nabla\left(H^2\rho\frac{d\mu}{d\rho}\right) - \frac{H^2}{2}\nabla\mu, \tag{1}$$

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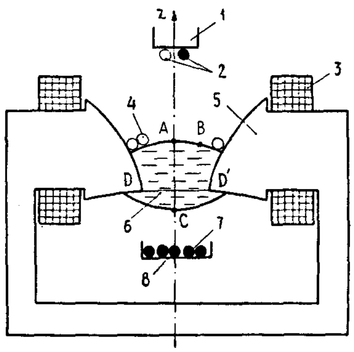

Fig. 1

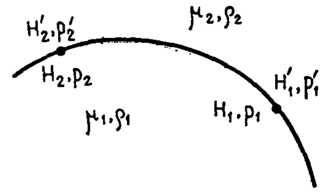

Fig. 2

where H is the strength of the resultant field, while p, ρ, and μ are the pressure, density, and magnetic permeability of the magnetic material.

To determine the total force acting on a magnetic material, we must consider the pressure discontinuity at the boundary of two magnetic materials, given by the expression [5]

$$p-p'=\left(\frac{1}{2}H^2\rho\frac{d\mu}{d\rho}-\frac{1}{2}H'^2\rho'\frac{d\mu'}{d\rho'}\right)-\left[\frac{\mu(\mu-\mu')}{2\mu'}H_n^2+\frac{\mu-\mu'}{2}H_t^2\right], \tag{2}$$

where the prime denotes quantities relating to the second magnetic material, while H_n and H_t are the normal and tangential field components at the interface.

For a weightless incompressible magnetic fluid in static conditions it is easy to obtain from (1) the expression

$$p-\frac{H^2}{2}\rho\frac{d\mu}{d\rho}=\text{const}. \tag{3}$$

Expressions (1), (2), and (3) are obviously suitable for calculation of the forces acting on the fluid. Taking into account the gravitational force, acting opposite to the direction of the z axis, we obtain in static conditions, instead of (1) and (3),

$$\rho\nabla(gz)=-\nabla p+\frac{1}{2}\nabla\left(H^2\rho\frac{d\mu}{d\rho}\right)-\frac{1}{2}H^2\nabla\mu; \tag{4}$$

$$p+\rho gz-\frac{1}{2}H^2\rho\frac{d\mu}{d\rho}=\text{const}, \tag{5}$$

where g is the gravitational acceleration.

We now take two points on the free surface separating the ferrofluid (Fig. 2) and the second medium, which have magnetic permeability and density equal to μ_1, ρ_1 and μ_2, ρ_2. Let the field and pressure at point 1 within the ferrofluid and the second medium be H_1, p_1 and H_1', p_1' and at point 2 be H_2, p_2 and H_2', p_2'.

The pressure p_2 and p_1 are obviously connected by relation (5), and the pairs of pressures p_2, p_2' and p_1, p_1' by relation (2). Using these relations, we obtain

$$p'_2-p'_1=\rho_1 g(z_1-z_2)+\frac{1}{2}\rho_2\frac{d\mu_2}{d\rho_2}(H'^2_2-H'^2_1)+\left[\frac{\mu_1(\mu_1-\mu_2)}{2\mu_2}(H_{2n}^2-H_{1n}^2)+\frac{\mu_1-\mu_2}{2}(H_{2t}^2-H_{1t}^2)\right]. \tag{6}$$

Taking into account that the pressure on the free surface is constant, and assuming that the second medium is air with $\mu=\mu_0$, we obtain the equation of the free surface:

$$z_2-z_1=\frac{1}{\rho g}\left[\frac{\mu(\mu-\mu_0)}{2\mu_0}(H_{2n}^2-H_{1n}^2)+\frac{\mu-\mu_0}{2}(H_{2t}^2-H_{1t}^2)\right]. \tag{7}$$

Method II. In this method the body force density has the form

$$\mathbf{f}=\mu_0(\mathbf{I}\nabla)\mathbf{H}, \tag{8}$$

where I is the magnetization of unit volume of the ferrofluid.

