ANALYSIS OF ELECTROMECHANICAL PROCESSES IN AN INDUCTIVE MHD MACHINE

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The subject of the present investigation is the electromechanical process which takes place in a system of hydraulically in-parallel coupled inductive machines when the external hydraulic circuit is changed. Also included is a stability analysis of the steady state in the autonomous self-excited regime. Transitional processes have been considered earlier for a different arrangement [1, 2].

The differential equations for liquid-metal MHD machines, as presented below, are based on the utilization of the concept of the resultant spatial vector, introduced in [3], which characterizes the three-dimensional wave amplitude and makes it possible to represent the instantaneous intensity values of the external and the inherent magnetic fields in the following manner:

$$\bar{\mathbf{H}}_{\mathbf{ex}}=i\hat{\mathbf{H}}_{\mathbf{ex}}e^{-i\alpha x};\quad \bar{\mathbf{h}}=i\hat{\mathbf{h}}e^{-i\alpha x}, \tag{1}$$

where $\hat{\mathbf{H}}_{ex}$ and $\hat{\mathbf{h}}$ are the resultant three-dimensional vectors, which are arbitrary functions of time, being related with the instantaneous values of phase currents through the relationships:

$$\hat{\mathbf{H}}_{\mathbf{ex}}=\gamma_{u}\hat{\mathbf{i}}_{s};\quad \hat{\mathbf{h}}=\gamma_{u}\hat{\mathbf{i}}_{r}, \tag{2}$$

where

$$\gamma_u=\frac{3}{2}\,\frac{w_{\phi}k_{c}}{\pi p\delta k_{1m}};\quad \hat{\mathbf{i}}_s=\frac{2}{3}(i_a+\mathbf{a}i_b+\mathbf{a}^2i_c)$$

is the resultant vector of the primary coil current, and $\hat{\mathbf{i}}_r$ is the resultant current vector in the liquid metal, leading to a current in the excitation coil.

The legitimacy of the formulation expressed by (1) is based on the assumptions that the inductor is infinitely long, that the three-phase coil is symmetrical, and that the spatial phase displacement angles are equal. The latter is indicated in the expression for the current vector $\hat{\mathbf{i}}_s$ in terms of unit vectors:

$$\mathbf{a}=-\frac{1}{2}+i\frac{\sqrt{3}}{2};\quad \mathbf{a}^2=-\frac{1}{2}-i\frac{\sqrt{3}}{2}.$$

Using Kirchhoff's second law for the primary circuit and Maxwell's equations for the secondary, while assuming that only the z component of the three-dimensional field vectors $\hat{\mathbf{H}}_{ex}$ and $\hat{\mathbf{h}}$ are present and that they vary with the x coordinate and time (the transverse fringe effect in the case of a plane machine and the virtual filling of the gap with liquid metal are taken into account similarly, as in [1]) we can now write down the system of differential equations.

For the windings of the primary circuit we have:

$$\bar{\mathbf{U}}_s=r_{\text{M}}\hat{\mathbf{i}}_s+L_1\frac{d\hat{\mathbf{i}}_s}{dt}+\frac{d\bar{\mathbf{\Psi}}_{ss}}{dt}, \tag{3}$$

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where $\Psi_{ss}=\gamma_{\psi}(\hat{\mathbf{H}}_{ex}+k_{tfc}\hat{\mathfrak{h}})$ is the resultant vector of linkage; $\gamma_{\psi}=2w_{\psi}k_c\ \mu d/\alpha, d$, channel width; $\hat{\mathbf{U}}_s={}^2/_3(U_a+\mathbf{a}U_b+\mathbf{a}^2U_c)$ is the resultant phase voltage vector.

For the liquid-metal circuit we get:

$$\frac{d\hat{\mathfrak{h}}}{dt}+\frac{d\hat{\mathbf{H}}_{ex}}{dt}+\left(\frac{\alpha^2}{\sigma\mu}-i\alpha v\right)\hat{\mathfrak{h}}-i\alpha v\hat{\mathbf{H}}_{ex}=0. \tag{4}$$

Equations (3) and (4) are supplemented by the equation of motion for the liquid metal:

$$\frac{dv}{dt}=\frac{F}{m}\ \frac{\tau p\alpha\mu}{2i}(\hat{\mathbf{H}}_{ex}\hat{\mathfrak{h}}^*-\hat{\mathfrak{h}}\hat{\mathbf{H}}^*_{ex})-\frac{F}{m}p(v). \tag{5}$$

where F is the channel cross section; m, mass, as determined in [1]; and p(v), external load characteristic, including frictional losses.

