ROTATION OF A MAGNETORHEOLOGIC SUSPENSION IN A CONSTANT MAGNETIC FIELD. I. EQUATION OF MOTION AND BOUNDARY CONDITIONS AT THE FREE SURFACE

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UDC 532.135:538.4

Numerous experimental and theoretical results have been presently obtained on the effect of magnetic fields on hydrodynamic flow characteristics of noncolloidal magnetorheologic suspensions (MRS) with a reconstructed external field structure [1-3]. The application of a rotating magnetic field on such a system has been investigated experimentally only [4]. At the same time, a number of interesting effects, related to particle rotation, was observed in numerous studies, devoted to the effect of a rotating magnetic field on the hydrodynamics of colloidal magnetic fluids (MF) [5-14]. In particular, the possibility was shown of exciting a hydrodynamic flow by a uniform rotating magnetic field. The basic physical mechanism of the magnetic field effect on the motion of the medium is the appearance of an internal torque, generated by the nonparallel vectors of magnetization and field intensity.

For an MF these problems have been solved in the one-dimensional statement: in the unbounded sizes by a cylindrical vessel, and in the gap between two neighboring cylinders. The role of the MF—air separation boundary has not been investigated. The retarding role of the vessel bottom has not been analyzed either.

The MRS specifics lead to further complications in analyzing its behavior in magnetic fields. This is related to the field effect on the microstructure of the medium, and to the reciprocal effect of the microstructure on the field characteristics inside the MRS.

In the present study all these factors are included in investigating the dynamic behavior of MRS in rotating magnetic fields.

Consider the behavior of MRS, consisting of magnetically soft particles, placed in a cylindrical vessel of a nonmagnetic material. The MRS is rotated as a whole with angular velocity Ω. The whole system is placed in an external magnetic field, whose intensity vector $\mathbf{H}$ is directed perpendicularly to the rotation axis of the fluid (Fig. 1). In the magnetic field the particles form aggregates (of ellipsoidal shape in the first approximation), whose geometric parameters depend both on the characteristics of the magnetic and hydrodynamic fields, and on the role of the medium and phase dispersions.

Translated from Magnitnaya Gidrodinamika, No. 2, pp. 89-94, April-June, 1987. Original article submitted October 27, 1986.

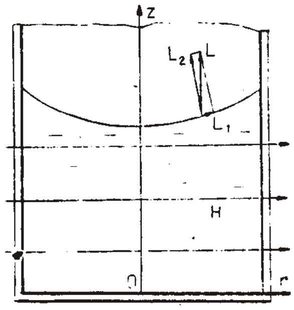

Fig. 1. Flow scheme.

In sufficiently strong magnetic fields the aggregates are oriented under a fixed angle to the field. The value of this angle α is determined by equating the oppositely directed hydromagnetic and magnetic rotational moments. The value of the hydrodynamic rotational moment is found from the condition that within the first approximation we have aggregate rotation with angular velocity Ω with respect to the medium dispersion (within first approximation the fluid in the cylinder performs solid body rotation). The hydrodynamic rotational moment is then [15]

$$M_{hyd} = 16\pi(a^2+b^2)\Omega\eta_0/[3(a^2\alpha_0+b^2\beta_0)]. \tag{1}$$

Here a and b are the major and minor axes of the ellipsoid, and α_0, β_0 are elliptic integrals, having an analytic representation in the case of an axially symmetric ellipsoid:

$$\alpha_0 = -(A+r_e/2)/[b^3(r_e^2-1)],\ \beta_0 = (A/2+r_e)/[b^3(r_e^2-1)],$$

$$A = \ln\{[r_e-(r_e^2-1)^{1/2}]/[r_e+(r_e^2-1)^{1/2}]\}/(r_e^2-1)^{1/2}.$$

The magnetic rotational moment L acting on the ellipsoidal aggregate [15] is

$$L = \frac{4}{3}\pi\mu_0\varkappa_a ab^2 G_y G_z[(1+\varkappa_a n_z)^{-1}-(1+\varkappa_a n_y)^{-1}]. \tag{2}$$