If I || H and rot H = 0 (current absent), then instead of [8] we obtain

$$\mathbf{f}=\mu_0(\mathbf{I}\nabla)\mathbf{H}. \tag{9}$$

Instead of Eq. (4) we now have

$$\rho\nabla(gz)=-\nabla p+\mu_0 I\nabla H. \tag{10}$$

Integrating (10), we obtain

$$\rho gz+p-\mu_0\int I\mathrm{d}H=\mathrm{const}. \tag{11}$$

Taking into account that the pressure on the free surface is constant, we obtain the equation of the free surface:

$$z_2-z_1=\frac{\mu_0}{\rho g}\int\limits_{H_1}^{H_2} I\mathrm{d}H. \tag{12}$$

This expression has been experimentally confirmed for a fluid surrounding a vertical current-carrying conductor.

The use of two methods of determination of the body forces in a ferromagnetic fluid — (1) and (8) — has led to discussion in the relevant literature [6]. The two methods have also been used to determine the free surface. Method I was used in [7], for instance.

In our opinion, the difference in expressions (1) and (8), and, concomitantly, (7) and (12) is due to the fact that expressions (1) and (7) contain the resultant field obtained after introduction of the magnetic material into the investigated region, whereas expressions (8) and (12) contain the external field that existed in the investigated region before introduction of the magnetic material. The derivation of Eq. (8) for the case where the external field is used is given in [5]. In the case of weak magnetic materials the resultant field does not differ greatly from the external field and Eq. (8) becomes Eq. (12) when $\mu\to\mu_0$. In fact, if we put $I=(\mu-\mu_0)/\mu_0$ in (12), then when μ = const we obtain

$$z_2-z_1=\frac{\mu-\mu_0}{2\rho g}(H_2^2-H_1^2). \tag{13}$$

Equation (8) tends to the same expression when $\mu\to\mu_0$.

Since the considered fluid can be regarded as a weak magnetic material, we will henceforth use formulas (7) and (12).

To use expression (12) we need to know the relation between I and H. It has been shown in several investigations that for a ferrofluid the relation between I and H is given by the Langevin formula

$$I=qI_S\left[\operatorname{cth}\alpha H-\frac{1}{\alpha H}\right] \tag{14}$$

where $\alpha=\mu_0 I_S V/kT$, q is the volume concentration of magnetic material in the ferrofluid, I_S is the magnetization of the continuous magnetic material, V is the volume of a single particle, k is Boltzmann's constant, and T is the temperature.

Substituting (14) in (12), we obtain

$$z_2-z_1=\frac{qkT}{\rho gV}\ln\frac{H_1\operatorname{sh}\alpha H_2}{H_2\operatorname{sh}\alpha H_1}. \tag{15}$$

In the particular case where H is large expression (14) tends to $I=I_{HS}=qI_S$, and Eq. (15) tends to the expression

$$z_2-z_1=\frac{\mu_0 I_{HS}}{\rho g}(H_2-H_1). \tag{16}$$

2. Determination of Position of Ferrofluid Suspended in Working Region of Separator. It is obvious that the position of the points A, B, and C, situated on the free surface (see

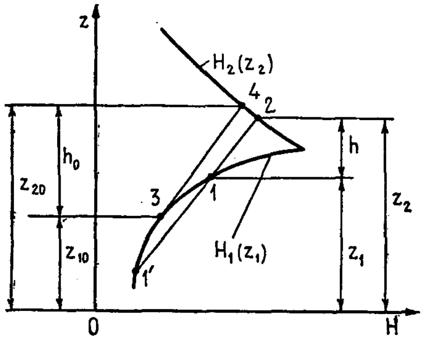

Fig. 3

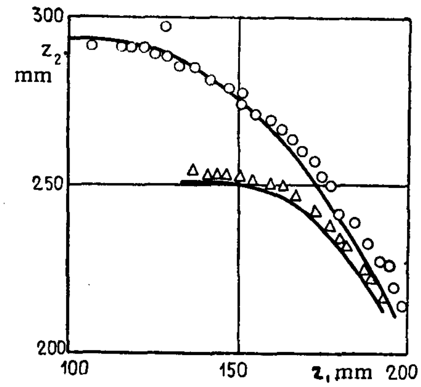

Fig. 4

Fig. 1) of the ferrofluid, and the field at these points will satisfy either Eq. (15) or, when I = const, Eq. (16).