1. We consider a transitional process which arises in a system of cylindrical inductive pumps connected in parallel into a single common collector, when one of them is disconnected and closed off by means of a valve (Fig. 1). In the present treatment it is assumed that the three-phase system of the pump winding and of the synchronous power supply generator is symmetrical both in the transitional and in the stationary regimes. As a consequence of this assumption, the resultant three-dimensional intensity vectors for the current and the phase voltage can be represented in the form:

$$\hat{\mathbf{H}}_{ex}=\dot{\mathbf{H}}e^{i\omega t};\quad \hat{\mathfrak{h}}=\dot{\mathfrak{h}}e^{i\omega t};\quad \hat{\imath}_s=\dot{I}e^{i\omega t};\quad \hat{\mathbf{U}}_s=U_m e^{i\omega t}. \tag{6}$$

where $\dot{\mathbf{H}}$, $\dot{\mathbf{I}}=I_r+iI_a$; $\dot{\mathfrak{h}}=h_r+ih_a$ are the complex amplitudes to be determined. They change slowly with time over a period $2T/\omega$; U_m is the voltage amplitude determined by processes in the synchronous generator.

In analogy with [2], Eqs. (3) and (4), taking into account (2) and (6), will now assume the form:

$$\frac{dI_r}{dt}=\frac{U_m}{L_1}-\frac{r_м}{L_1}I_r+\frac{\omega L_1+\gamma_\psi\gamma_н\alpha v}{L_1}I_a-\frac{\gamma_\psi\alpha^2}{L_1\sigma\mu}h_r-\frac{\gamma_\psi\alpha v}{L_1}h_a; \tag{7}$$

$$\frac{dI_a}{dt}=-\frac{\omega L_1+\gamma_\psi\gamma_н\alpha v}{L_1}I_r-\frac{r_м}{L_1}I_a-\frac{\gamma_\psi\alpha v}{L_1}h_r+\frac{\gamma_\psi\alpha^2}{L_1\sigma\mu}h_a; \tag{8}$$

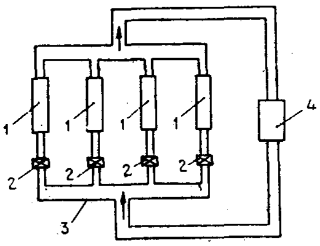
$$\frac{dh_r}{dt}=-\frac{\gamma_н U_m}{L_1}-\frac{r_м\gamma_н}{L_1}I_r-\gamma_н\alpha v\left(1+\frac{\gamma_\psi\gamma_н}{L_1}\right)I_a$$
$$-\left(1+\frac{\gamma_\psi\gamma_н}{L_1}\right)\frac{\alpha^2}{\sigma\mu}h_r+\left[\omega-\alpha v\left(1+\frac{\gamma_н\gamma_\psi}{L_1}\right)\right]h_a; \tag{9}$$

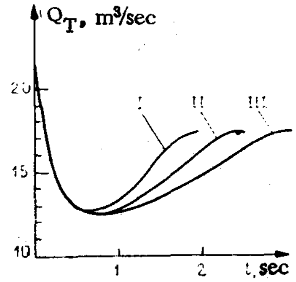
$$\frac{dh_a}{dt}=\gamma_н\alpha v\left(1+\frac{\gamma_\psi\gamma_н}{L_1}\right)I_r+\frac{r_м\gamma_н}{L_1}I_a-\left(1+\frac{\gamma_\psi\gamma_н}{L_1}\right)\frac{\alpha^2}{\sigma\mu}h_a-\left[\omega-\alpha v\left(1+\frac{\gamma_\psi\gamma_н}{L_1}\right)\right]h_r. \tag{10}$$