Here κ_a is the aggregate susceptibility, G_y and G_z are the components of the effective field intensity in the suspension for a coordinate system attached to the ellipsoid (the y axis is directed along the minor axis of the ellipsoid, and the z axis along the major axis), n_z, n_y are demagnetization factors:

$$n_z = (1-e^2)\{\ln[(1+e)/(1-e)]-2e\},\ n_y = (1-n_z)/2;$$

$$e = \sqrt{1-r_e^{-2}};\quad r_e = a/b.$$

To find the internal effective field inside the MRS the macroscopic characteristics were determined by averaging the microscopic characteristics over particle locations and orientations. Aggregate interaction was taken into account within the dipole–dipole approximation by the Lorentz–Lorentz (self-consistent field) method [16, 17]. In this case the order of the MRS structure was taken into consideration: the monodisperse aggregates were oriented under the identical angle α. As a result the components of the effective field vector are

$$G_z = 2H\cos\alpha/(A_4+A_1),\ G_y = -2H\sin\alpha/(A_2+A_3). \tag{3}$$

Here

$$A_1 = 1+2\varphi_a\varkappa_a/[3(1+\varkappa_a n_z)],\ A_2 = 1+2\varphi_a\varkappa_a/[3(1+\varkappa_a n_y)],$$

$$A_3 = 1-\varphi_a\varkappa_a/[3(1+\varkappa_a n_y)],\ A_4 = 1-\varphi_a\varkappa_a/[3(1+\varkappa_a n_z)],$$

α is the angle between the major axis of the aggregate and the magnetic field direction, and ϕ_a is the aggregate concentration, related to the disperse phase concentration by the relation $\phi_a = \phi/\Phi$ (Φ is the extent of aggregate filling, equal to 0.5 in typical situations [2, 3]).

From expressions (1)-(3) follows the first coupling equation between the unknown structure parameters (the extent of anisodiametry r_e and the orientation angle α):

$$4(r_e^2+1)/\{[2r_e^2(1-A')+A']G'_zG'_y[(1+\varkappa_a n_z)^{-1}-(1+\varkappa_a n_y)^{-1}]\}-S_1=0. \tag{4}$$

Here $S_1 = \mu_0\kappa_a H^2/(\eta_0\Omega)$, $G'_z = |G_z|/H$, $G'_y = |G_y|/H$, $A' = r_e(r_e + A/2)/(r_e^2 - 1)$.

In the absence of hydrodynamic field perturbations (for fluid rotation as a whole) the tensor of excess (magnetic) stresses σ^M in the fluid is [18]

$$\sigma^M = -{}^1/_2 N\lambda[\langle \mathbf{p}\otimes \tilde{L}\mathbf{p}\rangle + \langle \tilde{L}\mathbf{p}\otimes\mathbf{p}\rangle] - {}^1/_2 N\tilde{L},$$

where $\lambda = (a^2 - b^2)/(a^2 + b^2)$, N is the number of aggregates per unit volume ($N = \phi_a/({}^4/_3\pi ab^2)$), $\langle\ldots\rangle$ denotes averaging over directions, and $\mathbf{p}$ is the unit vector of aggregate orientation.

In a cylindrical coordinate system r, ϕ, z (Fig. 1) the nonvanishing components of the stress tensor are

$$\sigma_{\varphi r}{}^M = L'(\lambda\cos 2\varphi\cos 2\alpha - 1),\quad \sigma_{r\varphi}{}^M = L'(\lambda\cos 2\varphi\cos 2\alpha + 1), \tag{5}$$

where $L' = -LN/2 > 0$.