The position of points A and C determines the thickness and height of the suspended layer.

Let the function H(z) be assigned analytically as $H_2(z_2)$ in the region above D–D' and $H_1(z_1)$ in the region below D–D'. If we denote the thickness of the ferrofluid layer as h and the coordinates of the lower and upper points of the layer as z_1 and z_2, then to find the position of a ferrofluid of thickness h, using Eq. (16), we have the system

$$H_2(z_2)-H_1(z_1)=\frac{\rho g}{\mu_0 I_{HS}}h;\quad z_2-z_1=h. \tag{17}$$

Accordingly, for Eq. (15) we have

$$\frac{qkT}{\rho g V}\ln\frac{H_1(z_1)\,\mathrm{sh}\,\alpha H_2(z_2)}{H_2(z_2)\,\mathrm{sh}\,\alpha H_1(z_1)}=h;\quad z_2-z_1=h. \tag{18}$$

If function H(z) is assigned in the form of an experimentally obtained curve, systems (17) and (18) can be solved graphically.

Through point 1, lying on the curve $H_1(z_1)$ in Fig. 3 we draw a straight line corresponding to system (17) until it intersects the curve $H_2(z_2)$ at point 2. The ordinates of the points 1 and 2 then give the position of a ferrofluid of thickness h. The same operation is performed in the second case, but the curve corresponding to system (18), instead of the straight line, must be drawn through point 1.

To the above we must add the following. Since the line $H_1(z_1)$ is usually a curve, it can intersect the straight line (17) or the curve (18) at several points. If we are dealing with a suspension of solid magnetic material, this would mean that for one ordinate z_2 there would be several ordinates z_1 (points 1 and 1' in Fig. 3). For a magnetic fluid the problem becomes single-valued if we reject the possibility of negative pressure in the fluid.

We draw the straight line (17) (Fig. 3) through point 3 in the curve $H_1(z_1)$ so that the straight line (17) at this point is a tangent to this curve. The same must be done in the second case, but curve (18) is used.

It can easily be shown that at the obtained point of contact the body force acting on a magnetic material is equal to the body force of gravity and, hence, at all points lying below point 3 the pressure in the fluid will be negative.

Thus, we concluded that point 3 is the lowest point to which the suspended fluid can descend, while point 4 (Fig. 3) is the corresponding highest point. The thickness h_0 is thus the maximal thickness of the suspended fluid.

In the case of analytical assignment of function H(z) the maximal thickness for systems (17) and (18) can be determined by means of the following systems of equations:

$$H_2(z_{20})-H_1(z_{10})=\frac{\rho g}{\mu_0 I_{HS}}h_0;\quad z_{20}-z_{10}=h_0; \tag{19}$$

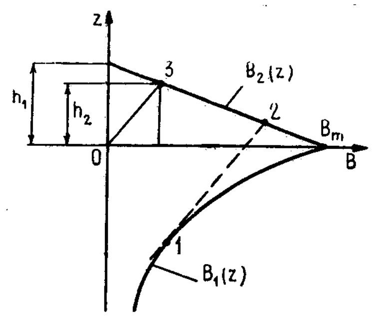

Fig. 5

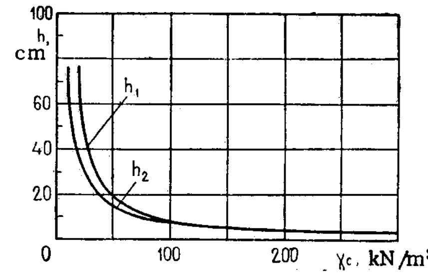

Fig. 6

$$\left.\frac{dH_1}{dz}\right|_{z=z_{10}}=\frac{\rho g}{\mu_0 I_{HS}};\quad \frac{qkT}{\rho g V}\ln\frac{H_1(z_{10})\,\mathrm{sh}\,\alpha H_2(z_{20})}{H_2(z_{20})\,\mathrm{sh}\,\alpha H_1(z_{10})}=h_0;$$