Equations (6) to (8) must be supplemented by the equations of motion for the liquid metal in the operating and the shutoff pumps. Assuming that the pressure losses are proportional to the square of the flow rate, i.e., $p(v) = R_pQ^2 + R_TQ_T^2$, we obtain from Eq. (5):

$$\frac{dv}{dt}=\frac{F\tau p\sigma B_p^2 v}{m}\left[k_l\left(\frac{\omega\tau}{\pi}-1\right)-c_0\right]-\frac{F}{m}(R_pQ^2+R_TQ_T^2), \tag{11}$$

where Q is the volumetric pump flow rate; Q_T, volumetric flow rate through the load; R_p, hydraulic resistance of the pump and the delivery pipe; R_T, hydraulic resistance of the common collector and the load; and k_l, attenuation coefficient which accounts approximately for the longitudinal fringe effect. The parameter B_p is computed from the relationship:





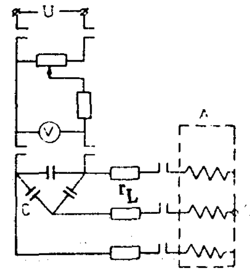


Fig. 1 Fig. 2 Fig. 3

Fig. 1. Schematic of the pumping system: 1) pumps; 2) shutoff valves; 3) common collector; 4) load.

Fig. 2. Total flow rates as functions of time: I) closure at 2 sec; II) closure at 2.5 sec; III) closure at 3 sec.

Fig. 3. Electrical pump schematic: C) capacitors connected in a triangular scheme; r_L) load resistance in a phase; A) machine windings.

$$B_p = |\dot{B}_p| = \mu|\gamma_{\text{н}}\dot{I} + \nu_e\dot{h}|,$$

where, as in [2], ν_e is a coefficient designed to approximately account for the reduction of the contribution from the component h(t) due to the longitudinal fringe effect.

The electromagnetic pressure in the switched-off pump has zero magnitude; however, at the instant of the pump shutoff blockage begins in the delivery pipe leading to an increase in its hydraulic resistance. Assuming, as before, that the pressure losses are proportional to the square of the flow rate, we obtain an equation of motion for the liquid metal in the switched-off pump:

$$\frac{dv_1}{dt} = -\frac{F}{m}[\text{sign}\, v_1\, R_{vol}(v_1 F)^2 + R_T Q_T^2], \tag{12}$$

where v_1 is the flow velocity of the liquid metal in the switched-off pump; $R_{vol} = R_p + R_v$; R_v, hydraulic resistance of the valve.

In this investigation we are considering a reversing valve having the shape of a plane plate, which closes off the cross section of the delivery pipe when swivelled uniformly around an axis coinciding with its diameter. The hydraulic resistance of a small valve as a function of the swing angle (assuming that the valve is closing uniformly) has been given in [3].

The equations obtained so far must be supplemented with the equation for the total inflow across the load:

$$Q_T = N(vF) + v_1 F, \tag{13}$$

where N is the number of working pumps, and by the equation for the synchronous motor supplying power to the pumps. In accordance with [2], this equation can be stated as:

$$U_m^2 - U_m(x_d + x_q)I_a + x_d x_q(I_a^2 + I_r^2) = E_m I_r x_q \sqrt{1 + \left(\frac{U_m - x_q I_a}{x_q I_r}\right)^2}, \tag{14}$$

where $x_d = L_d\omega$ and $x_q = L_q\omega$ are the frequency-dependent reactances, and E_m is the electromotive force produced by the excitation current in the stator windings of the synchronous motor.

The system of six differential Eqs. (7) to (12) together with Eqs. (13) and (14) was numerically solved on the electronic computer M-222. The initial values I_{ro}, I_{ao}, h_{ro}, h_{ao} and the hydraulic pump resistance were evaluated for the stationary regime.