During fluid rotation, due to centrifugal forces there occurs surface deformation of the fluid–air separation, acquiring the shape of a paraboloid of revolution. The aggregates occupy a position in which their projection in the direction of the magnetic field is maximum, i.e., they lie in a plane perpendicular to the axis of symmetry of the cylinder. In that case, in the direction of the magnetic field the fluid exerts per unit volume a rotational moment **L**, equal to the difference $\sigma_{\phi r}{}^M - \sigma_{r\phi}{}^M$, and directed along the z axis (Fig. 1). For the surface layer it is decomposed into two components: $\mathbf{L}_1$, the moment in the plane tangential to the separation surface, and $\mathbf{L}_2$, the moment perpendicular to the separation surface. Since the separation surface is free of stress, the magnetic stresses due to the external field lead to the appearance of hydrodynamic stresses. This creates a deviation from solid body rotation of the suspension motion. The fluid motion due to the perpendicular component of the moment vector ($\mathbf{L}_2$) is mutually compensated. Microscopic fluid rotation, generated by the presence of the parallel component of the moment vector (the component $\mathbf{L}_1$) to a shear macroscopic fluid motion, directed along the cylinder rotation.

For this hydrodynamic situation (the presence of a macroscopic shear flow) the excess stress tensor in the suspension is [18]

$$\tilde{\sigma} = 2\eta_0\tilde{D} + \eta_0\varphi_a\{-\rho\langle\mathbf{p}\tilde{D}\mathbf{p}\rangle\tilde{E} + 2\alpha'\tilde{D} + 2\beta D(3\langle\mathbf{p}\otimes\mathbf{p}\rangle - \tilde{E}) + \xi[\langle\mathbf{p}\otimes\tilde{D}\mathbf{p}\rangle + \langle\tilde{D}\mathbf{p}\otimes\mathbf{p}\rangle] + \chi\langle\mathbf{p}\otimes\mathbf{p}(\mathbf{p}\tilde{D}\mathbf{p})\rangle\}$$
$$- N\lambda/2[\langle\mathbf{p}\otimes\tilde{L}\mathbf{p}\rangle + \langle\tilde{L}\mathbf{p}\otimes\mathbf{p}\rangle] - N\langle\tilde{L}\rangle/2.$$

Here $\tilde{D}$ is the tensor of deformation velocities, $\tilde{E}$ is a unit tensor, D is the rotational diffusion coefficient, and ρ, α', β, ξ, χ are constants determined by the ellipsoid shape (whose explicit forms are given in [18]).

The intensity of the fluid shear motion, being a secondary flow, is not high. On the background of solid body rotation we have weak deviations of the azimuthal velocity from $v_\phi = -\Omega r$, with quite high angular velocity. Therefore, one can average the stress value during a period of rotation, i.e., over the angle ϕ. In the cylindrical coordinate system these mean components of the stress tensor are then equal to

$$\begin{gathered}
\sigma_{rr} = 2\eta_0 D_{rr} + \eta_0\varphi_a[-\rho(D_{rr}+D_{\varphi\varphi})/2 + \xi D_{rr} + 2\alpha' D_{rr} + \chi(3D_{rr}+D_{\varphi\varphi})/8],\\
\sigma_{r\varphi} = \sigma'_{r\varphi} + \bar{\sigma}_{r\varphi}{}^M,\\
\sigma_{zz} = 2\eta_0 D_{zz} + \eta_0\varphi_a[-\rho(D_{rr}+D_{\varphi\varphi})/2 + 2\alpha' D_{zz}],\\
\sigma_{\varphi\varphi} = 2\eta_0 D_{\varphi\varphi} + \eta_0\varphi_a[-\rho(D_{rr}+D_{\varphi\varphi})/2 + \xi D_{\varphi\varphi} + 2\alpha' D_{\varphi\varphi} + \chi(D_{rr}+D_{\varphi\varphi})/8];\\
\sigma_{\varphi r} = \sigma'_{\varphi r} + \bar{\sigma}_{\varphi r}{}^M,\quad \sigma_{\varphi z} = \sigma'_{\varphi z}.
\end{gathered} \tag{6}$$