$$z_{20}-z_{10}=h_0;\quad \frac{dH_1}{dz}qI_S\left[\mathrm{cth}\,\alpha H_1-\frac{1}{\alpha H_1}\right]\Big|_{z=z_{10}}=\frac{\rho g}{\mu_0}. \qquad (20)$$

3. Experimental Determination of Position of Suspended Ferrofluid. For the experiments we used a ferrofluid with the following properties: saturation magnetization I_H = 7000 A/m at field H = 3 · 10^5 A/m, density 0.96 g/cm³, particle diameter (50–60) · 10^{-10} m. The experiments were conducted in a C-shaped magnetic system with a gap 40 mm broad at the narrow part and 400 mm long with a magnetic field gradient of ∿10 T/m, which was practically constant at a height of 60 mm. The curve H(z) in the working gap was determined experimentally for two values of current in the electromagnet coils, so that in one case the field in the working region corresponded to saturation of the ferrofluid.

When the current was flowing through the electromagnet coils the ferrofluid was poured slowly into the working gap. Its free surface corresponded approximately to that shown in Fig. 1; the thickness of the layer increased with increase in the supply of the magnetic fluid.

When the next batch of ferrofluid had been poured in we measured the height z_1 of point C (see Fig. 1) and the height z_2 of point A. These values are marked as circles in Fig. 4 for saturation of the ferrofluid and as triangles for the second case. Figure 4 also shows the curves plotted by the graphic method shown in Fig. 3. As Fig. 4 shows, the difference between the experimental and calculated data did not exceed 10%.

4. Determination of Maximal Thickness of Layer of Ferrofluid as a Function of Specific Weight of Rock Being Separated. For effective separation it is desirable to have a constant body force acting on the ferrofluid in the upper part of the working gap:

$$\mu_0 I\frac{dH}{dz}=-c. \qquad (21)$$

In this case the specific weight of the rock to be separated is

$$\gamma_c=\gamma+c, \qquad (22)$$

where γ is the specific weight of the ferrofluid.

In the case of saturation we obtain from (21)

$$B=-\frac{c}{I_{HS}}z+B_m, \qquad (23)$$

where B_m is the maximal induction in the gap. In Fig. 5 this straight line is denoted by $B_2(z)$. In Fig. 5 the x axis corresponds to the line D–D' (see Fig. 1). If the ferrofluid is not suspended in the gap, but rests on the bottom, the thickness of the separation layer will obviously be given by the value of h_1 (Fig. 5). When B = 0 it follows from (22) and (23) that

$$h_1 = \frac{B_m I_{HS}}{\gamma_c - \gamma}. \tag{24}$$

In the case of a suspended fluid the relation between B and z in the lower part of the working gap is given by the curve $B_1(z)$ (Fig. 5). Here the thickness of the suspended layer is determined, as was shown earlier, by the tangent (19) to the curve $B_1(z)$ (straight line 1-2). To obtain a maximal value, the above-mentioned straight line must be drawn through the coordinate origin (straight line 0-3). Thus, the required thickness h_2 of the separation layer (Fig. 5) is given by the solution of the following system:

$$B = -\frac{c}{I_{HS}} z + B_m; \quad B = \frac{\rho g}{I_{HS}} z. \tag{25}$$

The solution of (25) gives

$$h_2 = \frac{B_m I_{HS}}{\gamma_c}. \tag{26}$$

Figure 6 gives the curves (24) and (26) for a ferrofluid with parameters I_{HS} = 7000 A/m. γ_f = 10 kN/m^3 for B_m = 1.1 T.

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