The following initial values were selected for the computation:

$$Q_0=5.54\,\mathrm{m/sec};\ F=0.382\,\mathrm{m}^2;\ p=5;\ \tau=0.221\ \mathrm{m};$$
$$\sigma=5.88\cdot10^6\ (\Omega\cdot\mathrm{m})^{-1},\quad m=1900\ \mathrm{kg};\ c_0=0.005;\ k_l=0.9;\ N=3;$$
$$\gamma_\psi=0.519\cdot10^4;\ \gamma_H=108.5;\ \alpha=14.2;\ \omega=236\ \mathrm{sec}^{-1}$$
$$F_1=1.1\,\mathrm{m}^2;\ L_1=7.65\cdot10^{-4}\ \mathrm{H};\ r_M=0.03\ \Omega.$$

Computations were carried out for three valve closing times. Results are presented in Fig. 2. It is apparent from Fig. 2 that for all t_{cl} values the total flux Q_T initially drops rapidly over a time span of 0.6 sec on account of the shunting by the circuit of the closed-off valve, after which it increases smoothly to a value of Q_T = 17.5 m³/sec. The minimum flux Q_T = 12.7 m³/sec constitutes 73% of the new steady-state Q_T value. It should be noted that neither the duration of the rapid flow rate decrease, nor the minimum flow rate through the load, nor the new steady-state flow rate are actually dependent on the valve closing time. This can be explained by realizing that during the initial moment the hydraulic resistance of the valve increases slowly, since during the rotation of the closing valve's disk in the initial time span the blocked area of the pipe is small and is practically independent of the time of the complete valve closure.

2. An autonomous self-excited system is established if a liquid-metal generator is inserted into the circuit shown in Fig. 1, which operates on the pressure developed by the pumps. Figure 3 shows the electrical schematic of a three-phase inductive MHD generator, wherein each phase contains a capacitance C and resistance r_L, both connected in series. The self-excitation arises at the instant of connecting the previously charged condensers (Fig. 3). The electromechanical processes which then evolve can be described in terms of a system of differential Eqs. (3) to (5) supplemented by a relationship between the voltage vector in the generator and the load parameters in the form:

$$\hat{\mathbf{U}}_s=-\hat{\mathbf{i}}_s r_L-\frac{1}{C}\int\hat{\mathbf{i}}_s\,dt \tag{15}$$

and the initial conditions:

$$\text{for } t=0\quad v=v_0;\quad \hat{\mathbf{i}}=\hat{\mathbf{h}}_0=0,$$

$\hat{\mathbf{U}}_{s0}=\hat{\mathbf{U}}_{c0}=\mathbf{i}U/\sqrt{3}$, since $\hat{\mathbf{U}}_{c0}=U_{a0}+\mathbf{a}U_{b0}+\mathbf{a}^2U_{c0}$, while $U_{a0}=0$; $U_{b0}=U/2$; $U_{c0}=-U/2$ in accordance with Fig. 3.

We note that the p(v) term in (5) in the generator regime includes the characteristic of the booster and the friction losses in the liquid-metal circuit.

The problem can be solved using the asymptotic methods of the vibration theory [5-7].

It is assumed that the mechanical transitional process develops substantially slower than the electromagnetic, and that the oscillation is nearly harmonic. Then, following [6], it is possible first to investigate Eqs. (3) and (4), which characterize "rapid motion," as being purely linear, assuming v = const. After this is done, utilizing the form of the solution, it is possible to set up the equations for the first approximation in the vibration theory, while taking into account the true current dependence of the parameters and to solve these equations together with (5).

In the first step, therefore, the problem consists in finding a solution of the linear differential Eqs. (3) and (4), taking into account Eqs. (2) and (15), expressed in the following form:

$$(L_1+M)\frac{d\hat{\mathbf{i}}_s}{dt}+Mk_{\mathrm{tfc}}\frac{d\hat{\mathbf{i}}_r}{dt}+(r_M+r_L)\hat{\mathbf{i}}_s+\frac{1}{C}\int\hat{\mathbf{i}}_s\,dt=0;$$
$$\frac{d\hat{\mathbf{i}}_s}{dt}+\frac{d\hat{\mathbf{i}}_r}{dt}+\left(\frac{\alpha^2}{\sigma\mu}-i\alpha v_0\right)\hat{\mathbf{i}}_r-i\alpha v_0\hat{\mathbf{i}}_s=0, \tag{16}$$

where M = $\gamma_{\text{н}}\gamma_{\psi}$ is the mutual induction for the initial conditions mentioned earlier.