The shear stresses are here equal to

$$\sigma'_{\varphi r} = \sigma'_{r\varphi} = 2\eta_1 D_{r\varphi},\quad \sigma_{\varphi z} = \sigma_{z\varphi} = 2\eta_2 D_{\varphi z},\quad \sigma'_{rz} = \sigma'_{zr} = 2\eta_2 D_{rz}, \tag{7}$$

while the magnetic stresses $\bar{\sigma}_{\phi r}{}^M$ and $\bar{\sigma}_{r\phi}{}^M$ are determined by Eqs. (5)

$$\bar{\sigma}_{\varphi r}{}^M = -L',\quad \bar{\sigma}_{r\varphi}{}^M = L'. \tag{8}$$

The effective viscosities appearing in expression (7) are

$$\eta_1 = \eta_0[1 + \varphi_a(\alpha' + \xi/2 + \chi/8)],\quad \eta_2 = \eta_0[1 + \varphi_a(\alpha' + \xi/4)], \tag{9}$$

where

$$\alpha' = (ab^4\alpha'_0)^{-1},\quad \xi = 4/[(a^2+b^2)ab^2\beta'_0] - 2/(ab^4\alpha'_0).$$

$$\chi = 2\alpha''_0/(ab^4\alpha'_0\beta''_0) - 8/[ab^2(a^2+b^2)\beta'_0] + 2/(ab^4\alpha'_0);$$

α'_0, β'_0, α''_0, β''_0 are elliptic integrals, having analytic representations in the case of an axially symmetric ellipsoid:

$$\alpha'_0 = r_e^4[2r_e^2 - 5 - 3A'/(2r_e)]/[4a^3b^2(r_e^2-1)],$$
$$\beta'_0 = 2r_e^2(1 + r_e^2/2 + 3r_eA'/4)/[a^3b^2(r_e^2-1)^2],$$
$$\alpha''_0 = 2r_e^2[r_e^2/4 + 1/8 + (4r_e^2-1)/(16r_e)],$$
$$\beta''_0 = -2r_e^2[^3/_2 + (2r_e^2+1)A'/(4r_e)]/[ab^2(r_e^2-1)^2],$$
$$A' = \ln\{[r_e - (r_e^2-1)^{1/2}]/[r_e + (r_e^2-1)^{1/2}]\}/(r_e^2-1)^{1/2}.$$

In strong magnetic fields, when $r_e \gg 1$, the following asymptotic representations are valid

$$A' = -(2\ln r_e + \ln 4)/4,\ \alpha'_0 = (2ab^4)^{-1},\ \alpha''_0 = (2ab^2)^{-1},$$
$$\beta'_0 = (a^3b^2)^{-1},\ \beta''_0 = (2\ln r_e - 3 + \ln 4)/a^3. \tag{10}$$

Limiting expressions for the coefficients follow then from (9), (10)

$$\alpha' \to 2,\ \xi \to 0,\ \chi \to 2r_e^2/(2\ln r_e - 3 + \ln 4) - 4. \tag{11}$$

The calculation accuracy by Eqs. (11) increases with increasing extent of anisodiametry. Thus, for $r_e = 10$, according to relations (10) $\chi = 62$ (the exact result is 58), while for $r_e = 100$ we have 2629 and 2752, respectively. Taking into account that for typical MRS the aggregate concentration can reach 0.1-0.2 [1], we obtain that in strong fields $\eta_1 \gg \eta_2$.