A solution of (16), which may be nonattenuating for the stated parameters, has the form:

$$\hat{\mathbf{i}}_s=\frac{i(p_1+p_0-i\alpha v_0)\,Ue^{p_1t}}{\sqrt{3}L_0(p_1-p_2)(p_1-p_3)}=\hat{\mathbf{I}}_s e^{i\omega t} \tag{17}$$

(two other solutions attenuate). In this equation $p_1=\delta+i\omega$, p_2, p_3 are the roots of the characteristic equation, and $p_0 = \alpha^2/\sigma\mu$,

$$\delta=-\frac{-Mk_{\text{tfc}}[1-(\alpha v_0)^2L_\Sigma C]-2p_0R_sCL_\Sigma}{4p_0CL_\Sigma^2};\quad \omega=\frac{1}{\sqrt{L_\Sigma C}};\quad L_\Sigma=L_1+M. \tag{18}$$

Using assumptions employed for $\hat{\mathbf{i}}_s$, from (16) one obtains:

$$\hat{\mathbf{i}}_r=-\frac{i(p_1-i\alpha v_0)\,Ue^{p_1t}}{\sqrt{3}L_0(p_1-p_2)(p_1-p_3)}=\hat{\mathbf{I}}_r e^{i\omega t}. \tag{19}$$

Two equations correspond to solution (19):

$$\frac{dI}{dt}=\delta I \text{ and } \omega=\omega_{\text{н}}+\frac{d\Phi}{dt}, \tag{20}$$

where

$$I=I_m e^{\delta t};\quad I_m=\frac{U\sqrt{A^2+B^2}}{\sqrt{3}L_0},$$

$$A+iB=\frac{i(p_1+p_0-i\alpha v_0)}{(p_1-p_2)(p_1-p_3)};\quad \Phi=\operatorname{arctg}\frac{B}{A}; \tag{21}$$

when the actual current dependence of the electrical parameters for δ and ω entering Eq. (18) is taken into account, the nonlinear Eqs. (20) constitute the first approximation equations in the vibration theory. We shall work out the solution (17), (19), and (20) for the case when the velocity is a weak function of time.

Computations have shown that for a realistic set of parameters for a MHD machine the B value is extremely small. Based on this finding, we assume that the $\widehat{\mathbf{I}}_s$ vector possesses only an effective component I having the initial value I_m.

After substitution of (2), (17), and (19) into (5) and after the introduction of a correction for the nonuniformity of the velocity profile over the channel height in accordance with [1], the equation of motion assumes the form:

$$\frac{dv}{dt}=\frac{F}{m}p(v)-\frac{F}{2m}p\tau\sigma k_{\text{tfc}}\gamma_{\text{н}}^2\frac{[\alpha(c_0+1)v-\omega]I^2}{(1+\delta/p_0)^2+(\omega-\alpha v)^2/p_0^2}, \tag{22}$$

which supplements the equation for the current amplitude (22) for

$$\delta=\frac{-Mk_{\text{tfc}}(1-\alpha^2L_\Sigma Cv^2)-2p_0R_sCL_\Sigma}{4p_0CL_\Sigma^2},\quad \omega=\frac{1}{\sqrt{L_\Sigma C}};$$

$$L_\Sigma=L_1+M;\quad R_s=r_{\text{м}}+r_{\text{L}} \tag{23}$$

and the initial condition:

$$I_m=\frac{8U\alpha v_0CL_0}{\sqrt{3}(3+\alpha^2v_0^2CL_0)[(2\alpha^2v_0^2CL_0+1)L_\Sigma+L_0]};$$

$$L_0=L_1+M-k_{\text{tfc}}M. \tag{24}$$

For the booster characteristic we assign a nonlinear function of velocity in the form:

$$p(v)=a-bv^2, \tag{25}$$

assuming, at the same time, that the electrical parameters do not depend on the current.

The investigation of the equations is continued by bringing in powerful methods of the vibration theory, which lead to the determination of the steady state and its stability under the action of small perturbations.

Eliminating the angular frequency (23) by introducing dimensionless parameters and designations:

$$\bar{v}=\frac{\alpha v}{\omega};\quad \bar{t}=\frac{F\alpha}{m\omega}t;\quad \bar{b}=\frac{b\omega^2}{a\alpha^2};\quad \bar{I}=\sqrt{\frac{k\omega}{a}}I;$$

$$k=\frac{p\tau\sigma k_{\text{tfc}}\gamma_{\text{H}}^2}{2}\omega(c_0+1),\quad Rm_s=\frac{\mu\sigma\omega}{\alpha^2},$$

where ω is determined by (23), $s_{\text{cr}}=-\dfrac{R_sCL_\Sigma\alpha^2}{\mu\sigma Mk_{\text{tfc}}}$ [9],

$$k_\sigma=Mk_{\text{tfc}}/4L_\Sigma;\quad n=Rm_sk_\sigma(1-2s_{\text{cr}});\quad q=Rm_sk_\sigma;\quad \xi=m\omega^2/F a\alpha, \tag{26}$$

the system of Eqs. (22) to (25) is transformed into the form:

$$\frac{d\bar{v}}{d\bar{t}}=1-\bar{b}\bar{v}^2-\frac{[\bar{v}-1/(c_0+1)]\bar{I}^2}{(1-n+q\bar{v}^2)^2+Rm_s^2(1-\bar{v})^2} \tag{27}$$

$$\frac{d\bar{I}}{d\bar{t}}=\xi(q\bar{v}^2-n)\bar{I}. \tag{28}$$

The possible steady states will be located at:

$$N(\bar{I}_N=0;\quad \bar{v}_N=\sqrt{1/\bar{b}});\quad M\left(\bar{I}_M=\sqrt{\frac{[1-\bar{b}(1-s_{\text{cr}})^2](1+Rm_s^2s_{\text{cr}}^2)}{c_0/(c_0+1)-s_{\text{cr}}}};\right.$$

$$\left.\bar{v}_M=\sqrt{\frac{n}{q}}\approx 1-s_{\text{cr}}\right).$$

The linearized system of equations for small oscillations assumes the form:

$$\frac{d\bar{v}}{d\bar{t}}=-\left\{2\bar{b}\bar{v}_i+\frac{[1-\varepsilon\bar{v}_i+\varepsilon/(c_0+1)]\bar{I}_i^2}{(1-n+q\bar{v}_i^2)^2+Rm_s^2(1-\bar{v}_i)^2}\right\}\bar{v}-\frac{2[\bar{v}_i-1/(c_0+1)]\bar{I}_i}{(1-n+q\bar{v}_i^2)^2+Rm_s^2(1-\bar{v}_i)^2}\bar{I};$$

$$\frac{d\bar{I}}{d\bar{t}}=2\xi q\bar{v}_i\bar{I}_i\bar{v}+(q\bar{v}_i^2-n)\xi\bar{I}, \tag{29}$$

where

$$\varepsilon=\frac{4q\bar{v}_i(1-n+q\bar{v}_i^2)+2Rm_s(1-\bar{v}_i)}{(1-n+q\bar{v}_i^2)^2+Rm_s^2(1-\bar{v}_i)^2}.$$

At the point N the "variations" can be written in the following manner:

$$\bar{v}_N=C_ve^{-2\sqrt{\bar{b}}\bar{t}};\quad \bar{I}_N=C_Ie^{(q/\bar{b}-n)\bar{t}}. \tag{30}$$

For the steady state at the point N to be stable, the following condition must be satisfied for positive values of q and n, taking into account (26):

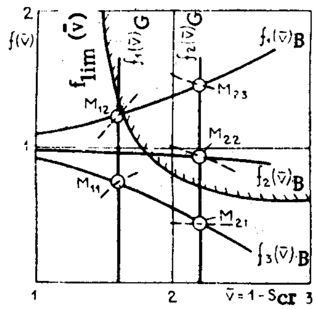


Fig. 4. Variation of f with $\bar{v}$; calculation parameters: $Rm_s = 1$, $c_0 = 0.05$, $\omega = 314$, and $k_\sigma = 0.08$.

$$1/\sqrt{\bar{b}} < 1 - s_{cr}, \tag{31}$$

since the attenuation decrement in velocity for a diminishing characteristic of the running device is always negative. By selecting the proper slope of the booster characteristic, it is possible to obtain the condition for the growth of the self-excitation for linear electrical circuit parameters $(1/\sqrt{\bar{b}} > 1 - s_{cr})$.