Secondary flows generated during vessel rotation have weak intensity, therefore the characteristic Reynolds numbers are quite small (Re < 1). This makes it possible to neglect inertial terms in the equations of motion. It follows from the hydrodynamic flow analysis that in this situation only the velocity component v_ϕ is nonvanishing. The equation for its value v'_ϕ averaged over the angle $v'_\phi = 1/2\pi \int_0^{2\pi} v_\phi d\phi$ is (primes are omitted)

$$\frac{\eta_1}{r^2}\frac{\partial}{\partial r}\left[r^3\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right)\right] + \eta_2\frac{\partial^2 v_\varphi}{\partial z^2} = 0. \tag{12}$$

To close the problem it is necessary to formulate boundary conditions. The most complicated is the condition for v_ϕ at the free surface. No surface forces must occur there. The shape of the surface for fluid rotation with a free boundary is given by the equation $z = \Omega^2r^2/(2g)$, while the unit vector normal to the surface has the projections $n_r = -\Omega^2r(gA)$, $n_\phi = 0$, $n_z = A^{-1}$, where $A = \sqrt{1 + (\Omega^2r/g)^2}$.

The condition of absence of forces at the free surface is (for the sake of simplicity in solving the problem, the boundary condition at the deformed surface is taken to be that on the plane z = 0, which is valid for $\Omega^2R/g \ll 1$)

$$f_r|_{z=0} = (\sigma_{rr}n_r + \sigma_{r\varphi}n_\varphi + \sigma_{rz}n_z)|_{z=0} = 0,$$
$$f_\varphi|_{z=0} = (\sigma_{\varphi r}n_r + \sigma_{\varphi\varphi}n_\varphi + \sigma_{\varphi z}n_z)|_{z=0} = 0,$$
$$f_z|_{z=0} = (\sigma_{zr}n_r + \sigma_{z\varphi}n_\varphi + \sigma_{zz}n_z)|_{z=0} = 0.$$

Due to the fact that among the magnetic stresses, according to (5) the only nonvanishing components are $\bar{\sigma}_{\phi r}{}^M$ and $\bar{\sigma}_{r\phi}{}^M$, one must take into account only the second of these relations, which by the shape of components of the unit vector to the normal reduces to

$$\frac{\Omega^2 r}{g}\left[\bar{\sigma}_{\varphi r}{}^M + \eta_1 r\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right)\right] = \eta_2\frac{\partial v_\varphi}{\partial z}\ \text{for}\ z = 0, \tag{13}$$

where $\bar{\sigma}_{\phi r}{}^M$, η_1, η_2 are determined from (8) and (9), respectively.

In strong magnetic fields the radial viscosity η_1, characteristic of flows across the radius, is much larger than the axial viscosity η_2. Therefore, a hydrodynamic boundary layer is formed in the region adjacent to the free surface. Taking this into account, and performing the transition to dimensionless variables according to the equations

$$\bar{v} = -\frac{\Omega r + v_\varphi}{V}, \quad \bar{r} = r/R, \quad \bar{z} = \sqrt{\eta_1/\eta_2}\, z/R, \quad b = \sqrt{\eta_1}\,\Omega^2 R/(\eta_2 g), \tag{14}$$

where the characteristic velocity of the secondary flow is

$$V = -\bar{\sigma}_{\varphi r}{}^{M}\Omega^2 R^2/\sqrt{\eta_1\eta_2}g \quad (\sigma_{\varphi r}{}^{M} < 0), \tag{15}$$

the problem (12)-(14) can be reduced to

$$\frac{1}{\bar{r}^2}\frac{\partial}{\partial \bar{r}}\left[\bar{r}^3\frac{\partial}{\partial \bar{r}}\left(\frac{\bar{v}}{\bar{r}}\right)\right] + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} = 0,$$
$$-b\bar{r}^2\frac{\partial}{\partial \bar{r}}\left(\frac{\bar{v}}{\bar{r}}\right) + \frac{\partial \bar{v}}{\partial \bar{r}} = \bar{r} \quad \text{for} \quad \bar{z} = 0. \tag{16}$$

To close the problem (16) it is necessary to determine the dependence of the magnetic stresses $\bar{\sigma}_{\phi r}{}^{M}$, as well as of the viscosities η_1 and η_2 on the field intensity, the angular velocity of rotation, and the characteristics of the phase and medium dispersions. This will be carried out in the second part of this study.

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