Following [10], the variation at the point $\bar{M}$ is sought in the form $\bar{\vartheta}_M = C_v e^{\lambda t}$; $\bar{I}_M = C_I e^{\lambda t}$ by means of a substitution into (31). In this instance $\lambda_{1,2} = -\frac{\alpha_1}{2}\left(1 \mp \sqrt{1 - \frac{4\alpha_2\beta_1}{\alpha_1^2}}\right)$ are the roots of the characteristic equation, wherein, taking into account the current and velocity values at the point $\bar{M}$, we have:

$$\alpha_1 = 2\bar{b}(1 - s_{cr}) + \frac{1 - \bar{b}(1 - s_{cr})^2}{c_0/(c_0+1) - s_{cr}}(1 - \gamma) \tag{32}$$

for

$$\gamma = \frac{4Rm_s^2[c_0/(c_0+1) - s_{cr}][k_\sigma - (2k_\sigma+1)s_{cr}/2]}{1 + Rm_s^2 s_{cr}^2};$$

$$\alpha_2 = 2\sqrt{\frac{[c_0/(c_0+1) - s_{cr}][1 - \bar{b}(1 - s_{cr})^2]}{1 + Rm_s^2 s_{cr}^2}};$$

$$\beta_1 = 2q(1 - s_{cr})\sqrt{\frac{[1 - \bar{b}(1 - s_{cr})^2](1 + Rm_s^2 s_{cr}^2)}{c_0/(c_0+1) - s_{cr}}}; \quad \beta_2 = 0.$$

Inasmuch as in the generator regime the term $4\alpha_2\beta_1/\alpha_1^2$ is always positive, the stability of the regime at the point $\bar{M}$ is determined by the sign of the coefficient α_1, such that a positive value of α_1 corresponds to the state of stable equilibrium.

From the condition $\alpha_1 = 0$ we determine the limiting value:

$$\bar{b}_{lim} = \frac{1 - \gamma}{(1 - s_{cr})\{2[c_0/(c_0+1) - s_{cr}] - (1 - \gamma)\ \ (1 - s_{cr})\}} \tag{33}$$

and then, using Eqs. (33), (25), and (26), the equation of the limiting curve:

$$f_{lim}(\bar{v}) = \frac{2[c_0/(c_0+1) - s_{cr}]}{2[c_0/(c_0+1) - s_{cr}] - (1 - \gamma)(1 - s_{cr})}. \tag{34}$$

An analysis of this equation discloses that up to the slip value s_{cr}^{o} the zone of unstable operation lies in the region of the rising p(Q) characteristic of the booster, however, starting with $s_{cr} > s_{cr}^{o}$ the unstable operation zone changes over to the region of the decreasing booster characteristic.

In Fig. 4 the zone of unstable operation is located to the right of the curve $f_{lim}(\bar{v})$. The latter is drawn under the assumptions that ω = const, Rm_s = const. Plotted in Fig. 4 are the characteristics of the booster $(f_1(\bar{v})_B - f_3(\bar{v})_B)$ and the generator $(f_1(\bar{v})_G, f_2(\bar{v})_G)$. Steady-state conditions are possible at the points M_{11}, M_{22}, M_{12}, M_{21}, and M_{23}.

Due to the inertia of the self-oscillating system, the velocity fluctuations do not produce any instantaneous current changes. This allows us to presume that at the initial instant the process develops along the $\bar{I}$ = const characteristic. In Fig. 4 the $\bar{I}$ = const characteristics in the vicinity of the point under consideration are shown as dashed line sections.

These characteristics indicate that the velocity variation at points M_{11}, M_{12}, and M_{21} are accompanied by a negative increment in pressure, tending to return the system to the starting state, while at points M_{23} and M_{22} the pressure increase is positive and acts to augment the velocity fluctuation, leading to a loss of stability.

At $Rm_s = 0$ the nonstable operation zone disappears in the region of positive booster characteristic. A rise in Rm_s leads to a reduction in the stable operational zone for the generator.

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